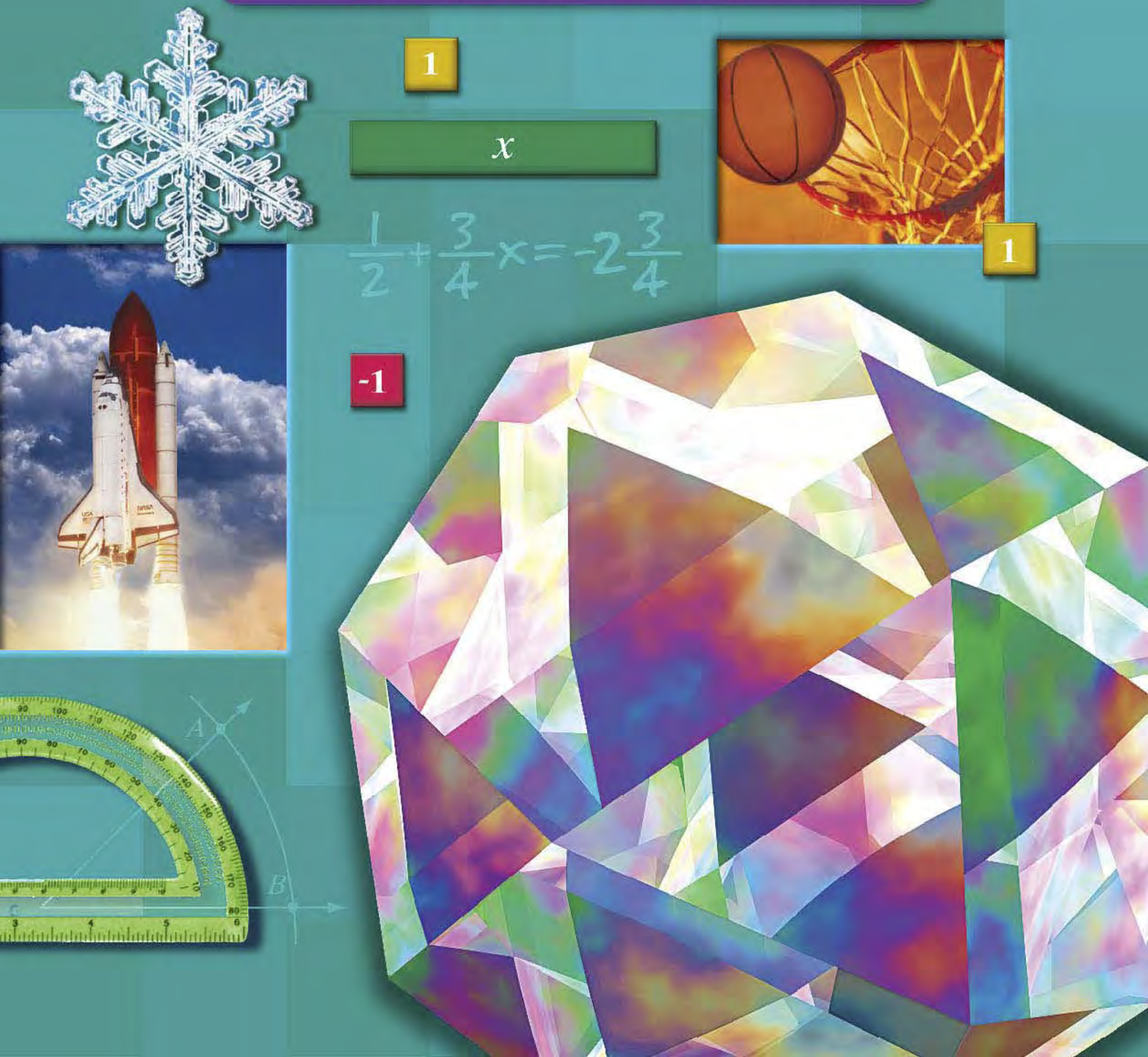
SADLIER-OXFORD

# Fundamentals of Algebra

SOURCEBOOK



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Printed in the United States of America
ISBN: 978-0-8215-8207-7
3456789/14 13 12 11 10 09

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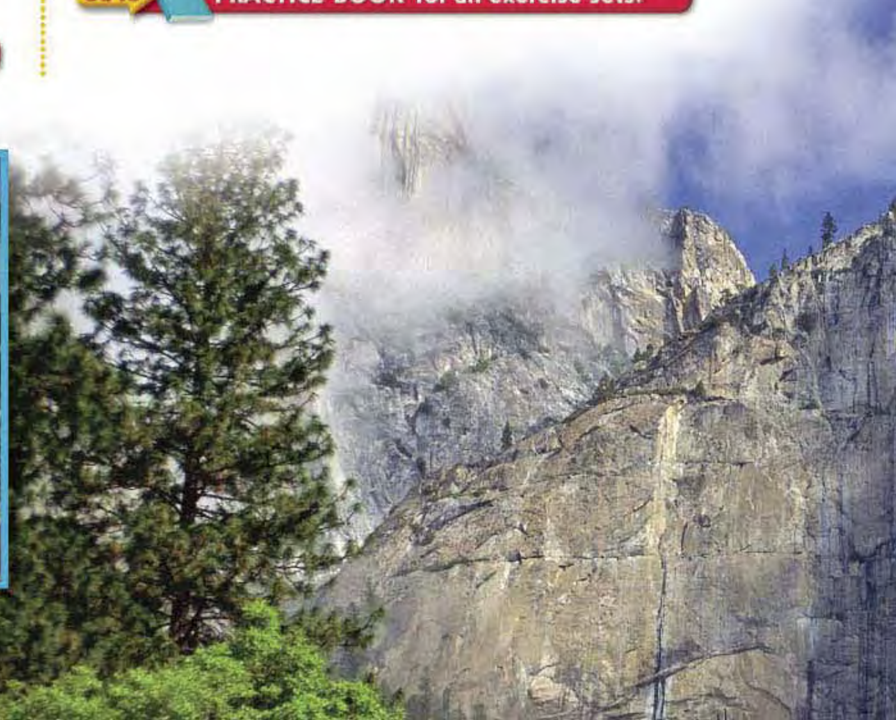
# Contents



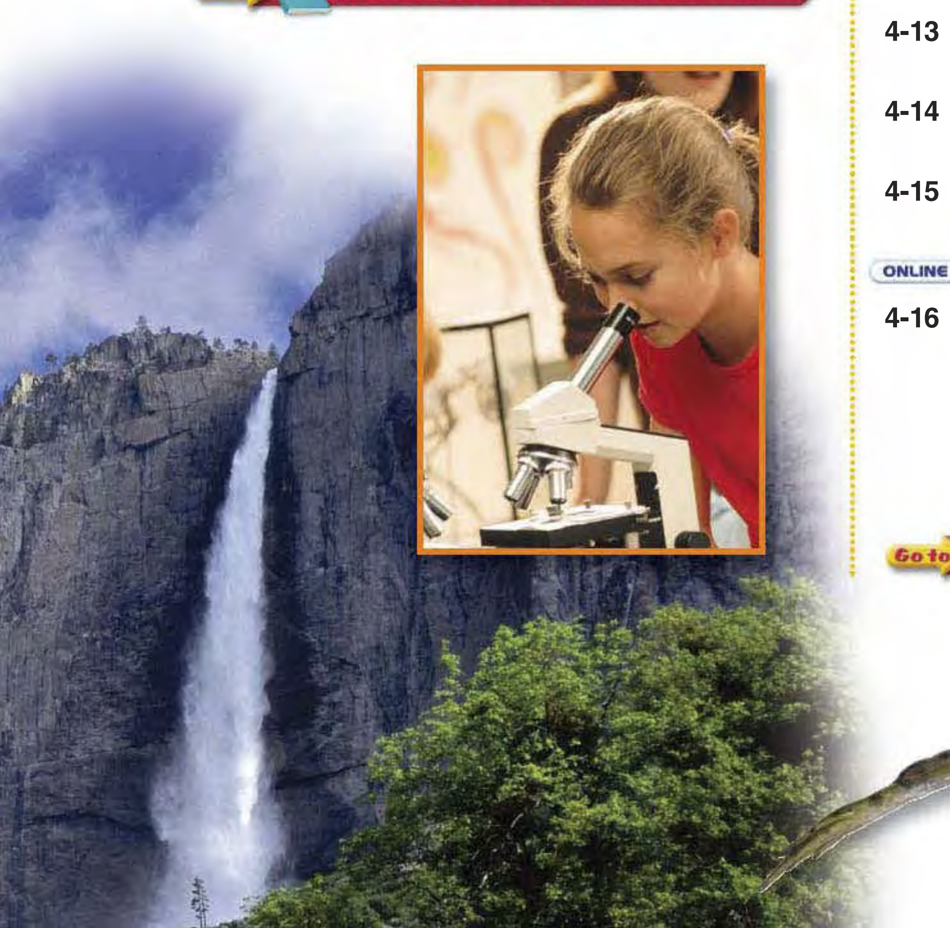


## Chapter 3

# Inequalities



Go to PRACTICE BOOK for all exercise sets.



## Chapter 4

# Rational Numbers: Decimals



Go to PRACTICE BOOK for all exercise sets.

## Chapter 5

# Rational Numbers: Fractions



Go to PRACTICE BOOK for all exercise sets.

## Chapter 6

# Ratio and Proportion



Go to PRACTICE BOOK for all exercise sets.

## Chapter 7

# Percent and Consumer Applications



Go to PRACTICE BOOK for all exercise sets.

## Chapter 8

# Data Analysis and Statistics



Go to PRACTICE BOOK for all exercise sets.


## Chapter 9

# Two-Dimensional Geometry



Go to PRACTICE BOOK for all exercise sets.

## Chapter 10

# Two-Dimensional Geometry and Measurement Applications



Go to PRACTICE BOOK for all exercise sets.

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# Three-Dimensional Geometry



Go to PRACTICE BOOK for all exercise sets.

## Chapter 12

# Probability



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Chapter 13

## Patterns, Relations, and Functions



Go to PRACTICE BOOK for all exercise sets.

Chapter 14

## Polynomials, Equations, and Inequalities



Go to PRACTICE BOOK for all exercise sets.

## End-of-Book Materials

### Skills Update



# Fundamentals of Algebra

**Dear Students,**

You are about to begin a very important year in your study of mathematics. As you prepare for your future Algebra course, there are significant foundational concepts and skills that will be important for you to master. Your progress in using problem-solving skills will also play a valuable role in your success in Algebra. We have written this book to strengthen your mathematical foundations as well as your problem-solving skills.

Many students think that success in math is just a matter of talent. Yet we know from research studies that success in math results from the effort you make and the learning habits that you develop. You probably realize that most math concepts build on what you have learned earlier. You can use this knowledge to your advantage by making sure you understand each concept before moving on to the next. Ask yourself how new concepts relate to ones you have learned earlier. Think about how problems you encounter are like ones you have solved earlier. Making these connections will help you to become more confident in your ability to solve problems. To help you develop your problem-solving skills, we have included lessons on problem-solving strategies in every chapter. Make a habit of thinking of these strategies as important tools you can use in your everyday mathematics work as well as in your everyday life.

The study of mathematics offers more than computational and problem-solving skills, as you will learn in the *Enrichment* feature in each chapter. These *Enrichment* topics will illuminate the math curriculum and give valuable insight into the special beauty of mathematics.

We know that this book can help you to value math, to develop confidence about your mathematical work, and to learn to reason and communicate more effectively. We wish you the best this year.

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## Problem-Solving Steps

**Read** Read to understand what is being asked.

**Plan** Select a strategy.

**Solve** Apply the strategy.

**Check** Check to make sure your answer makes sense.

# Your Problem-Solving Adventure

Practically every moment of our lives, we are faced with decisions about what to do and how to do it. Sometimes these decisions require a lot of thought; other times, they are instinctive reactions. But in almost all cases, they can be considered to be problem-solving experiences.

Mastery in problem solving depends on critical thinking. To think critically, you must be able to organize your thoughts. The problem-solving steps outlined above will help you do just that.

### Consider the following problem.

**Problem:** Your school principal asks you to organize a single elimination basketball tournament, one in which one loss eliminates a team. If 25 teams enter the tournament, how many games have to be played in order to have a winner?

**Solution:** A typical solution would require modeling the tournament and then counting how many games are actually played until there is one winner. However, using a problem-solving strategy called *Adopt a Different Point of View*, you can answer this problem instantly. Instead of modeling the tournament and looking at the winners of each game as the teams progress to a championship, consider the losers. How many losers must there be to have a champion among 25 teams? Clearly, with one winner, there must be 24 losers. The number of games necessary to have 24 losers is 24. Problem solved!

With this example, you can see the power that some problem-solving strategies can have. Our objective in this book is to introduce you to ten problem-solving strategies that will be invaluable not only when you work with mathematics, but also in other situations, both in and out of school. One caution: Just reading these problem-solving strategy sections diligently will not guarantee that these methods will become part of your regular thought processes. You must apply these problem-solving techniques as often as you can so that they *do* become part of your regular thought processes.

### Problem-Solving Strategies

# Using Your SourceBook

**Skills update** references direct you to previously learned skills that will be called on as you work through the current lesson.

Clear, concise mathematical **objectives** list the points you will cover in the lesson.

Teaching the lesson using more than one **method** addresses the fact that not everybody learns the same way. No matter what the method, the instruction is developed **step by step**.

**Key concepts** summarize important terms, definitions, and mathematical properties to be used as helpful reinforcements.

Additional **examples** model variations of the lesson objectives.

Algebra 1-3

*Update your skills. See page 409 VI.*

## Add Integers

**Objective** To model addition of integers • To add integers with like and unlike signs • To add more than two integers

Kyle lost 5 points in the first round of an electronic game and lost 3 points in the second round. What was the total number of points Kyle lost after two rounds? What integer represents the loss?

To find the total points lost, add: $-5 + (-3)$

▶ You can use tiles and a number line to add integers with like signs.

Add: $-5 + (-3)$

**Method 1** Use Tiles

Let ■ = −1.

• Model each addend.

■■■■■ + ■■■

$-5 \quad + \quad (-3)$

• Join the tiles.

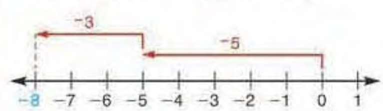
■■■■■■■■

$-8$

**Method 2** Use a Number Line

• Start at zero.

• Move left to add negative integers.




$-5 + (-3) = -8$

Kyle lost a total of 8 points after two rounds. The integer representing this loss is −8.

▶ You can also use absolute value to add integers with like signs.

Add: $-58 + (-31)$

$|-58| + |-31|$

$58 + 31$

$89$

So $-58 + (-31) = -89$. ← Both addends are negative, so the sum is negative.

**Key Concept**

**Add Integers with Like Signs**

• Add the absolute values of the addends.

• Use the sign of the addends for the sum.

**Examples**

1 Add: $67 + 29$

$|67| + |29|$ ← Add the absolute values.

$67 + 29$

$96$

So $67 + 29 = 96$. ← Both addends are positive, so the sum is positive.

2 Add: $-5 + (-9) + (-16)$

$[-5 + (-9)] + (-16)$ ← Add from left to right.

$-14 \quad + (-16)$ ← Add.

$-30$ ← Both addends are negative, so the sum is negative.

So $-5 + (-9) + (-16) = -30$.

6 Chapter 1

Grade 7 Student SourceBook

The *Online* logo points you to the Web-based components of the program. Here you will find helpful resources and engaging activities.

The use of **color** helps emphasize key steps in the instruction.

**Think** boxes model the reasoning and mental computation involved in computing and solving problems.

**Try These** prepares you for the types of exercises you will encounter in the Practice Book.

***Discuss and Write*** questions help concretize the concepts of the lesson as you write about them in your own words.

Follow the information in the ***Go To*** logo to find the exercise sets to complete in the Practice Book.

ONLINE www.progressinmathematics.com

Practice & Activities

▶ You can also use tiles and a number line to add integers with unlike signs.

Add: $6 + (-4)$

Let [orange tile] = 1 — Let [red tile] = −1 — Zero Pair: $1 + (-1) = 0$

**Method 1** Use Tiles

• Model each addend.

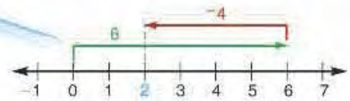
$+ \quad (-4)$

• Group and remove zero pairs.

$0 + 0 + 0 + 0 + 1 + 1 = 2$

**Method 2** Use a Number Line

• Start at zero.
• Move right to add positive integers.
• Move left to add negative integers.




So $6 + (-4) = 2$.

▶ You can also use absolute value to add integers with unlike signs.

Add: $-9 + 13$

$|13| - |-9|$

$13 - 9$

$4$

So $-9 + 13 = 4$. ← $|13| > |-9|$, so the sum is positive.

**Key Concept**

**Add Integers with Unlike Signs**

- Subtract the lesser absolute value from the greater absolute value.
- Use the sign of the addend with the greater absolute value for the sum.

**Examples**

**1** Add: $14 + (-34)$

$|-34| - |14|$ ← Subtract the absolute values.

$34 - 14$

$20$

So $14 + (-34) = -20$. ← $|-34| > |14|$, so the sum is negative.

**2** Add: $-3 + 8 + (-11)$

$8 + (-3) + (-11)$ ← Change the order to add like signs.

$8 + \quad (-14)$ ← Add.

$-6$ ← $|-14| > |8|$, so the sum is negative.

So $-3 + 8 + (-11) = -6$.

**Try These**

**Find the sum.**

**1.** $-6 + (-4)$ **2.** $-17 + 7$ **3.** $17 + (-7)$ **4.** $92 + (-26) + 3$

**5.** **Discuss and Write** Explain how you can add integers with like signs and with unlike signs. Give examples to support your explanation.

Go to PRACTICE BOOK Lesson 1-3 for exercise sets.



Grade 7 Student SourceBook

# Using Your Practice Book

Read the instruction in the **teaching display** before you begin the exercises.

Your SourceBook instruction is summarized and illustrated to help you with the exercises in the Practice Book.

The ***Use With*** logo helps you easily locate the SourceBook pages that correspond to the current Practice Book lesson.

## 1-3 Add Integers

Name ______________________________ Date ____________

To add integers with *like signs*, add the absolute values of the addends and use the sign of the addends for the sum.

Add: $-2 + (-6)$

Think: $|-2| + |-6| \longrightarrow 2 + 6 = 8$

$-2 + (-6) = -8$ The addends are negative, so the sum is negative.

Add: $2 + 6$

Think: $|2| + |6| \longrightarrow 2 + 6 = 8$

$2 + 6 = 8$ The addends are positive, so the sum is positive.

To add integers with *unlike signs*, subtract the lesser absolute value from the greater absolute value. Use the sign of the addend with the greater absolute value for the sum.

Add: $-2 + 6$

Think: $|6| - |-2| \longrightarrow 6 - 2 = 4$

$-2 + 6 = 4$ $|6| > |-2|$, so the sum is positive.

Add: $2 + (-6)$

Think: $|-6| - |2| \longrightarrow 6 - 2 = 4$

$2 + (-6) = -4$ $|-6| > |2|$, so the sum is negative.

**Add.**

| | | |
|---|---|---|
| **1.** $-4 + (-2)$ $-6$ | **2.** $7 + (-16)$ ______ | **3.** $-11 + 12$ ______ |
| **4.** $0 + (-3)$ ______ | **5.** $7 + (-15)$ ______ | **6.** $-7 + 5$ ______ |
| **7.** $-4 + (-6)$ ______ | **8.** $9 + (-6)$ ______ | **9.** $-17 + (-8)$ ______ |
| **10.** $-11 + (-16)$ ______ | **11.** $18 + (-3)$ ______ | **12.** $-14 + (-12)$ ______ |
| **13.** $16 + 17$ ______ | **14.** $-13 + (-13)$ ______ | **15.** $-11 + 19$ ______ |
| **16.** $-45 + 45$ ______ | **17.** $-45 + 12$ ______ | **18.** $23 + (-18)$ ______ |
| **19.** $-14 + (-34)$ ______ | **20.** $43 + 9$ ______ | **21.** $-39 + (-4)$ ______ |
| **22.** $19 + (-23)$ ______ | **23.** $47 + 29$ ______ | **24.** $35 + 56$ ______ |
| **25.** $-67 + 54$ ______ | **26.** $-14 + (-32)$ ______ | **27.** $28 + (-31)$ ______ |
| **28.** $-50 + 35$ ______ | **29.** $24 + (-19)$ ______ | **30.** $-81 + (-11)$ ______ |
| **31.** $-213 + (-327)$ ______ | **32.** $121 + (-232)$ ______ | **33.** $-453 + 112$ ______ |

Use with SOURCEBOOK Lesson 1-3, pages 6–7.

Chapter 1 5

Grade 7 Student Practice Book

The ***Online*** logo reminds you that there are more Practice exercises and activities on the Web site.

Your teacher may create customized worksheets and tests using the **Practice/Test Generator**.

**For More Practice Go To:**

ONLINE www.progressinmathematics.com | Practice/Test Generator

**Find the sum.**

**34.** $15 + 19 + (-23)$
$34 + (-23)$
**11**

**35.** $-9 + (-13) + (-17)$

**36.** $-12 + 12 + (-4)$

**37.** $-17 + (-49) + 5$

**38.** $15 + 78 + 34$

**39.** $-19 + 16 + (-42)$

**40.** $-23 + 14 + (-33)$

**41.** $-7 + (-19) + 32$

**42.** $102 + (-345) + 234$

**43.** $-78 + (-56) + 679$

**44.** $178 + (-129) + 96$

**45.** $-312 + (-154) + 283$

**Complete the addition table.**

| | Rule: + | −5 | 20 | −17 | 6 |
|---|---|---|---|---|---|
| **46.** | 12 | 12 + (−5) = 7 | | | |
| **47.** | −8 | | | | |
| **48.** | 15 | | | | |

In every lesson, you will have the opportunity to apply your **problem-solving** skills.

**Use mental math to compare. Write <, =, or >.**

**49.** $-7 + (-8)$ = $-8 + (-7)$

**50.** $-4 + 8$ ____ $4 + (-8)$

**51.** $-2 + (-3)$ ____ $2 + 3$

**52.** $15 + (-12)$ ____ $12 + (-15)$

**53.** $-22 + 22$ ____ $4 + (-4)$

**54.** $-33 + 0$ ____ $0 + 33$

**Solve. Check to verify your answers.**

**55. Meteorology** As of 2006, California's record high temperature was 179 degrees above its record low. If the record low temperature is −45°F, what is the state's record high temperature?

$-45° + 179° = 134°$
The record high temperature is 134°F.

**56. Sports** On the first four possessions of the game, the Blue Hawks football team made the following plays: a 1-yard loss, a 3-yard gain, a 4-yard loss, and no gain. How many yards did the football team lose or gain after the first four possessions?

***Write About It*** is just one of many end-of-lesson features that encourage you to use higher-order thinking skills in mental math, critical thinking, spiral review, test preparation, and others.

**WRITE ABOUT IT**

**57.** When adding two integers, how can you tell if a sum will be positive, negative, or zero without actually adding? Use examples to explain.



6 Chapter 1

**Grade 7 Student Practice Book**

# Your Enrichment Journey

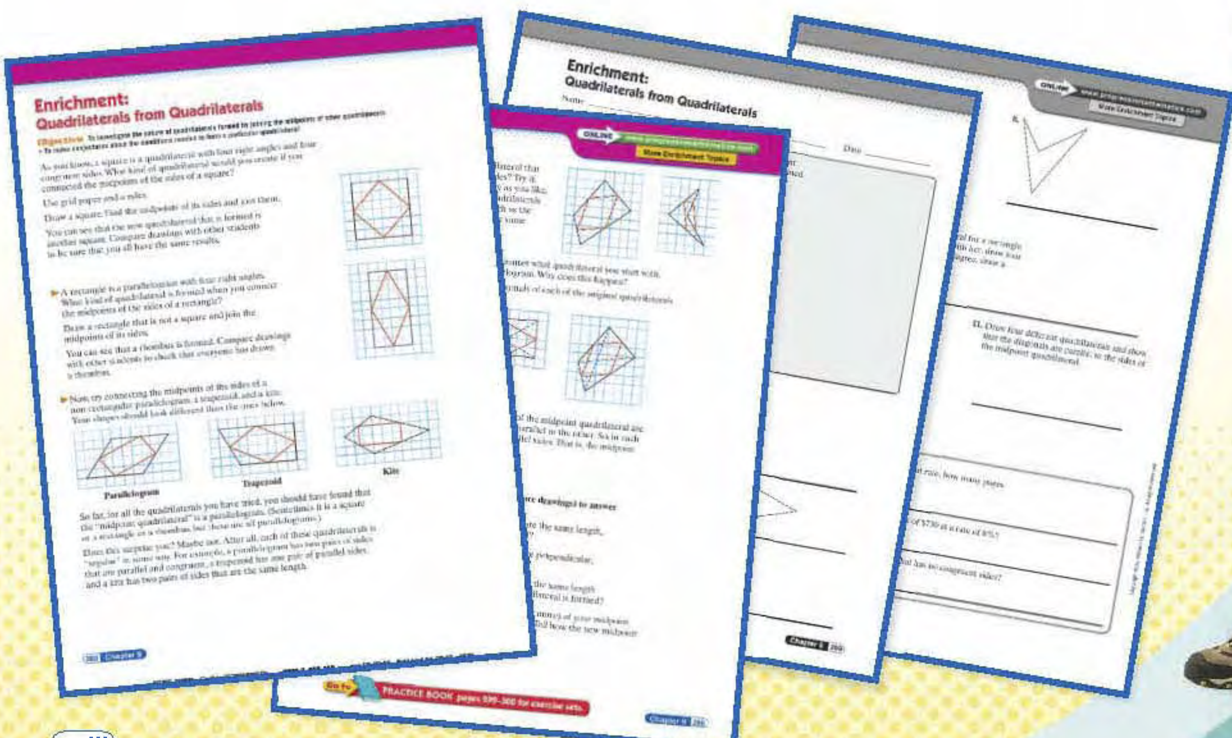
In this textbook, we provide some real topics of enrichment. You may wonder what we mean by this, since we are sure you have been told many times that you are being "enriched." Well, we are going to present topics that are not part of the regular curriculum, but are within the scope of your course of study. Some of these topics may be encountered in later years, but others may be missed if we do not include them now. In all cases, we hope you enjoy them. To really do that, however, you must read them with an extra degree of enthusiasm!

**Accelerate** **Expand** **Extend**

In each chapter the Enrichment topic is appropriate to the material you have been taught, and will enrich you in one of three ways: It will *accelerate* instruction to expose you to concepts that await you in the not-too-distant future; *expand* a topic presented; or *extend* a topic taught and apply it to a closely related theme. The common thread is that the topics can be briefly, yet reasonably, and completely presented to you in a friendly fashion.

Some topics present mathematics in a historical context, which adds a great deal of interest and insight to the topic, while others may be more of a challenge that is intended to be motivating. In all cases, we hope you will find these Enrichment topics enjoyable and, as a result, see mathematics as a subject that can be fun, as well as extremely practical and useful.

These Enrichment topics go a long way to ensuring that you will be able to see the beauty of mathematics. It is your teacher's goal to have you appreciate mathematics. We feel these Enrichment topics will help you reach that goal.



CHAPTER 1

# Integers

## In This Chapter You Will:

- Compare and order integers
- Model operations with integers
- Identify and use properties involving operations on integers
- Use the order of operations to simplify expressions
- Relate integers to the four quadrants of the coordinate plane
- Apply the strategy: *Guess and Test*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- The set of whole numbers includes 0 and the counting numbers (1, 2, 3, …).
- Operations are related in this way:

$9 + 3 = 12 \longrightarrow 12 - 3 = 9$
$3 + 9 = 12 \longrightarrow 12 - 9 = 3$
$7 \bullet 2 = 14 \longrightarrow 14 \div 2 = 7$
$2 \bullet 7 = 14 \longrightarrow 14 \div 7 = 2$

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 1–32

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES


## Critical Thinking

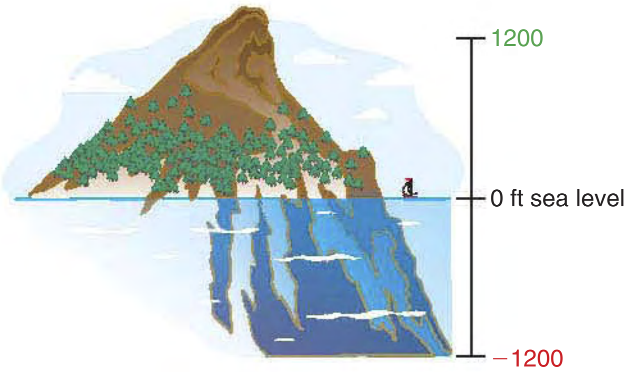
On Monday Griffin noticed that the temperature dropped from 17°F at 9:00 A.M. to 8°F at 9:00 P.M. On Tuesday he saw that the temperature dropped from 7°F at 9:00 A.M. to –4°F at 9:00 P.M. On which day did the temperature drop the most? By how much more did the temperature drop on that day?

Algebra

# 1-1 Integers and Absolute Value

**Objective** To identify integers • To identify opposites • To find the absolute value of an integer

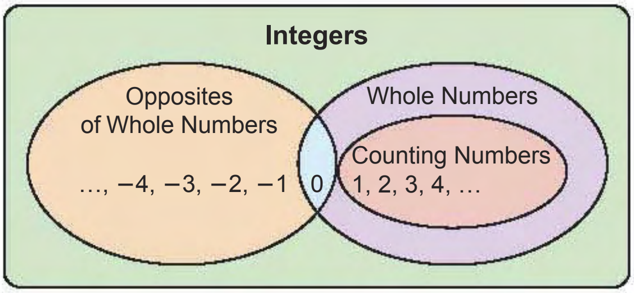
Judy climbed the Rock of Gibraltar to an elevation of 1200 feet above sea level. The maximum depth of the Strait of Gibraltar is more than 1200 feet below sea level.




You can write these numbers as *integers*.

► **Integers** are whole numbers and their opposites. They are positive, negative, or zero. The set of integers can be written as {. . . , −4, −3, −2, −1, 0, 1, 2, 3, 4, . . .}, where ". . ." means that the numbers go on indefinitely.

The set of integers contains:
the set of counting numbers, {1, 2, 3, 4, . . .}.
the set of whole numbers, {0, 1, 2, 3, 4, . . .}.
the set of *opposites* of whole numbers, {. . . , −4, −3, −2, −1, 0}.




- For sea level, or the starting point, use 0.

  sea level → 0

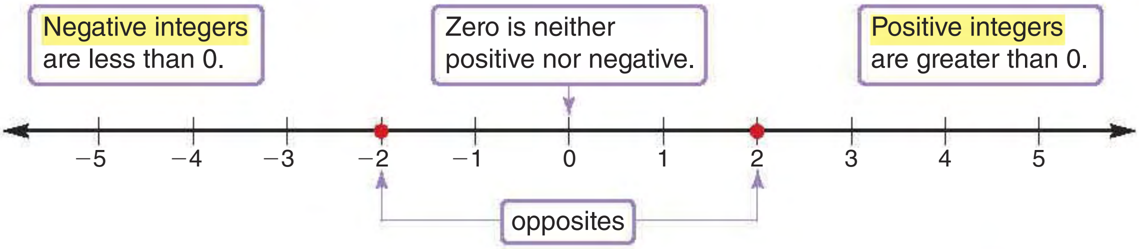
- For elevations above sea level, or numbers greater than 0, use a positive integer.

  1200 feet above sea level → +1200 or 1200

- For elevations below sea level, or numbers less than 0, use a negative integer.

  1200 feet below sea level → −1200

► The **opposite** of an integer is the integer that is the same distance from 0 on a number line but is located on the opposite side of 0. To *graph* an integer on a number line, draw a dot to locate the point on the number line that corresponds to the integer.




Positive 1200 and negative 1200 are opposites.

The opposite of positive 7 is negative 7. Write: −(7) or −7

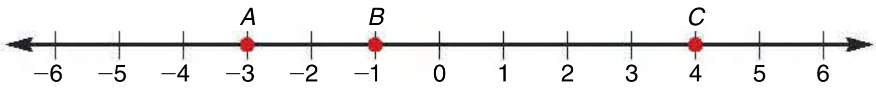
The opposite of negative 5 is positive 5. Write: −(−5) or 5

The opposite of 0 is 0. Zero is its own opposite. Write: 0

Positive integers are commonly written without the positive sign.

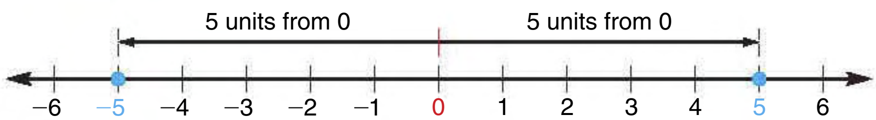
## Example

**1** For each lettered point on the number line below, name the integer and its opposite.




| | | |
|---|---|---|
| Point $A$ is at $-3$. | Point $B$ is at $-1$. | Point $C$ is at 4. |
| $-(-3) = 3$ | $-(-1) = 1$ | $-(4) = -4$ |

► The absolute value of an integer is its distance from zero on a number line. The symbol for absolute value is $| \ |$.




Both 5 and $-5$ are 5 units from 0. $|-5| = |5| = 5$

| | |
|---|---|
| The absolute value of positive 5 is 5. | Write: $|5| = 5$ |
| The absolute value of negative 5 is 5. | Write: $|-5| = 5$ |
| The absolute value of 0 is 0. | Write: $|0| = 0$ |

**Key Concept**

**Absolute Value**

Since distance cannot be negative, the *absolute value* of any number is always positive or 0.

## Try These

**Write an integer for each situation.**

**1.** \$10 earned **2.** 3° below 0° **3.** a loss of 5 pounds **4.** an increase of 10°

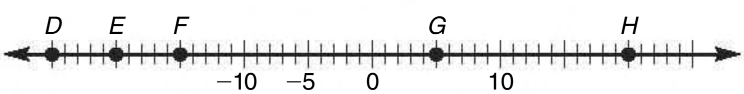
**Write the opposite of each.**

**5.** $-8$ **6.** 9 **7.** $-(16)$ **8.** $-(-7)$

**Write the integer for each.**

**9.** $|-7|$ **10.** $|6|$ **11.** $-|-9|$ **12.** $-|5|$

**Name the integer that corresponds to each lettered point on the number line.**




**13.** $D$ **14.** $E$ **15.** $F$ **16.** $G$

**17.** What integer corresponds to the point that is located 3 units to the right of point $H$?

**Graph each set of integers on a number line.**

**18.** $\{-7, -3, 0\}$ **19.** $\{-1, 1, 6\}$ **20.** $\{-2, -8, 0, 4\}$ **21.** $\{-10, 2, -9, 9\}$

**22.** **Discuss and Write** Name the integers between $-2$ and 2. Name the whole numbers between $-2$ and 2. Are your answers the same? Explain.

Go to PRACTICE BOOK Lesson 1-1 for exercise sets.

*Update your skills. See page 407 II.*

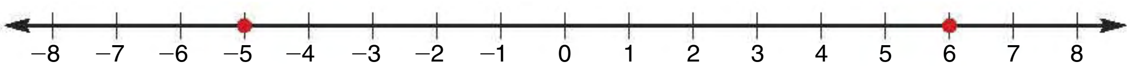
# Compare and Order Integers

**Objective** To compare integers, with and without a number line • To order integers

A meteorologist recorded temperatures in Juneau, Alaska, over a 2-day period. The temperatures are shown in the table at the right. Which day had a greater high temperature?

| Temperature (°F) | | |
|---|---|---|
| | Day 1 | Day 2 |
| High | 6 | −5 |
| Low | −1 | −7 |

► To find the greater high temperature, you can use a number line to compare the integers 6 and −5. Any number on a number line is greater than any of the numbers to its left.




Since 6 is to the right of −5 on the number line, 6 is greater than −5. So day 1 had a greater high temperature.

► You can also use the symbols <, =, and > to compare integers.

Compare: $-1 \ \underline{?}\ -7$

So $-1 > -7$.

**Think** −1 is to the right of −7 on the number line, so −1 is greater than −7.

**Remember:**
< means *is less than.*
= means *is equal to.*
> means *is greater than.*

## Examples

Compare. Write <, =, or >.

**1** $-9 \ \underline{?}\ 8$

$-9 < 8$

**Think** −9 is to the left of 8 on a number line, so −9 is less than 8.

**2** $5 \ \underline{?}\ -6$

$5 > -6$

**Think** 5 is to the right of −6 on a number line, so 5 is greater than −6.

**3** $-(-4) \ \underline{?}\ 4$

$4 = 4$

**Think** The opposite of −4 is 4.

**4** $-(-3) \ \underline{?}\ -(-5)$

$3 < 5$

**Think** The opposite of −3 is 3. The opposite of −5 is 5.

► You can also compare the absolute value of integers.

Compare: $|3| \ \underline{?}\ |-5|$ Write <, =, or >.

3 units from 0    5 units from 0

$3 < 5$

So $|3| < |-5|$.

**Remember:** The absolute value of any number is the distance the number is from 0 on a number line.

## Examples

Compare. Write <, =, or >.

**1** $|-6| \underline{\ ?\ } |4|$

$6 > 4$

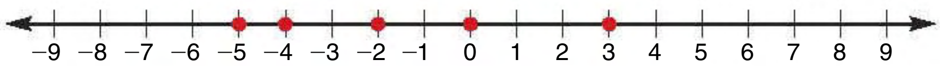
Think
$|-6| = 6$
$|4| = 4$

**2** $-|-3| \underline{\ ?\ } |-2|$

$-3 < 2$

Think
$|-3| = 3$
$|-2| = 2$

► A number line can be used to order integers from least to greatest and greatest to least. Use the number line to order 3, −4, 0, −2, and −5.




- To order from *least to greatest,* write the integers as they appear on the number line from left to right (least to greatest).
  From least to greatest: −5, −4, −2, 0, 3
- To order from *greatest to least*, write the integers as they appear on the number line from right to left (greatest to least).
  From greatest to least: 3, 0, −2, −4, −5

## Examples

**1** Order from least to greatest:

$-9, |-3|, -|-2|, -3$

Think
$-|-2| = -2$

Ordered from least to greatest:

$-9, -3, -|-2|, |-3|$

**2** Order from greatest to least:

$-8, |-4|, -|-6|, 2$

Think
$-|-6| = -6$

Ordered from greatest to least:

$|-4|, 2, -|-6|, -8$

## Try These

**Compare. Write <, =, or >.**

**1.** $-7 \underline{\ ?\ } 2$ **2.** $-5 \underline{\ ?\ } -9$ **3.** $0 \underline{\ ?\ } -8$ **4.** $-|-7| \underline{\ ?\ } -(-7)$

**Order from least to greatest.**

**5.** 22, −4, 0, 12, −19 **6.** −9, −3, 0, −2 **7.** $-|5|, -6, 11, 0$

**Order from greatest to least.**

**8.** 32, −18, 40, −20 **9.** $-12, 14, |-8|, -2$ **10.** $-6, |-1|, -1, 6$

**11.** Discuss and Write Is it possible for a negative number to have a greater absolute value than a positive number? Explain your answer.

Go to PRACTICE BOOK Lesson 1-2 for exercise sets.

Algebra

# 1-3 Add Integers

Update your skills. See page 409 VI.

**Objective** To model addition of integers • To add integers with like and unlike signs • To add more than two integers

Kyle lost 5 points in the first round of an electronic game and lost 3 points in the second round. What was the total number of points Kyle lost after two rounds? What integer represents the loss?

To find the total points lost, add: $-5 + (-3)$

► You can use tiles and a number line to add integers with like signs.

Add: $-5 + (-3)$

**Method 1** Use Tiles

Let ■ = −1.

- Model each addend.

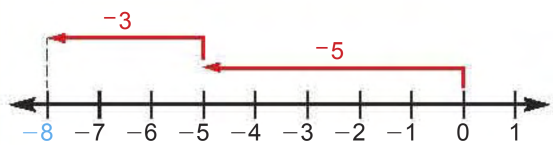
$-5 \quad + \quad (-3)$

- Join the tiles.

$-8$

**Method 2** Use a Number Line

- Start at zero.
- Move left to add negative integers.




$-5 + (-3) = -8$

Kyle lost a total of 8 points after two rounds. The integer representing this loss is $-8$.

► You can also use absolute value to add integers with like signs.

Add: $-58 + (-31)$

$|-58| + |-31|$

$58 + 31$

$89$

So $-58 + (-31) = -89$. ← Both addends are negative, so the sum is negative.

**Key Concept**

**Add Integers with Like Signs**
- Add the absolute values of the addends.
- Use the sign of the addends for the sum.

## Examples

**1** Add: $67 + 29$

$|67| + |29|$ ← Add the absolute values.

$67 + 29$

$96$

So $67 + 29 = 96$. ← Both addends are positive, so the sum is positive.

**2** Add: $-5 + (-9) + (-16)$

$[-5 + (-9)] + (-16)$ ← Add from left to right.

$-14 \quad + (-16)$ ← Add.

$-30$ ← Both addends are negative, so the sum is negative.

So $-5 + (-9) + (-16) = -30$.

► You can also use tiles and a number line to add integers with unlike signs.

Add: $6 + (-4)$

Let [orange tile] = 1 ... 

**Method 1** Use Tiles

- Model each addend.

$6 \quad + \quad (-4)$

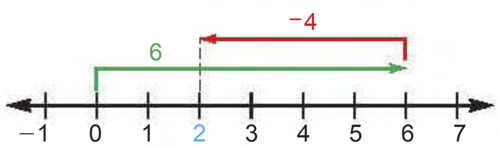
Let ■ = 1    ■ ■ ← Zero Pair
Let ■ = $-1$    $1 + (-1) = 0$

- Group and remove zero pairs.

$0 + 0 + 0 + 0 + 1 + 1 = 2$

**Method 2** Use a Number Line

- Start at zero.
- Move right to add positive integers.
- Move left to add negative integers.




So $6 + (-4) = 2$.

► You can also use absolute value to add integers with unlike signs.

Add: $-9 + 13$

$|13| - |-9|$

$13 - 9$

$4$

So $-9 + 13 = 4$. ← $|13| > |-9|$, so the sum is positive.

**Key Concept**

**Add Integers with Unlike Signs**

- Subtract the lesser absolute value from the greater absolute value.
- Use the sign of the addend with the greater absolute value for the sum.

## Examples

**1** Add: $14 + (-34)$

$|-34| - |14|$ ← Subtract the absolute values.

$34 - 14$

$20$

So $14 + (-34) = -20$. ← $|-34| > |14|$, so the sum is negative.

**2** Add: $-3 + 8 + (-11)$

$8 + (-3) + (-11)$ ← Change the order to add like signs.

$8 + (-14)$ ← Add.

$-6$ ← $|-14| > |8|$, so the sum is negative.

So $-3 + 8 + (-11) = -6$.

## Try These

**Find the sum.**

**1.** $-6 + (-4)$    **2.** $-17 + 7$    **3.** $17 + (-7)$    **4.** $92 + (-26) + 3$

**5. Discuss and Write** Explain how you can add integers with like signs and with unlike signs. Give examples to support your explanation.

Go to PRACTICE BOOK Lesson 1-3 for exercise sets.

Algebra

# 1-4

*Update your skills. See page 409 VI.*

# Subtract Integers

**Objective** To model subtraction of integers • To subtract integers with like and unlike signs

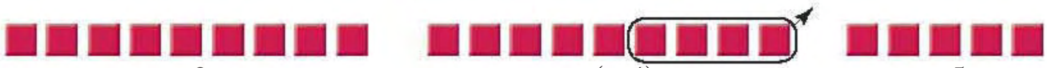
Alyssa and Heather are playing a board game they made up. A player who lands on blue adds −4 to her score. A player who lands on white subtracts −4 from her score. Alyssa lands on white. If her score was −9, what is her new score?

▶ You can use tiles or a number line to model subtraction of integers with like signs.

To find Alyssa's new score, subtract: $-9 - (-4)$

**Method 1** Use Tiles

**Remember:** ■ = −1



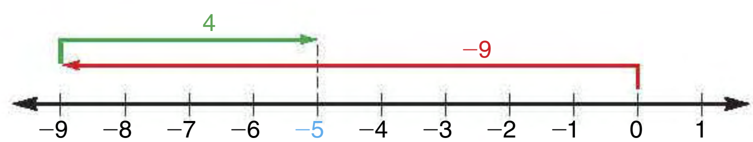
| −9 | − (−4) | −5 |
|---|---|---|
| Model Alyssa's old score, −9. | To subtract −4, remove four −1 tiles. | There are five −1 tiles left. |

**Method 2** Use a Number Line

- Start at 0, move left to −9.
- Move right to add 4.

**Think**

$-(-4) = 4$, so subtracting −4 is the same as adding 4.




So Alyssa's new score is −5.

▶ You can use opposites to subtract integers with like signs. Two integers that are opposites are called additive inverses of each other. The sum of an integer and its additive inverse is 0.

**Key Concept**

**Subtracting Integers**

To subtract an integer, add its opposite, or additive inverse.

$$a - b = a + (-b)$$

Subtract: $12 - 15$

$12 + (-15)$ ← Add the opposite of the number being subtracted.

$12 + (-15)$ ← $|-15| > |12|$ The result is negative.

$-3$

So $12 - 15 = -3$.

Subtract: $-7 - (-12)$

$-7 + (12)$ ← Add the opposite of the number being subtracted.

$-7 + 12$ ← $|12| > |-7|$ The result is positive.

$5$

So $-7 - (-12) = 5$.

► You can also use tiles and a number line to model subtraction of integers with unlike signs.

Subtract: $5 - (-2)$

**Remember:** ← zero pair
$1 + (-1) = 0$

**Method 1** Use Tiles

You cannot remove two $-1$ tiles because there are no $-1$ tiles.

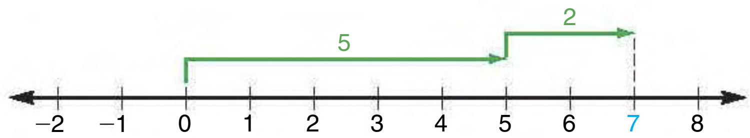
Adding zero pairs does not change the value. Now remove two $-1$ tiles.

The result is 7 tiles.

**Method 2** Use a Number Line

- Start at 0, move right to 5.
- Move right to add 2.

**Think**
$-(-2) = 2$, so subtracting $-2$ is the same as adding 2.




So $5 - (-2) = 7$.

► You also can use opposites to subtract integers with like signs.

Subtract: $-3 - 2$

$-3 + (-2)$ ← Add the opposite of the number being subtracted.

$-3 + (-2)$ ← Both addends are negative. The result is negative.

$-5$

So $-3 - 2 = -5$.

Subtract: $10 - (-3)$

$10 + (3)$ ← Add the opposite of the number being subtracted.

$10 + 3$ ← Both addends are positive. The result is positive.

$13$

So $10 - (-3) = 13$.

► Rename any absolute value expression before you subtract.

## Examples

**1**
$-|-5| - 3$
$-5 - 3$ ← $|-5| = 5$
$-5 + (-3)$
$-8$

**2**
$-|10| - |6|$
$-10 - 6$
$-10 + (-6)$
$-16$

**3**
$-|12| - |-22|$
$-12 - 22$
$-12 + (-22)$
$-34$

**4**
$|-8 - 10|$
$|-8 + (-10)|$
$|-18|$
$18$

## Try These

**Find the difference.**

**1.** $-5 - 8$ **2.** $5 - (-8)$ **3.** $6 - |15|$ **4.** $|-16| - (-9)$ **5.** $-|6 - (-4)|$

**6. Discuss and Write** Explain whether subtracting two negative integers always results in a negative integer. Give examples to support your explanation.

Go to PRACTICE BOOK Lesson 1-4 for exercise sets.

Algebra

# 1-5 Multiply Integers

Update your skills. See page 411 XI.

**Objective** To multiply integers with and without models


PROBLEM-SOLVING STRATEGY 3

The mercury in the barometer fell 3 cm per hour during the 4 hours before the storm struck. What integer represents the mercury-level change during those 4 hours?

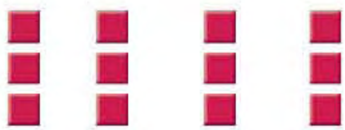
► To find the change, multiply: $4(-3)$

**Think** $-3$ represents "fell 3 cm."

$4(-3)$ means 4 groups of $-3$, so $4(-3) = (-3) + (-3) + (-3) + (-3)$.

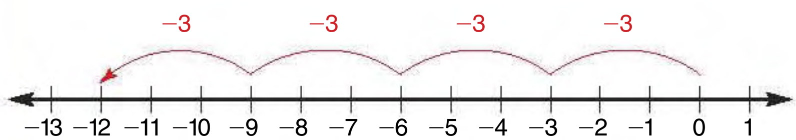
**Method 1** Use Tiles

$-3 + (-3) + (-3) + (-3)$



$4(-3) = -12$

**Method 2** Use a Number Line




$4(-3) = -12$

So the change in the number of centimeters of mercury is $-12$.

► You can use patterns to find the product of two negative integers. To multiply $(-4)(-3)$, start with $4(-3) = -12$, and continue the pattern.

$$\begin{aligned}
4(-3) &= -12 \\
3(-3) &= -9 \\
2(-3) &= -6 \\
1(-3) &= -3 \\
0(-3) &= 0 \\
(-1)(-3) &= 3 \\
(-2)(-3) &= 6 \\
(-3)(-3) &= 9 \\
(-4)(-3) &= 12
\end{aligned}$$

Notice that the product increases by 3.

**Key Concept**

**Multiply Integers**

- The product of two integers with *like* signs is positive.
  - *positive* • *positive* = positive
  - *negative* • *negative* = positive
- The product of two integers with *unlike* signs is negative.
  - *positive* • *negative* = negative
  - *negative* • *positive* = negative

► You can also use rules for multiplying integers.

## Examples

**1** $5 \bullet 2 = 10$ ← Factors have like signs. The product is positive.

**2** $-6(-2) = 12$ ← Factors have like signs. The product is positive.

**3** $4(-7) = -28$ ← Factors have unlike signs. The product is negative.

**4** $-3 \bullet 9 = -27$ ← Factors have unlike signs. The product is negative.

You can also use patterns to find products of more than two negative factors.

*Odd Number of Factors*

$(-4)(-4)(-4)$

$16(-4)$ ← $(-4)(-4) = 16$

$-64$ ← $16(-4) = -64$

*Even Number of Factors*

$(-4)(-4)(-4)(-4)$

$16(-4)(-4)$ ← $(-4)(-4) = 16$

$-64(-4)$ ← $16(-4) = -64$

$256$ ← $-64(-4) = 256$

**Key Concept**

**Multiply Two or More Negative Integers**

- When the number of negative factors is even, the product is positive.
- When the number of negative factors is odd, the product is negative.

## Examples

Simplify.

**1** $10(-5)(-2)$

$-50(-2)$

$100$ ← Two negative integers; the product is positive.

**2** $-12(-2) \bullet 2(-1)$

$24 \bullet 2(-1)$

$48(-1)$

$-48$ ← Three negative integers; the product is negative.

You can simplify expressions that contain absolute value symbols.

| | | | |
|---|---|---|---|
| $\|-8 \bullet 2\|$ | $\|-12\| \bullet 6$ | $-\|8 \bullet 3\|$ | $-(\|-6\| \bullet \|9\|)$ |
| $\|-16\|$ | $12 \bullet 6$ | $-24$ | $-(6 \bullet 9)$ |
| $16$ | $72$ | | $-54$ |

## Try These

**Tell whether each product is positive or negative.**

**1.** $(-2) \bullet 3(-5)(-7)$ **2.** $(-1)(-6)(-2)(-4)$ **3.** $(-3) \bullet 4(-1) \bullet 9$

**Find each product.**

**4.** $-3(-6)$ **5.** $9(-4)$ **6.** $-8(2)(-5)$

**7.** $(-2)(-3)(-1)(-4)$ **8.** $-5(-1)(-3)(-3)$ **9.** $-[3(-8)]$

**10.** $-(-4)(-2)$ **11.** $|-6 \bullet 9|$ **12.** $|-7| \bullet |-7|$

**13. Discuss and Write** Is the absolute value of a product in which one factor is $-a$ and the other factor is $b$ the same as a product in which one factor is the absolute value of $b$ and the other is the absolute value of $-a$? Explain, using $a = 3$ and $b = 11$ to support your answer.

Go to PRACTICE BOOK Lesson 1-5 for exercise sets.

Algebra
1-6

Update your skills. See page 412 XII.

# Divide Integers

**Objective** To divide integers with and without models

Mrs. Ferrara drains water from her above-ground pool. The height of the water decreases by 3 inches per minute. How many minutes does it take for the height of the water to decrease 15 inches?

► To find the number of minutes, divide: $-15 \div (-3)$

You can use repeated subtraction to model division.

**Think**
*To drain* means "to decrease," so use $-15$ to represent a decrease of 15 inches.

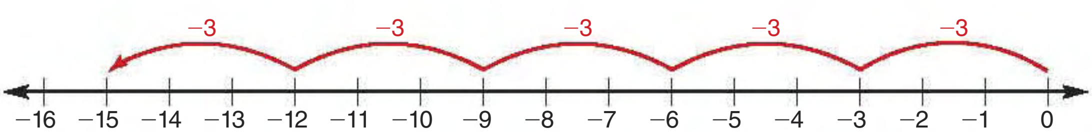
$-15 \div (-3)$ means $-15$ separated into equal groups of $-3$.

**Method 1** Use Tiles

$-15 - (-3) - (-3) - (-3) - (-3) - (-3)$

$-15 \div (-3) = 5$

**Method 2** Use a Number Line




$-15 \div (-3) = 5$

So it takes 5 minutes for the height of the water to decrease 15 inches.

► Multiplication and division are inverse operations; one undoes the other. You can use this relationship to determine the rules for dividing integers.

| Multiplication Sentence | Related Division Sentence |
|---|---|
| $3 \bullet 6 = 18$ | $18 \div 6 = 3$ |
| $3(-6) = -18$ | $-18 \div (-6) = 3$ |
| $-3(-6) = 18$ | $18 \div (-6) = -3$ |
| $-3 \bullet 6 = -18$ | $-18 \div 6 = -3$ |

The pattern above shows that the rules for dividing integers are similar to the rules for multiplying integers.

Since there is no related multiplication sentence for any division sentence with a divisor of 0, division by zero is said to be undefined. For example, $3 \div 0$ is undefined because there is no value of $n$ that makes $n \bullet 0 = 3$ true.

► You can also use rules for dividing integers.

**Key Concept**

**Divide Integers**

- The quotient of two integers with like signs is positive.
  *positive* ÷ *positive* = positive
  *negative* ÷ *negative* = positive
- The quotient of two integers with unlike signs is negative.
  *positive* ÷ *negative* = negative
  *negative* ÷ *positive* = negative
- The quotient is 0 if the dividend is zero.
  $0 \div n = 0$

## Examples

Divide.

**1** $54 \div 9$
$54 \div 9 = 6$ ← 54 and 9 have like signs. The quotient is positive.

**2** $-64 \div (-8)$
$-64 \div (-8) = 8$ ← $-64$ and $-8$ have like signs. The quotient is positive.

**3** $81 \div (-9)$
$81 \div (-9) = -9$ ← 81 and $-9$ have unlike signs. The quotient is negative.

**4** $-144 \div 12$
$-144 \div 12 = -12$ ← $-144$ and 12 have unlike signs. The quotient is negative.

► There are special rules for division that involves 1, −1, and 0.

- The quotient of any nonzero integer and 1 is that integer. $\frac{a}{1} = a$ or $a \div 1 = a$
- The quotient of any nonzero integer and −1 is the opposite of that integer. $\frac{a}{-1} = -a$ or $a \div (-1) = -a$
- Zero divided by any nonzero integer is zero. $\frac{0}{a} = 0$ or $0 \div a = 0$
- The division of any integer by zero is undefined. $a \div 0$ or $\frac{a}{0}$ is undefined.

## Try These

**Find the quotient.**

**1.** $36 \div (-6)$ **2.** $-60 \div (-10)$ **3.** $25 \div (-5)$ **4.** $\frac{9}{-1}$

Think: $\frac{9}{-1} = 9 \div (-1)$

**5.** $-[72 \div (-8)]$ **6.** $-(-4) \div (-2)$ **7.** $|-66 \div 11|$ **8.** $|-110| \div |-11|$

**9. Discuss and Write** Explain how the inverse operation can help you determine the sign of the quotient of two integers. Justify your answer with examples.

Go to PRACTICE BOOK Lesson 1-6 for exercise sets.

Algebra

# 1-7 Properties

**Objective** To identify properties of addition and multiplication

► These properties are true for operations with any integers. Let *a, b,* and *c* represent any integers.

### Commutative Property of Addition

Changing the *order* of the addends does *not* change the sum.

$a + b = b + a$

*Example:* $6 + (-2) = -2 + 6$

$4 = 4$ True

### Commutative Property of Multiplication

Changing the *order* of the factors does *not* change the product.

$ab = ba$

*Example:* $6(-2) = -2(6)$

$-12 = -12$ True

**Think** "order"

### Associative Property of Addition

Changing the *grouping* of the addends does *not* change the sum.

$(a + b) + c = a + (b + c)$

*Example:* $(-6 + 2) + 3 = -6 + (2 + 3)$

$-4 + 3 = -6 + 5$

$-1 = -1$ True

### Associative Property of Multiplication

Changing the *grouping* of the factors does *not* change the product.

$(ab)c = a(bc)$

*Example:* $(-6 \bullet 2)3 = -6(2 \bullet 3)$

$(-12)3 = -6(6)$

$-36 = -36$ True

**Think** "grouping"

### Identity Property of Addition

Adding 0 and any number does not change the value of the number.

$a + 0 = a$ or $0 + a = a$

*Examples:*

$-6 + 0 = -6$

or

$0 + (-6) = -6$

identity element of addition

### Identity Property of Multiplication

Multiplying 1 and any number does not change the value of the number.

$a \bullet 1 = a$ or $1 \bullet a = a$

*Examples:*

$-6(1) = -6$

or

$1(-6) = -6$

identity element of multiplication

**Think** "same number"

### Inverse Property of Addition

The sum of any integer and its additive inverse is 0.

$a + (-a) = 0$

*Example:* $3 + (-3) = 0$

**Think** 3 and $-3$ are opposites.

### Zero Property of Multiplication

The product of 0 and any number is 0.

$0 \bullet a = 0$ or $a \bullet 0 = 0$

*Example:* $0(-3) = 0$ or $(-3)0 = 0$

**Think** $n \bullet 0 = 0$

**Distributive Property of Multiplication over Addition**

Multiplying a sum by a number is the same as multiplying each addend by that number and adding the two products.

$a(b + c) = ab + ac$

*Example:*

$-3(-4 + 5) = -3(-4) + (-3)(5)$

$-3(1) = 12 + (-15)$

$-3 = -3$ True

**Distributive Property of Multiplication over Subtraction**

Multiplying a difference by a number is the same as multiplying each number in the subtraction expression by that number and subtracting the two products.

$a(b - c) = ab - ac$

*Example:* $-3(-4 - 5) = -3(-4) - (-3)(5)$

$-3[-4 + (-5)] = 12 - (-15)$

$-3(-9) = 12 + 15$

$27 = 27$ True

## Examples

Name the property used.

**1** $(-5 \bullet 2)(7) = -5 \bullet (2 \bullet 7)$ ← grouping changed

Associative Property of Multiplication

**2** $1(-3) = -3$ ← 1 is an identity element.

Identity Property of Multiplication

**3** $(-8 + 3) + 1 = -8 + (3 + 1)$ ← grouping changed

Associative Property of Addition

**4** $-1 + (-9) = -9 + (-1)$ ← order changed

Commutative Property of Addition

**5** $2[(-8) - (-7)] = 2(-8) - 2(-7)$

Distribute the same factor across both subtraction terms.

Distributive Property of Multiplication over Subtraction

**6** $-1(-6 + 0) = -1(-6) + (-1)(0)$

Distribute the same factor across both addends.

Distributive Property of Multiplication over Addition

## Try These

**Name the property used.**

**1.** $0(-9) = 0$

**2.** $-8 + 0 = -8$

**3.** $-5 \bullet 1 = -5$

**4.** $(-7 + 3) + 8 = -7 + (3 + 8)$

**5.** $-8(-9 - 6) = (-8)(-9) - (-8)(6)$

**Use the properties to find the value of *a*.**

**6.** $-7(-2 + 4) = -7(-2) + a(4)$

**7.** $3(2 - 5) = 3(2) - 3(a)$

**8.** **Discuss and Write** Do all the properties of addition work for subtraction? Do all the properties of multiplication work for division? Justify your answers with examples.

Go to PRACTICE BOOK Lesson 1-7 for exercise sets.

Algebra

# 1-8 Closure Property

**Objective** To identify the closure properties for any defined set of numbers

If performing an operation on *any* two numbers in a set *always* results in a number that is in that set, then the set is *closed* under that operation. This is called the Closure Property.

► To test if a set of numbers is closed under an operation:

1. Identify the set of numbers and the operation in the closure question.
2. Choose two numbers from the set.
3. Perform the given operation on those numbers to check for closure.

Is the result in the set of numbers?
If *yes*, then that set *could be* closed under that operation.
If *no*, then the set is *not* closed under that operation.
Whenever the answer is *no*, the test case is called a counterexample.

**Key Concept**

**Counterexamples and Confirming Examples**

Only one *counterexample* is needed to prove that a set is *not* closed under a given operation. However, one *confirming* example does *not* prove that a set *is* closed under a given operation. If all possibilities cannot be tested, then proof must be established in some other way.

► Explore the set of whole numbers for closure under addition and subtraction.

Is the sum of two odd whole numbers always odd?

closure question

*Set:* odd numbers

*Operation:* addition

*Check:* $1 + 5 = 6$
6 is even.

*Result:* Since 6 is *not* odd, the set of odd whole numbers is *not* closed under addition.

## Examples

**1** Is the set of whole numbers closed under addition?

*Set:* whole numbers

*Operation:* addition

*Check:* $1 + 0 = 1$
1 is a whole number.

*Result:* The set of whole numbers *could be* closed under addition.

**2** Is the set of whole numbers closed under subtraction?

*Set:* whole numbers

*Operation:* subtraction

*Check:* $6 - 10 = -4$
$-4$ is not a whole number.

*Result:* The set of whole numbers is *not* closed under subtraction.

Mathematicians agree that the set of whole numbers is closed under addition, but the set of whole numbers is not closed under subtraction.

► Explore the set of integers for closure under addition, subtraction, multiplication, and division.

## Examples

**1** Is the set of integers closed under addition?

*Set:* integers

*Operation:* addition

*Check:* $-15 + 15 = 0$
0 is an integer.

*Result:* The set of integers *could be* closed under addition.

**2** Is the set of integers closed under subtraction?

*Set:* integers

*Operation:* subtraction

*Check:* $-13 - (-4) = -9$
$-9$ is an integer.

*Result:* The set of integers *could be* closed under subtraction.

**3** Is the set of integers closed under division?

*Set:* integers

*Operation:* division

*Check:* $\frac{-2}{4} = \frac{-1}{2} = -0.5$
$-0.5$ is not an integer.

*Result:* The set of integers is *not* closed under division.

**4** Is the set of integers closed under multiplication?

*Set:* integers

*Operation:* multiplication

*Check:* $-5 \bullet 10 = -50$
$-50$ is an integer.

*Result:* The set of integers *could be* closed under multiplication.

Mathematicians agree that the set of integers is closed under addition, subtraction, and multiplication, but the set of integers is not closed under division.

## Try These

**True or false? Give examples to justify your answer.**

**1.** The set of numbers $-1$, 0, and 1 is closed under multiplication.

**2.** The set of the multiples of 10 is closed under division.

**3.** **Discuss and Write** If $7 = 2 + 5$ and $13 = 11 + 2$, are all the sums of two prime numbers between 0 and 25 prime?
(*Hint:* A prime number is a whole number greater than 1 having exactly two factors, itself and 1.)

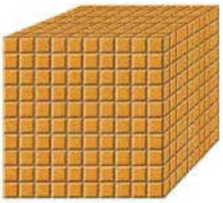
Go to PRACTICE BOOK Lesson 1-8 for exercise sets.

Algebra

# 1-9 Powers and Laws of Exponents

**Objective** To write the standard form for numbers that are given in exponential form • To write the exponential form for numbers that are given in standard form • To apply the multiplication and division laws of exponents • To identify the value of a number to the zero power • To simplify expressions with exponents

The volume of the cube at the right is $10 \bullet 10 \bullet 10$, or 1000, cubic units. How can you express the volume of the cube using exponents?



► To use exponents to express the volume of the cube, show how many times 10 is used as a factor.

An exponent tells you how many times a number, called the base, is used as a factor.

10 is used 3 times as a factor.

$$10 \bullet 10 \bullet 10 = 10^3$$

exponent

base

A number that is written with an exponent is called a power of the base.

factored form | standard form | exponential form

$$10 \bullet 10 \bullet 10 = 1000 = 10^3$$

Read $10^3$ as: ten to the third power,
the third power of ten, or
ten cubed.

Using exponents, you find that the volume of the cube is $10^3$ cubic units.

## Examples

Write the exponential form for each number given in factored form.

**1** $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = 3^5$

Think
3 is used as a factor 5 times.

**2** $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 2^8$

Think
2 is used as a factor 8 times.

**3** $-4(-4) \bullet 5 \bullet 5 \bullet 5 = (-4)^2 \bullet 5^3$

Think
$-4$ is used as a factor 2 times.
5 is used as a factor 3 times.

**4** $6 \bullet 7 \bullet 7 \bullet 7 = 6^1 \bullet 7^3$ or $6 \bullet 7^3$

$6^1 = 6$

Think
7 is used as a factor 3 times.

► You can write powers in standard form by expressing the power in factored form and then multiplying the factors.

Simplify: $(-5)^2 \bullet 10^2$

$(-5)^2 \bullet 10^2 = (-5)(-5) \bullet (10)(10)$ ← Express each power in factored form.

$= 25 \bullet 100$ ← Multiply.

$= 2500$

While $-5^2$ and $(-5)^2$ may appear to be the same, they actually have different meanings and different values.
$-5^2$ means "the opposite of five squared."
$-5^2 = -(5 \bullet 5) = -25$
$(-5)^2$ means "negative five squared."
$(-5)^2 = (-5)(-5) = 25$

► The Laws of Exponents can help you simplify expressions.

- To multiply two powers that have the same base, keep the base and add the exponents.

Simplify: $6^3 \bullet 6^5$

$6^3 \bullet 6^5 = (6 \bullet 6 \bullet 6) \bullet (6 \bullet 6 \bullet 6 \bullet 6 \bullet 6)$
$= 6^8$

Or $6^3 \bullet 6^5 = 6^{3+5}$
$= 6^8$

**Key Concept**
**Law of Exponents for Multiplication**
$a^m \bullet a^n = a^{m+n}$, where $a \neq 0$

- To divide two powers that have the same base, keep the base and subtract the exponents.

Simplify: $\frac{3^6}{3^2}$

$$\frac{3^6}{3^2} = \frac{3 \bullet 3 \bullet 3 \bullet 3 \bullet \cancelto{1}{3} \bullet \cancelto{1}{3}}{\cancelto{1}{3} \bullet \cancelto{1}{3}}$$
$= 3 \bullet 3 \bullet 3 \bullet 3$
$= 3^4$

Or $\frac{3^6}{3^2} = 3^{6-2} = 3^4$

**Key Concept**
**Law of Exponents for Division**
$a^m \div a^n = a^{m-n}$, where $a \neq 0$

- A number to the zero power always equals 1.

Simplify: $\frac{3^6}{3^6}$

$$\frac{3^6}{3^6} = \frac{\cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3}}{\cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3} \bullet \cancelto{1}{3}}$$
$= 1$

Or $\frac{3^6}{3^6} = 3^{6-6} = 3^0 = 1$

Apply the Law of Exponents for Division.

**Key Concept**
**Law of Exponents for Zero**
$a^0 = 1$, where $a \neq 0$

## Try These

**Write each in simplest exponential form.**

**1.** $6 \bullet 6 \bullet 6 \bullet 7 \bullet 7$ **2.** $-1(-1)(-2)(-3)(-3)$ **3.** $2 \bullet 2^2 \bullet 2^4$ **4.** $\frac{5^6}{5^2}$

**Write each in factored form. Then simplify.**

**5.** $4^3$ **6.** $2^0$ **7.** $10^9 \div 10^5$ **8.** $5^1 \bullet 5^3$

**9. Discuss and Write** Explain how patterns and place value can be used to show that any nonzero number to the zero power is equal to 1.

Go to PRACTICE BOOK Lesson 1-9 for exercise sets.

Algebra

# 1-10 Order of Operations

**Objective** To use the order of operations to simplify numerical expressions with grouping symbols and exponents • To use a calculator to check solutions

When more than one operation is used in a mathematical expression, you need to know which operation to perform first so there is only one result. The order of operations is a set of rules that are used to simplify mathematical expressions with more than one operation.

Todd and Ana both simplified this expression:

$6^2 - (8 + 2 \bullet 2) \div 2^2$

Todd's answer was 4, and Ana's answer was 33. Which student was correct?

► To simplify the expression, use the order of operations.

$6^2 - (8 + 2 \bullet 2) \div 2^2$

$6^2 - (8 + 2 \bullet 2) \div 2^2$ ← Simplify within parentheses. First multiply 2 • 2, then add.

$6^2 - 12 \div 2^2$ ← Simplify the exponents.

$36 - 12 \div 4$ ← Divide.

$36 - 3$ ← Subtract.

$33$

Ana's answer was correct.

**Key Concept**

**Order of Operations**

- Following the order of operations, first simplify operations within grouping symbols.
- Simplify exponents.
- Multiply or divide from left to right.
- Add or subtract from left to right.

► Sometimes an expression contains more than one set of grouping symbols. When this happens, begin simplifying with the innermost set.

Simplify: $[(32 + 43) \div (-15)] \bullet 2^3$

$[(32 + 43) \div (-15)] \bullet 2^3$ ← Simplify within parentheses.

$[75 \div (-15)] \bullet 2^3$ ← Simplify within the brackets.

$-5 \bullet 2^3$ ← Simplify the exponent.

$-5 \bullet 8$ ← Multiply.

$-40$

**Grouping Symbols**

parentheses ( )

brackets [ ]

**Think**

$2^3 = 2 \bullet 2 \bullet 2$

## Examples

**1** Simplify: $12(-2)^3 + (-10)$

$12(-2)^3 + (-10)$ ← Simplify the exponent.

$12[(-2)(-2)(-2)] + (-10)$

$12(-8) + (-10)$ ← Multiply.

$-96 + (-10)$ ← Add.

$-106$

**2** Simplify: $[(-24 \div 3)(-20 \div 4)] \div 2$

$[(-24 \div 3)(-20 \div 4)] \div 2$ ← Compute within parentheses. Work from left to right.

$[(-8)(-5)] \div 2$ ← Multiply.

$40 \div 2$ ← Divide.

$20$

► A fraction bar is also a grouping symbol that indicates division. Simplify the expression in the dividend and the divisor before dividing.

## Examples

Find the value of each expression.

**1** $\dfrac{3^2(34-2)}{-4(-1)}$

$\dfrac{3^2(34-2)}{-4(-1)}$ ← Evaluate the dividend.

$\dfrac{3^2(34-2)}{-4(-1)}$ ← Compute within parentheses.

$\dfrac{3^2(32)}{-4(-1)}$ ← Simplify the exponent.

$\dfrac{9(32)}{-4(-1)}$ ← Multiply.

$\dfrac{288}{-4(-1)}$ ← This is the value of the dividend.

$\dfrac{288}{-4(-1)}$ ← Multiply to evaluate the divisor.

$\dfrac{288}{4}$ ← This is the value of the divisor.

$\dfrac{288}{4} = 72$ ← Divide the dividend by the divisor to find the value of the expression.

**2** $\dfrac{-2^3(30-24) \div 3}{4 \div (-2) + (-6)}$

$\dfrac{-2^3(30-24) \div 3}{4 \div (-2) + (-6)}$ ← Evaluate the dividend.

$\dfrac{-2^3(30-24) \div 3}{4 \div (-2) + (-6)}$ ← Compute within parentheses.

$\dfrac{-2^3(6) \div 3}{4 \div (-2) + (-6)}$ ← Simplify the exponent.

$\dfrac{-8(6) \div 3}{4 \div (-2) + (-6)}$ ← Multiply.

$\dfrac{-48 \div 3}{4 \div (-2) + (-6)}$ ← Divide.

$\dfrac{-16}{4 \div (-2) + (-6)}$ ← This is the value of the dividend.

$\dfrac{-16}{4 \div (-2) + (-6)}$ ← To evaluate the divisor, divide and then add.

$\dfrac{-16}{-8}$ ← This is the value of the divisor.

$\dfrac{-16}{-8} = 2$ ← Divide the dividend by the divisor.

► You can use a calculator to check your work. Check the answer from Example 1 above. You will need to enclose the divisor within parentheses.

**Check:** $\dfrac{3^2(34-2)}{-4(-1)} \stackrel{?}{=} 72$

The caret key, [^], is used to enter exponents.

Press [^] [(] [−] [)] [÷] [(] [(−)] [×] [(−)] [)] [ENTER] 72

## Try These

**Simplify.**

**1.** $3(-16) - 60 \div 12$

**2.** $\dfrac{(16+4) \div 5 \bullet 8}{2^3}$

**3.** $\dfrac{(20 \div 10)(26-5)}{6}$

**4.** $\dfrac{(2+4)^3}{2^3 + 4^3}$

**5. Discuss and Write** Why is it important to have rules for simplifying an expression that contains more than one operation?

Go to PRACTICE BOOK Lesson 1-10 for exercise sets.

Algebra

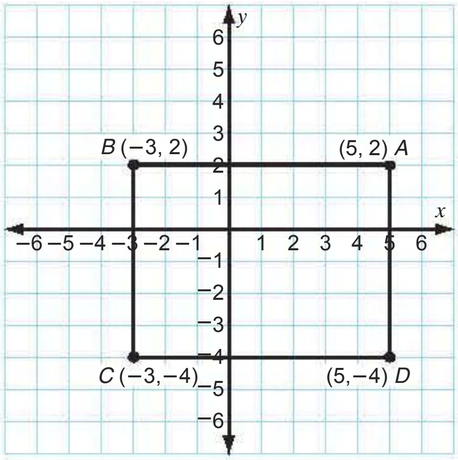
1-11

# The Coordinate Plane

**Objective** **To identify and graph points located in all four quadrants, on the *x*-axis, and on the *y*-axis • To identify the quadrant or axis location of an ordered pair**

Russ used graph paper to draw rectangle *ABCD* on the grid at the right. The *coordinates* that give the location of points *A, B, C,* and *D* are *A* (5, 2), *B* (−3, 2), *C* (−3, −4), and *D* (5, −4). How can Russ determine which of the following points lie inside the rectangle?

*E* (1, −2), *F* (4, 5), *G* (5, 0), *H* (0, −5), *O* (0, 0)




► The coordinate plane is a grid system formed by two number lines that cross at their zero points. The point at which the number lines cross is called the origin. The horizontal number line is the *x*-axis. The vertical number line is the *y*-axis.

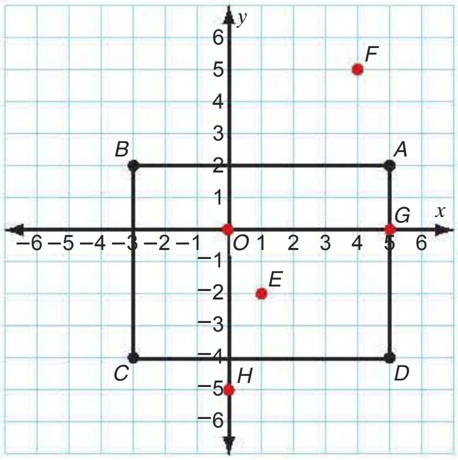
Each point on the grid can be represented by an *ordered pair* of numbers (*x*, *y*) called the *coordinates* of the point. The *x*-coordinate is the first number in the ordered pair. It locates the point by telling how many units to the left or right of the origin the point is. The *y*-coordinate is the second number in the ordered pair. It locates the point by telling how many units above or below the origin the point is.

The coordinates that locate a point are called an ordered pair because the *x*-coordinate is always first, and the *y*-coordinate is always second.

► To determine which points lie inside the rectangle, draw a coordinate grid on graph paper. Graph points *A, B, C, D,* and connect them to form a rectangle. Then graph the other points.

**Key Concept**

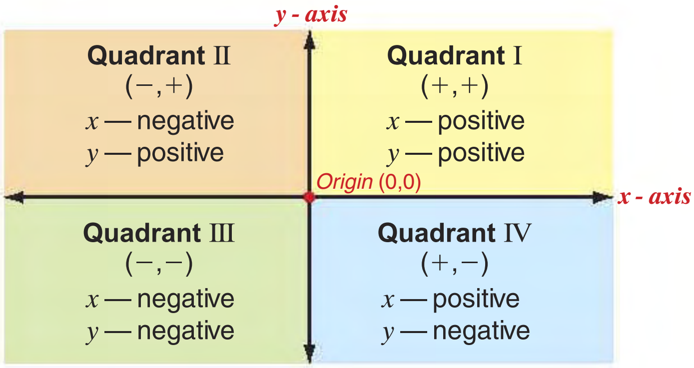
**Graph Coordinates**
- Start at the origin, (0, 0).
- Move horizontally along the *x*-axis the number of units indicated by the *x*-coordinate. Move to the right for a positive number, and move to the left for a negative number.
- From that location, move vertically the number of units indicated by the *y*-coordinate. Move up for a positive number, and move down for a negative number.
- Draw a dot, and label the point with its coordinates.




Point *E* is inside the rectangle.
Point *F* is outside the rectangle.
Point *G* lies on the rectangle.
Point *H* is outside the rectangle.
Point *O* is inside the rectangle.

So points *E* and *O* lie inside rectangle *ABCD*.

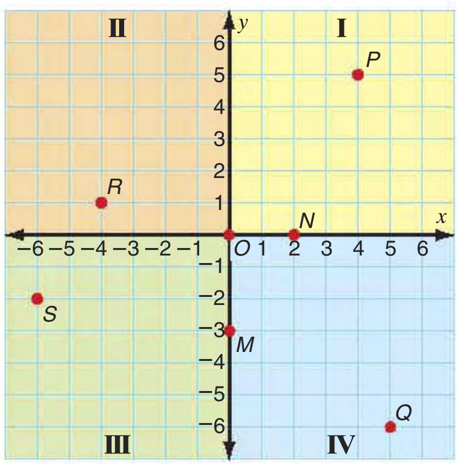
▶ The $x$-axis and the $y$-axis divide the coordinate plane into four quadrants.




Points on the $x$-axis or the $y$-axis are not in any of the quadrants. The origin is on both the $x$-axis and the $y$-axis.

## Example

**1** Each point shown on the graph below is located either in one of the four quadrants or on an axis. Locate each point on the graph by giving its ordered pair and by indicating in which quadrant or on which axis the point lies.




**Points**

$P$ (4, 5) ← (+, +) Quadrant I

$Q$ (5, −6) ← (+, −) Quadrant IV

$R$ (−4, 1) ← (−, +) Quadrant II

$S$ (−6, −2) ← (−, −) Quadrant III

$M$ (0, −3) ← on the $y$-axis

$N$ (2, 0) ← on the $x$-axis

$O$ (0, 0) ← on both the $x$-axis and the $y$-axis

## Try These

**Graph and connect the points.**

**1.** $M$ (−2, −3), $N$ (0, −1), $P$ (1, −3)

**2.** $Q$ (−1, 4), $R$ (−1, −2), $S$ (3, −2), $T$ (3, 4)

**3.** $U$ (−2, 5), $V$ (3, 5), $W$ (6, 0), $X$ (3, −5), $Y$ (−2, −5), $Z$ (−5, 0)

**Name the quadrant or axis that is the location for each point.**

**4.** $A$ (−1, −2) **5.** $B$ (−1, 4) **6.** $C$ (0, −5) **7.** $D$ (2, −2) **8.** $E$ (4, 3) **9.** $F$ (3, 0)

**10.** What do the points at these locations have in common? (3, 2), (5, 2), (19, 2)
What do the points at these locations have in common? (4, 6), (4, 18), (4, 21)

**11.** **Discuss and Write** Explain how to graph points $A$ and $B$ in exercises 4 and 5 above on a coordinate grid.

Go to PRACTICE BOOK Lesson 1-11 for exercise sets.

# 1-12 Problem-Solving Strategy: Guess and Test

**Objective** To solve problems using the strategy *Guess and Test*

**Problem 1:** Exactly 440 tickets to a play were sold. Seats were priced at only two levels: $18 or $10. The total revenue on tickets sales was $6640. How many $18 tickets were sold?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Some tickets sold for $18, others for $10.
There were 440 tickets sold in all.
The combined value of all tickets sold was $6640.

**Question:** How many $18 tickets were sold?

**Problem-Solving Strategies**
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**
Since there are a finite number of possibilities, you can try to find the solution by using the strategy *Guess and Test.*

**Solve** **Apply the strategy.**
Begin by guessing that half of the tickets (that is, $\frac{1}{2} \bullet 440$ tickets $= 220$ tickets) were $18 tickets and the other half were $10 tickets.

Calculate the revenue for all the tickets:
$220 \bullet \$18 + 220 \bullet \$10 = \$6160$

This is less than the actual revenue of $6640. So adjust the guess, increasing the number of $18 tickets, and calculate the revenue again. Continue guessing, testing, and adjusting until you get the correct answer. Make a table like the one below to keep track of the results.

| Number × $18 Tickets | Number × $10 Tickets | Revenue | Comment |
|---|---|---|---|
| 220 | 220 | $6160 | Too low; try more $18 tickets. |
| 300 | 140 | $6800 | Too high; the number of $18 tickets must be between 220 and 300. |
| 260 | 180 | $6480 | Too low; the number of $18 tickets must be between 260 and 300. |
| 280 | 160 | $6640 | Correct! |

So 280 of the $18 tickets were sold.

**Check** **Check to make sure your answer makes sense.**
If 280 tickets were sold at $18 each, it would generate $280 \bullet \$18$, or $5040, in revenue.
The remaining tickets ($440 - 280$, or 160), sold for $10 each, provide additional revenue of $160 \bullet \$10$, or $1600.
Therefore, the total revenue is $\$5040 + \$1600$, or $6640.

**Problem 2:** The width of a rectangle is the same as the side length of a square. The length of the rectangle is twice the side length of the square. The total of their areas is 588 cm$^2$. If the dimensions of the figures are whole numbers, what are those dimensions?

**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** The width of a rectangle is the same as the side length of a square.
The length of the rectangle is twice the side length of the square.
The combined area is 588 cm$^2$.

**Question:** What are the whole-number dimensions of the square and the rectangle in centimeters?

**Plan** **Select a strategy.**

Because all of the involved lengths are whole numbers and because there are only a finite number of possibilities, you can try to find the solution using the strategy *Guess and Test*.

**Solve** **Apply the strategy.**

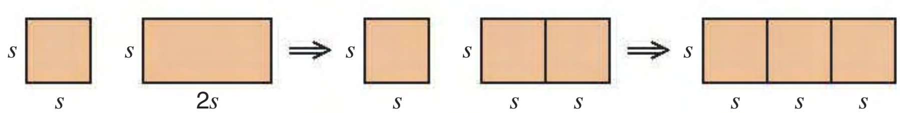
Start by guessing the side length of the square. From this, determine the dimensions of the rectangle, the area of each figure, and the combined area. Based on the result, adjust the guess and test again. Continue until you have the answer. The table below shows one series of guesses.

| Side Length of Square | Width of Rectangle | Length of Rectangle | Area of Square | Area of Rectangle | Combined Area | Comment |
|---|---|---|---|---|---|---|
| 10 | 10 | 20 | 100 | 200 | 300 | Too low |
| 20 | 20 | 40 | 400 | 800 | 1200 | Too high |
| 15 | 15 | 30 | 225 | 450 | 675 | Too high |
| 12 | 12 | 24 | 144 | 288 | 432 | Too low |
| 14 | 14 | 28 | 196 | 392 | 588 | Correct! |

The square has sides of length 14 cm, and the rectangle has dimensions 14 cm by 28 cm.

**Check** **Check to make sure your answer makes sense.**

Draw a picture to show that the combined areas of the square and rectangle are the same as the area of 3 of the squares.




If the side of the square is $s$, then its area is $s^2$.

$3s^2 = 588 \text{ cm}^2 \rightarrow s^2 = \frac{1}{3} \bullet 588 \text{ cm}^2 = 196 \text{ cm}^2 \rightarrow s = 14 \text{ cm}$

The width of the rectangle is $s$, and the length is $2s$. So the width is 14 cm, and the length is $2 \bullet 14$ cm, or 28 cm. The dimensions are correct!

**Think**
The number whose square is 196 is 14. ($196 = 14^2$)

Go to PRACTICE BOOK Lesson 1-12 for exercise sets.

# Enrichment: Sequence Sums

**Objective** To explore Gauss's method for finding the sum of the first 100 counting numbers
• To apply Gauss's method in finding sums of number sequences

Carl Friedrich Gauss (April 30, 1777–February 23, 1855) was one of the world's greatest mathematicians. He made many discoveries in both pure and applied mathematics. Even as a child, he had a strong understanding of number and function and could do complex mental calculations.

► A story about 7-year-old Carl Friedrich Gauss tells that he quickly found the answer to a problem his teacher thought would keep everyone in class busy for quite a while. That problem was to find the sum of all the numbers from 1 through 100.

The task is to find this sum: $1 + 2 + 3 + 4 + 5 + \ldots + 96 + 97 + 98 + 99 + 100$

You can list and add the numbers, you can use a calculator or computer, or you can look for a pattern that might get to the solution even faster than a calculator. Look for a pattern that might lead to the solution.

- The sum of the first and the last numbers in the sequence $(1 + 100)$ is 101.
- The sum of the second and the next-to-last numbers in the sequence $(99 + 2)$ is 101, and so on.
- There are 50 pairs of numbers with that sum in the sequence.
- To find the sum of all the numbers from 1 through 100, multiply the number of pairs (50) by the sum of the numbers in each of pair (101) to obtain 5050.

► You can use a similar method to find the sums of other number sequences if you can find pairs of numbers that have the same sum. For example, find the sum of the even numbers from 2 through 400.

$2 + 4 + 6 + 8 + 10 + \ldots + 392 + 394 + 396 + 398 + 400$

- The sum of the first number and the last number in the sequence is 402.
- There are 100 pairs of numbers with that sum in the sequence.
- Multiply 402 by 100 to find the sum of the numbers in the sequence.

So the sum of the even numbers from 2 through 400 is 40,200.

## Try These

**1.** Find the sum of the numbers from 1 through 25,000.

**2.** Find the sum of the odd numbers from 5 through 95.

**3.** Find the sum of the multiples of 3 from 3 through 300.

**4.** **Discuss and Write** Choose problem 2 or 3 and tell how you found your answer.

Go to PRACTICE BOOK page 25 for exercise sets.

# Test Prep: Multiple-Choice Questions

## Strategy: Understand Distractors

Multiple-choice questions have one correct answer choice and several incorrect answer choices called distractors. The incorrect answer choices are often common errors. They are called distractors because they are believable and can distract you from selecting the correct answer.

Look at the sample test item.

**Sample Test Item**

Simplify the expression below.

$(5 + 5)^2 \div 2 - 5 \bullet 3$

**A.** 135
**B.** 35
**C.** 10
**D.** −5

**Read the whole test item, including the answer choices.**

- Underline important words.
  Simplify the expression.
  To *simplify* the expression, use the order of operations.
- Restate the question in your own words.
  What answer do you get when you simplify $(5 + 5)^2 \div 2 - 5 \bullet 3$?

**Test-Taking Tips**

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Solve the problem.**

- Apply appropriate rules, definitions, and properties.
  Use the order of operations to simplify the expression.

**Think**

Order of Operations

| | |
|---|---|
| **1.** Grouping Symbols | **2.** Exponents |
| **3.** Multiply or divide from left to right. | **4.** Add or subtract from left to right. |

$(5 + 5)^2 \div 2 - 5 \bullet 3$ ← Compute within parentheses.

$10^2 \div 2 - 5 \bullet 3$ ← Simplify the exponent.

$100 \div 2 - 5 \bullet 3$ ← Multiply or divide from left to right.

$50 - 15$ ← Subtract.

$35$

**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices. Watch out for distractors.

**A.** 135 ← Did not follow the order of operations, just worked from left to right. Eliminate this choice.

**B.** 35 ← The exponent was simplified correctly and the order of operations was used. This is the correct choice!

**C.** 30 ← Did not follow the order of operations. Eliminate this choice.

**D.** −5 ← Did not simplify the exponent correctly. $10^2$ was simplified as 20, not 100. Eliminate this choice.

**Try These** Item 1 is partially worked out for you.

**Choose the correct answer.**

**1.** Which of the following statements is *not* true?

**A.** $5(12) = 5(10) + 5(2)$

**B.** $(5)(5)(5) = 3(5)$

**C.** $(5 + 12) + 6 = 5 + (12 + 6)$

**D.** $-5 + 12 = 12 + (-5)$

**Read the whole test item, including the answer choices.**

- Underline important words.
  Which of the following statements is *not* true?
- Restate the question in your own words.
  Which statement is false?

**Solve the problem.**

- Apply appropriate rules, properties, and definitions.
  Find the answer choice that does *not* correctly apply properties or rules. (Remember that you are looking for the statement that is *not* true.)
  *Hint:* If you do not remember the property or rule, you can evaluate both sides of the statements given in the answer choices.

### Test-Taking Tips

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Item Analysis**

**Choose the answer. Explain how you eliminated answer choices.**

- Analyze and eliminate answer choices.

**A.** $5(12) = 5(10) + 5(2)$

**B.** $(5)(5)(5) = 3(5)$

**C.** $(5 + 12) + 6 = 5 + (12 + 6)$

**D.** $-5 + 12 = 12 + (-5)$

**2.** If $a$ is a positive integer and $b$ is a negative integer, which will *always* be positive?

**F.** $a + b$

**G.** $a - b$

**H.** $a \times b$

**J.** $a \div b$

**3.** What is the value of $3^2 + 3^3$?

**A.** 15

**B.** 30

**C.** 36

**D.** 243

**4.** Which of the following ordered pairs represents a point in the second quadrant?

**F.** $(3, -2)$

**G.** $(-3, -2)$

**H.** $(3, 2)$

**J.** $(-3, 2)$

**5.** What is the absolute value of $-8$?

**A.** $-8$

**B.** $-\frac{1}{8}$

**C.** $\frac{1}{8}$

**D.** 8

Go to PRACTICE BOOK pages 27–28 for exercise sets.

# Expressions and Equations

CHAPTER 2

## In This Chapter You Will:

- Write and simplify numerical and algebraic expressions
- Solve addition, subtraction, multiplication, and division equations
- Model and solve two-step equations
- Use a calculator to key in and solve formulas
- Apply the strategy: *Organize Data*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- Negative integers are less than 0. Zero is neither positive nor negative. Positive integers are greater than 0.
- Any number on a horizontal number line is greater than any number to its left.
- The additive inverse of an integer is the integer that is the same distance from 0 on a number line but is located on the opposite side of 0.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 33–60

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

### Critical Thinking

Cedric downloaded 10 blues songs and 15 rock songs. He also downloaded two EPs containing 4 songs each. Later he erased 2 of the blues songs, 3 of the rock songs, and one of the EPs. If Cedric already had 200 songs on his computer, how many songs did he have when he finished downloading and erasing songs?

Algebra 2-1

# Mathematical Expressions

**Objective** To translate word phrases into numerical expressions or algebraic expressions • To translate numerical expressions or algebraic expressions into word phrases

Anna, Bonita, Carla, and Dina are playing basketball. In a game, Carla's score of 6 points is three times Dina's score. Anna's score is 3 points less than Bonita's. Write a *mathematical expression* to express each girl's score.

**Key Concept**

**Expressions and Variables**

A mathematical expression can be a numerical expression or an algebraic expression.

- A numerical expression contains numerals and indicates at least one mathematics operation.
- An algebraic expression contains one or more variables and mathematics operations. It may also contain numerals. A variable is a symbol, usually a letter, used to represent a number.
- A mathematical expression does *not* contain the symbols $<$, $=$, or $>$.

Carla's score → $6$ ← numerical expression

Dina's score → $6 \div 3$ or $\frac{6}{3}$ ← numerical expressions

Since Bonita's score is not given, you cannot write a numerical expression for Bonita's score or for Anna's score. However, you can write an algebraic expression. Choose a variable and explain what it means.
Let $b$ = Bonita's score.

Bonita's score → $b$ ← algebraic expression

Anna's score → $b - 3$ ← algebraic expression

3 points less than Bonita's

► You can write mathematical expressions from word phrases.

| Word Phrase | Mathematical Expression | Expression Type |
|---|---|---|
| four times the sum of five and zero | $4(5 + 0)$ | numerical |
| two more than the product of 8 squared and 10 | $(8^2 \bullet 10) + 2$ | numerical |
| half the difference between 12 and 7 | $\frac{1}{2}(12 - 7)$ or $\frac{12 - 7}{2}$ | numerical |
| one third the sum of a number *n* and 7 | $\frac{1}{3}(n + 7)$ or $\frac{n + 7}{3}$ | algebraic |
| the sum of a number *n* and 8 divided by 2 | $\frac{n + 8}{2}$ | algebraic |
| 4 more points than Cal scored ($c$) | $c + 4$ | algebraic |
| twice as many DVDs as Bob has ($d$) | $2d$ | algebraic |

You can also write word phrases from mathematical expressions.

| Mathematical Expression | Word Phrases |
|---|---|
| Addition Expression $-\|-3\| + a$ | • the opposite of the absolute value of negative 3 plus $a$<br>• the sum of the opposite of the absolute value of negative 3 and $a$<br>• the opposite of the absolute value of negative 3 increased by $a$<br>• $a$ added to the opposite of the absolute value of negative 3<br>• $a$ more than the opposite of the absolute value of negative 3 |
| Subtraction Expression $\|b\| - 7$ | • the difference between the absolute value of $b$ and 7<br>• 7 subtracted from the absolute value of $b$<br>• the absolute value of $b$ minus 7<br>• 7 less than the absolute value of $b$<br>• the absolute value of $b$ less 7<br>• the absolute value of $b$ decreased by 7 |
| Multiplication Expression $5 \bullet n^3$ or $5n^3$ | • 5 times $n$ cubed<br>• 5 times $n$ to the third power<br>• the product of 5 and $n$ cubed<br>• $n$ cubed multiplied by 5 |
| Division Expression $-9 \div x$ or $\frac{-9}{x}$ | • negative nine divided by $x$<br>• the quotient of negative 9 and $x$ |

## Try These

**Write each as a numerical expression.**

**1.** the sum of 1.5 and 3
**2.** 8 increased by $-5$
**3.** half the product of 6 and 3

**Write each as an algebraic expression. Use *n* as the variable.**

**4.** a number decreased by 3.5
**5.** 10 divided by twice a number
**6.** $-7$ less than half a number
**7.** two years younger than Bailey
**8.** one third of the students
**9.** 4 multiplied by the sum of a number and 5
**10.** the difference when a number doubled is subtracted from 3

**Write a word phrase for each expression.**

**11.** $-3n$
**12.** $\frac{x - 12}{2}$
**13.** $4a - |-6|$
**14.** $5(a + 1)$
**15.** $\frac{10}{2n}$
**16.** $2n + 3$

**17. Discuss and Write** Does the phrase "5 subtracted from $x$" mean the same as the phrase "the difference between 5 and $x$"? Explain.

Go to PRACTICE BOOK Lesson 2-1 for exercise sets.

Algebra

# 2-2 Simplify and Evaluate Algebraic Expressions

**Objective** To translate one- and two-step verbal expressions into algebraic expressions • To evaluate algebraic expressions • To simplify algebraic expressions by combining like terms • To simplify algebraic expressions by using properties

Elena normally practices on her keyboard for a certain number of hours each week. In preparation for her concert, she plans to increase her weekly practice time by 4 hours. The week before the concert she plans to double that time. What algebraic expression represents the numbers of hours she plans to practice the week before her concert?

► To write an algebraic expression, choose a variable to represent the unknown. Then write the algebraic expression to represent the situation.

Let $h$ = the number of hours Elena normally practices.

double $\quad 2 \bullet (h + 4) \quad$ increase of 4 hours

So the algebraic expression $2 \bullet (h + 4)$ or $2(h + 4)$ represents the number of hours Elena will practice the week before her concert.

► If a value of a variable is given, you can *evaluate* an algebraic expression. To evaluate the algebraic expression, substitute the value given for the variable and compute according to the order of operations.

If Elena normally practices 5 hours a week, how many hours will she practice the week before her concert?

$2(h + 4)$

$2(5 + 4)$ ← Substitute 5 for $h$.

$2(9)$ ← Follow the Order of Operations. Simplify the expression within the parentheses first.

$18$

Elena will practice playing on her keyboard for 18 hours the week before her concert.

## Examples

**1** Evaluate $\frac{a + b}{c}$ for $a = 12$, $b = 3$, and $c = -5$.

$\frac{a + b}{c} = \frac{12 + 3}{-5}$ ← Substitute the given values.

$= \frac{15}{-5}$ ← Follow the Order of Operations.

$= -3$

**2** Evaluate $5x^3y^2$ for $x = 2$ and $y = -3$.

$5x^3y^2 = 5x^3 \bullet y^2$

$= 5(2)^3 \bullet (-3)^2$

$= 5(8) \bullet 9$

$= 40 \bullet 9$

$= 360$

**Think** $5x^3 = 5 \bullet x \bullet x \bullet x$

► An algebraic expression can have one or more **terms**. Terms are separated by plus or minus signs. You can simplify an algebraic expression by combining *like terms*. **Like terms** are those with the same variables raised to the same power. Terms with no variables are also like terms. An expression is in **simplest form** when it has no like terms.

$3xy + 4 + 6xy$ ← 3 terms → $2x + 5xy - 7x^2$

like terms: $3xy$, $6xy$ — unlike terms: $2x$, $5xy$, $7x^2$

► A term that does not contain a variable is called a **constant**.
The numerical factor of a term containing a variable is called a **coefficient**.

5 is the coefficient of $5x$. 1 is the coefficient of $x$.

$5x + 3 + x - 8^2$ constants

► It is sometimes necessary to use properties to simplify an expression.

## Examples

**1** Simplify: $5x - 2y + 3x + 8$

$5x + 3x - 2y + 8$ ← Use the Commutative Property.

$8x - 2y + 8$ ← Combine like terms $(5x + 3x)$.

**2** Simplify: $-9(x - y) + 3y$

$-9x - (-9y) + 3y$ ← Use the Distributive Property.

$-9x + 9y + 3y$ ← $-(-9y) = +9y$

$-9x + 12y$ ← Combine like terms $(9y + 3y)$.

**3** Simplify: $4n + 3n + |-5| + 2n + 6$

$4n + 3n + 2n + |-5| + 6$ ← Use the Commutative Property.

$9n + 11$ ← Combine like terms $(4n + 3n + 2n)$ and $(5 + 6)$.

**4** Simplify: $31 + 9n^2s + 5 + n^2s$

$31 + 5 + 9n^2s + n^2s$ ← Use the Commutative Property.

$31 + 5 + (9 + 1)n^2s$ ← Use the Distributive Property.

$36 + 10n^2s$ ← Combine like terms $(31 + 5)$ and $(9 + 1)$.

## Try These

**Evaluate each expression when $a = 3$, $b = -2$, and $c = 5$.**

**1.** $abc$  **2.** $a^2 - |-c|$  **3.** $c^3 - 4ab^2$  **4.** $b(a + c)$

**Simplify each expression by combining like terms.**

**5.** $3m^2n + |-8| + 2m^2n$  **6.** $4w - w + 3$  **7.** $xy^2 + 5 + 3xy^2 - 2xy^2 + |-7|$

**8. Discuss and Write** When evaluating an expression, is it easier to combine terms first or to substitute values first? Justify your answer with examples.

Go to PRACTICE BOOK Lesson 2-2 for exercise sets.

Algebra

# 2-3 Equations

**Objective** To identify algebraic equations and numerical equations
• To identify equations as true or false, closed or open

Mr. Chu is a computer programmer. He charges \$50 per hour for his services. If he works for 3 hours, how much will he earn?

► To find how much Mr. Chu will earn, write an *equation*.

**Key Concept**

**Equations**

An equation is a statement that contains an equal sign (=) showing that two mathematical expressions are equal.

An equation can be either numerical or algebraic.

A numerical equation contains only numbers and operations.

An algebraic equation contains one or more variables and at least one mathematics operation. It may also contain numbers.

Let $n$ = the number of dollars Mr. Chu will earn.

hours worked   hourly rate

$3 \bullet \$50 = n$ ← amount Mr. Chu will earn

$3 \bullet \$50 = n$ ← algebraic equation

$\$150 = n$

Mr. Chu will earn \$150 for 3 hours of work.

► You can write numerical and algebraic equations from word sentences.

| Word Sentence | Equation | Equation Type |
|---|---|---|
| Two squared plus six equals ten. | $2^2 + 6 = 10$ | numerical |
| The opposite of three decreased by negative ten is seven. | $-3 - (-10) = 7$ | numerical |
| The product of six and five is equal to thirty. | $6(5) = 30$ | numerical |
| The difference of two to the third power and negative four is twelve. | $2^3 - (-4) = 12$ | numerical |
| A number ($y$) divided by nine is equal to four. | $\frac{y}{9} = 4$ | algebraic |
| Eighty-one is equal to ten less than the square of a number ($x$). | $81 = x^2 - 10$ | algebraic |
| The absolute value of negative six plus a number ($n$) is equal to six. | $\|-6\| + n = 6$ | algebraic |
| The opposite of the absolute value of negative three equals three plus a number ($z$). | $-\|-3\| = 3 + z$ | algebraic |

▶ An equation can be a *closed sentence* or an *open sentence*.

- A closed sentence is a numerical statement. It does *not* contain variables. Closed sentences are either true or false.

| | |
|---|---|
| $6 + 4 \stackrel{?}{=} 10$ | $6 + 5 \stackrel{?}{=} 12$ |
| $10 = 10$ | $11 \neq 12$ ≠ means "is not equal to." |
| So $6 + 4 = 10$ is true. | So $6 + 5 = 12$ is false. |

- An open sentence is an algebraic statement. It *does* contain one or more variables. Open sentences are neither true nor false.

$a + 2 = 7$ is an open sentence.

▶ You can determine whether a given value for a variable is a *solution* of an equation. A solution is a value for the variable that makes an algebraic equation true. An algebraic equation may have more than one solution.

Determine whether $x = 4$ and $x = -5$ are solutions of the equation $x^2 + 4 = 20$.

| | |
|---|---|
| $4^2 + 4 \stackrel{?}{=} 20$ ← Substitute 4 for $x$. | $(-5)^2 + 4 \stackrel{?}{=} 20$ ← Substitute −5 for $x$. |
| $16 + 4 \stackrel{?}{=} 20$ ← Simplify. | $25 + 4 \stackrel{?}{=} 20$ ← Simplify. |
| $20 = 20$ True | $29 \neq 20$ |
| 4 is a solution of the equation. | −5 is not a solution of the equation. |

## Try These

**Write an equation for each. Label each equation as *numerical* or *algebraic*.**

**1.** Six less than the absolute value of a number is equal to twelve.

**2.** Four times negative five is equal to negative twenty.

**3.** One more than twice the opposite of a number is equal to six.

**4.** A number divided by 7 is 21.

**5.** Toni is 3 feet tall. She is half the height of her father.

**Identify whether the equation is an *open* or *closed* sentence.**

**6.** $n + 8 = 9$

**7.** $\frac{20}{4} + 5 = 10$

**8.** $x^2 = 14$

**Determine whether the given value for the variable is a solution of the equation. Write *true* or *false*. Show the steps to support your conclusion.**

**9.** $-3x = 18$ when $x = -6$

**10.** $53 - 26 = y$ when $y = 25$

**11.** $\frac{-36}{z} = -|18|$ when $z = 2$

**12.** Discuss and Write Write two word sentences for the equation $2x + 4 = -24$.

Go to PRACTICE BOOK Lesson 2-3 for exercise sets.

Algebra 2-4

Update your skills. See page 409 VI.

# Solve Addition Equations

**Objective** **To apply the Subtraction Property of Equality to solve algebraic addition equations with integers • To combine numerical terms to simplify addition equations with integers**

PROBLEM-SOLVING STRATEGY 4

If Baltimore's Fort McHenry Tunnel were 1176 feet longer, it would be as long as New York's Brooklyn Battery Tunnel. The Brooklyn Battery Tunnel is 9117 feet long. How long is the Fort McHenry Tunnel?


► To find the length of the Fort McHenry Tunnel, write and solve an addition equation.

Let $x$ = the length of the Fort McHenry Tunnel.

| Fort McHenry Tunnel (ft) | | 1176 feet longer | | Brooklyn Battery Tunnel (ft) | |
|---|---|---|---|---|---|
| $x$ | $+$ | $1176$ | $=$ | $9117$ | ← addition equation |

► Use the Subtraction Property of Equality to solve an addition equation. Addition and subtraction are inverse operations because they "undo" each other.

**Key Concept**

**Subtraction Property of Equality**
When you subtract the same number from both sides of an equation, you get a true statement.
If $a = b$, then $a - c = b - c$.

**Solve:** $x + 1176 = 9117$

$x + 1176 - 1176 = 9117 - 1176$ ← Subtract 1176 from both sides to "undo" the addition and to isolate $x$.

$x = 7941$

**Check:** $x + 1176 = 9117$

$7941 + 1176 \stackrel{?}{=} 9117$ ← Substitute 7941 for $x$.

$9117 = 9117$ True

So 7941 is the solution of the addition equation $x + 1176 = 9117$.

The Fort McHenry Tunnel is 7941 feet long.

**Think**
Whenever you subtract a number from one side of an equation, you must subtract the same number from the other side to keep the equation in balance.
The new equation is equivalent to the original equation.

## Example

**1** **Solve:** $x + (-5) = 12$

$x + (-5) - (-5) = 12 - (-5)$ ← Subtract $-5$ from both sides to isolate $x$.

$x = 17$

**Check:** $x + (-5) = 12$

$17 + (-5) \stackrel{?}{=} 12$ ← Substitute 17 for $x$.

$12 = 12$ True

► Equations that have the same solution are called equivalent equations.

$x + 1176 = 9117$ and $x = 7941$ are equivalent equations.

▶ Whether the variable in an addition equation is on the right or left side of the equal sign, you use the Subtraction Property of Equality to solve the equation.

**Solve:** $|-6| = 24 + x$

**Remember:** $|-6| = 6$

$6 - 24 = 24 - 24 + x$ ← Subtract 24 from both sides to isolate $x$.

$-18 = x$

**Check:** $|-6| = 24 + x$

$|-6| \stackrel{?}{=} 24 + (-18)$ ← Substitute $-18$ for $x$.

$6 = 6$ True

▶ Sometimes you need to combine like numerical terms when solving an equation.

**Solve:** $12 + d + 5 + (-8) = 41$

$d + 12 + 5 + (-8) = 41$ ← Use the Commutative Property.

$d + 9 = 41$ ← Combine like numerical terms.

$d + 9 - 9 = 41 - 9$ ← Subtract 9 from both sides to isolate $d$.

$d = 32$

**Check:** $12 + d + 5 + (-8) = 41$

$12 + 32 + 5 + (-8) \stackrel{?}{=} 41$ ← Substitute 32 for $d$.

$41 = 41$ True

▶ You can use estimation and mental math to check if a solution is reasonable.

**Solve:** $436 + x = 575$

First estimate by rounding: $440 + x = 580$ ← 436 rounds up to 440, and 575 rounds up to 580.

$440 + 140 = 580$ ← Think: 440 plus what number is 580?

$x \approx 140$ ← $\approx$ means "is approximately equal to"

Then solve: $436 + x = 575$

$436 - 436 + x = 575 - 436$ ← Subtract 436 from both sides to isolate $x$.

$x = 139$

**Think**
139 is close to the estimate of 140. The answer is reasonable.

## Try These

**Solve and check.**

**1.** $x + 9 = 21$ **2.** $52 + s = 37$ **3.** $12 = y + (-2)$ **4.** $|-7| + w + 1 = -11$

**Estimate each solution. Then find the actual solution of the equation.**

**5.** $m + 67 = 77$ **6.** $46 + h = 83$ **7.** $-85 = k + (-4)^2$ **8.** $4 + z + 7 = 0$

**9. Discuss and Write** Compare and contrast the steps you would use to solve $x + 5 = 12$ and $x + (-5) = 12$. Explain your answer.

Go to PRACTICE BOOK Lesson 2-4 for exercise sets.

Algebra

2-5

Update your skills. See page 409 VI.

# Solve Subtraction Equations

**Objective** **To apply the Addition Property of Equality to solve algebraic subtraction equations with integers • To write a related sentence to solve for the subtrahend • To combine numerical terms to simplify subtraction equations with integers**

Lowell Middle School held a vote on whether or not to change the name of the school basketball team to the Gold Stars. Forty-two students were absent at the time of the vote. The 914 students present all voted "Yes." How many students are enrolled at Lowell Middle School?

► To find the number of students enrolled, write and solve a subtraction equation.

Let $s$ = the number of students enrolled at Lowell Middle School.

| students enrolled | | students absent | is equal to | students present | |
|---|---|---|---|---|---|
| $s$ | $-$ | $42$ | $=$ | $914$ | ← subtraction equation |

► To solve a subtraction equation, use the Addition Property of Equality. Remember that addition is the inverse operation of subtraction.

**Key Concept**

**Addition Property of Equality**
When you add the same number to both sides of an equation, you get a true statement.
If $a = b$, then $a + c = b + c$.

**Solve:** $s - 42 = 914$

$s - 42 + 42 = 914 + 42$ ← Add 42 to both sides to isolate $s$.

$s = 956$

**Check:** $s - 42 = 914$

$956 - 42 \stackrel{?}{=} 914$ ← Substitute 956 for $s$.

$914 = 914$ True

So 956 is the solution of the subtraction equation $s - 42 = 914$. There are 956 students enrolled in Lowell Middle School.

**Think**
Whenever you add a number to one side of an equation, you must add the same number to the other side to keep the equation in balance.
The new equation is equivalent to the original equation.

► When the number being subtracted is negative, follow the rules for subtracting integers. Then use the Subtraction Property of Equality.

**Remember:** To subtract an integer, add the opposite, or additive inverse, of the number being subtracted.

**Solve:** $m - (-8) = 34$

$m + 8 = 34$ ← Rewrite the equation by following the rules for subtracting integers.

$m + 8 - 8 = 34 - 8$ ← Use the Subtraction Property of Equality.

$m = 26$

**Check:** $m - (-8) = 34$

$26 - (-8) \stackrel{?}{=} 34$ ← Substitute 26 for $m$.

$26 + 8 \stackrel{?}{=} 34$

$34 = 34$ True

►Sometimes you need to simplify expressions before solving an equation.

## Examples

**1** **Solve:** $2^2 = x - 15$

$4 = x - 15$ ← Simplify the power.

$4 + 15 = x - 15 + 15$ ← Add 15 to both sides to isolate $x$.

$19 = x$

**Check:** $2^2 = x - 15$

$4 \stackrel{?}{=} 19 - 15$ ← Simplify the power and substitute 19 for $x$.

$4 = 4$ True

**2** **Solve:** $n - 4 - 9 = 13$

$n - 4 - 9 = 13$ ← Combine like terms.

$n - 13 = 13$ ← Simplify.

$n - 13 + 13 = 13 + 13$ ← Add 13 to both sides to isolate $n$.

$n = 26$

**Check:** $n - 4 - 9 = 13$

$26 - 4 - 9 \stackrel{?}{=} 13$ ← Substitute 26 for $n$.

$22 - 9 \stackrel{?}{=} 13$ ← Simplify.

$13 = 13$ True

►To solve an equation when the number being subtracted is missing, write a related subtraction sentence.

**Solve:** $9 - b = 7$

$9 - 7 = b$ ← Write a related sentence.

$2 = b$

**Check:** $9 - b = 7$

$9 - 2 \stackrel{?}{=} 7$ ← Substitute 2 for $b$.

$7 = 7$ True

►You can use estimation and mental math to check if the solution of an equation is reasonable.

**Solve:** $x - 21 = 117$

First estimate by rounding: $x - 20 = 120$ ← 21 rounds down to 20, and 117 rounds up to 120.

$140 - 20 = 120$ ← Think: What number minus 20 is 120?

$x \approx 140$

Then solve: $x - 21 = 117$

$x - 21 + 21 = 117 + 21$ ← Add 21 to both sides to isolate $x$.

$x = 138$

**Think**
138 is close to the estimate of 140.
The answer is reasonable.

## Try These

**Solve and check.**

**1.** $-7 - x = 12$  **2.** $g - (-14) = 11$  **3.** $11 = y - (-4)^2$  **4.** $-|-8| = f - 13$

**5.** **Discuss and Write** Are $x - (-3) = 9$ and $x = 6$ equivalent equations? Explain why or why not.

Go to PRACTICE BOOK Lesson 2-5 for exercise sets.

Update your skills. See pages 411 XI, 412 XII.

# Solve Multiplication Equations

**Objective** To apply the Division Property of Equality to solve algebraic multiplication equations with integers • To combine numerical terms to simplify multiplication equations

PROBLEM-SOLVING STRATEGY 1

Maya earns \$15 each time she works at the concession stand during a game. Her goal is to earn \$345 to buy the DVD player she wants. At how many games will Maya need to work in order to buy the DVD player?

► To find the number of games at which Maya needs to work, write and solve a multiplication equation.

Let $g$ = the number of games at which Maya will need to work.

| amount earned | | number of games | | cost of DVD player |
|---|---|---|---|---|
| \$15 | • | $g$ | = | \$345 ← multiplication equation |

► To solve a multiplication equation, use the Division Property of Equality. Multiplication and division are inverse operations because they "undo" each other.

**Solve:** $15g = 345$ **Remember:** 15 • $g$ means $15g$.

$$\frac{\cancel{15}^{1}g}{\cancel{15}_{1}} = \frac{\cancel{345}^{23}}{\cancel{15}_{1}}$$ ← Divide both sides by 15 to "undo" the multiplication and to isolate $g$. Simplify.

$$g = 23$$

**Check:** $15g = 345$

$15 \bullet 23 \stackrel{?}{=} 345$ ← Substitute 23 for $g$.

$345 = 345$ True

So 23 is the solution of the multiplication equation $15g = 345$. Maya needs to work at 23 games to earn enough money to buy the DVD player.

**Key Concept**

**Division Property of Equality**

When you divide both sides of an equation by the same nonzero number, the result is a true statement.

If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

**Think**

Whenever you divide one side of an equation by a number, you must divide the other side by the same number to keep the equation in balance.

The new equation is equivalent to the original equation.

## Examples

**1** **Solve:** $-a = 78$ **Remember:** $-a = -1 \bullet a$

$$\frac{-1 \bullet a}{-1} = \frac{78}{-1}$$ ← Divide both sides by $-1$ to isolate $a$.

$$\frac{-\cancel{1}^{1} \bullet a}{-\cancel{1}_{1}} = \frac{78}{-1}$$ ← Simplify.

$$a = -78$$

**Check:** $-a = 78$

$-(-78) \stackrel{?}{=} 78$ ← Substitute $-78$ for $a$.

$78 = 78$ True

**2** **Solve:** $|-54| = 9x$

$54 \div 9 = 9x \div 9$ ← Divide both sides by 9 to isolate $x$ and simplify.

$6 = x$

**Check:** $|-54| = 9x$

$54 \stackrel{?}{=} 9(6)$ ← Substitute 6 for $x$.

$54 = 54$ True

**3** Kaela opens a savings account and decides to deposit \$28 each week. How long will it take her to save \$700?

Let $w$ = the number of weeks.

**Solve:** $28w = 700$

$28 \bullet w \div 28 = 700 \div 28$ ← Divide both sides by 28 to isolate $w$. Then simplify.

$w = 25$

**Check:** $28w = 700$

$28(25) \stackrel{?}{=} 700$ ← Substitute 25 for $w$.

$700 = 700$ True

So it will take 25 weeks for Kaela to save \$700.

► Sometimes you need to combine like terms before solving an equation.

**Solve:** $n + 3n = 28$

$4n = 28$ ← Combine like terms $(1n + 3n)$.

$\frac{4 \bullet n}{4} = \frac{28}{4}$ ← Divide both sides by 4 to isolate $n$ and simplify.

$n = 7$

**Check:** $n + 3n = 28$

$7 + 3(7) \stackrel{?}{=} 28$ ← Substitute 7 for $n$.

$7 + 21 \stackrel{?}{=} 28$ ← Simplify.

$28 = 28$ True

## Try These

**Solve and check.**

**1.** $4x = 124$

**2.** $5s + s = -48$

**3.** $96 = -3y$

**4.** $-24x = |72|$

**5.** $6z + z = -29 - 13$

**6.** $144 = -12n$

**7.** $-225 = -14t - t$

**8.** $-x = 36$

**9.** $x - 3x = 32$

**10.** $3 + (-5) = 14z$

**11.** $(-2)^2 = -x$

**12.** $4x = -|24|$

**13. Discuss and Write** What property would you use to solve $-5a = 30$? Explain your reasoning.

Go to PRACTICE BOOK Lesson 2-6 for exercise sets.

Algebra

# 2-7 Solve Division Equations

**Objective** To apply the Multiplication Property of Equality to solve algebraic division equations with integers

PROBLEM-SOLVING STRATEGY 7

The Explorers basketball team scored an average of 68 points per game during seven games. What was the total number of points scored in the seven games?

► To find the total number of points scored in the seven games, write and solve a division equation.

Let $n$ = the total number of points scored in seven games.

$$n \div 7 = 68$$ ← division equation

7: number of games played; 68: average points scored per game

► To solve a division equation, use the Multiplication Property of Equality. Remember that multiplication and division are inverse operations.

**Key Concept**

**Multiplication Property of Equality**
When you multiply both sides of an equation by the same number, you get a true statement.
If $a = b$, then $ac = bc$.

**Solve:** $\frac{n}{7} = 68$ ← $n \div 7$ means $\frac{n}{7}$

$(7)\frac{n}{7} = 68(7)$ ← Multiply both sides by 7 to isolate $n$.

$\frac{7 \bullet n}{1 \bullet 7} = \frac{\cancel{7}^{1} n}{\cancel{7}_{1}} = 476$ ← Simplify.

$n = 476$

**Think**
Whenever you multiply one side of an equation by a number, you must multiply the other side by the same number to keep the equation in balance.
The new equation is equivalent to the original equation.

**Check:** $\frac{n}{7} = 68$

$\frac{476}{7} \stackrel{?}{=} 68$ ← Substitute 476 for $n$.

$\frac{\cancel{476}^{68}}{\cancel{7}_{1}} \stackrel{?}{=} 68$ ← Simplify.

$68 = 68$ True

So 476 is the solution of the division equation.
The Explorers scored a total of 476 points in seven games.

## Examples

**1** **Solve:** $-4 = \frac{n}{5}$

$(5)(-4) = \frac{n}{5}(5)$ ← Multiply both sides by 5 to isolate $n$.

$-20 = \frac{n}{\cancel{5}_{1}}(\cancel{5}^{1})$ ← Simplify.

$-20 = n$

**Check:** $-4 = \frac{n}{5}$

$-4 \stackrel{?}{=} \frac{-20}{5}$ ← Substitute $-20$ for $n$.

$-4 = -4$ True

**2** **Solve:** $\frac{x}{-12} = |-3|$

$(-12)\frac{x}{-12} = 3(-12)$ ← Multiply both sides by $-12$ to isolate $x$.

$(\cancel{-12})\frac{x}{\cancel{-12}} = -36$ ← Simplify.

$x = -36$

**Check:** $\frac{x}{-12} = |-3|$

$\frac{-36}{-12} \stackrel{?}{=} 3$ ← Substitute $-36$ for $x$.

$3 = 3$ True

**3** **Solve:** $z \div (-6) = -22$

$[z \div (-6)](-6) = -22(-6)$ ← Multiply both sides by $-6$ to isolate $z$.

$z = 132$

**Check:** $z \div (-6) = -22$

$132 \div (-6) \stackrel{?}{=} -22$ ← Substitute 132 for $z$.

$-22 = -22$ True

**4** **Solve:** $\frac{1}{3}a = 16$

$\frac{a}{3} = 16$ ← Write $\frac{1a}{3}$ as $\frac{a}{3}$.

$(3)\frac{a}{3} = 16(3)$ ← Multiply both sides by 3 to isolate $a$.

$a = 48$

**Check:** $\frac{1}{3}a = 16$

$\frac{1}{3}(48) \stackrel{?}{=} 16$ ← Substitute 48 for $a$.

$16 = 16$ True

**5** Leslie and 6 friends went out to dinner. The 7 friends divided the cost of the dinner equally. Leslie's share was \$48. What was the total cost of dinner?

Let $t$ = the total cost of the dinner.

**Solve:** $\frac{t}{7} = 48$

$(\cancel{7})\frac{t}{\cancel{7}} = (7)48$ ← Multiply both sides by 7 to isolate $t$. Simplify.

$t = 336$

**Check:** $\frac{t}{7} = 48$

$\frac{\cancel{336}^{48}}{\cancel{7}_{1}} \stackrel{?}{=} 48$ ← Substitute 336 for $t$. Simplify.

$48 = 48$ True

So the total cost of the dinner was \$336.

## Try These

**Solve and check. Use the Multiplication Property of Equality.**

**1.** $\frac{x}{12} = 3$ **2.** $y \div 8 = 56$ **3.** $5 = \frac{w}{-13}$ **4.** $93 = \frac{s}{-25}$

**5.** $\frac{m}{-7} = -9$ **6.** $\frac{e}{-11} = -11$ **7.** $\frac{1}{6}u = -40$ **8.** $\frac{a}{10} = -|-13|$

**9. Discuss and Write** Explain why you use the Multiplication Property of Equality to solve division equations. Give examples to justify your answer.

Go to PRACTICE BOOK Lesson 2-7 for exercise sets.

# Solve Two-Step Equations

**Objective** To model solving two-step algebraic equations with integers
• To solve two-step algebraic equations using the properties of equality

Isabel has \$15 to spend at the amusement park. Admission is \$3 and ride tickets are \$2 each. How many ride tickets can Isabel buy?

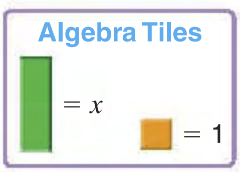

► To find how many ride tickets, write and solve a *two-step equation*.

A two-step equation is an equation that involves two operations.

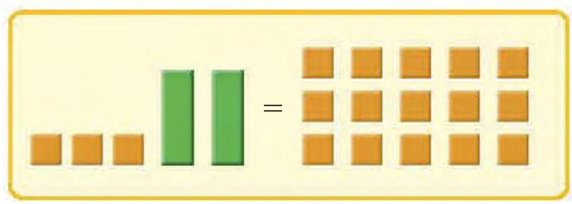
Let $x$ = the number of ride tickets.

| admission fee | plus | cost of ride tickets | is equal to | total cost | |
|---|---|---|---|---|---|
| \$3 | + | \$2$x$ | = | \$15 | ← two-step equation |

You can use algebra tiles to model and solve two-step equations.

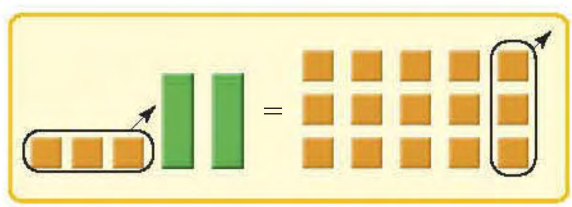



**1** Model $3 + 2x = 15$.



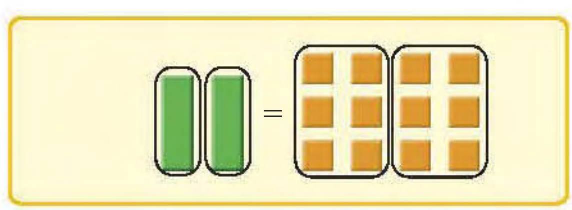
**2** Apply the Subtraction Property of Equality. Remove 3 yellow tiles from each side.

$$3 - 3 + 2x = 15 - 3$$



**3** Apply the Division Property of Equality. Divide the tiles on each side into 2 equal groups.

$$2x = 12$$
$$\frac{2x}{2} = \frac{12}{2}$$
$$x = 6$$



There are 6 yellow tiles in each group. Isabel can buy 6 ride tickets.

► You can solve two-step equations without using tiles. To solve a two-step equation, use inverse operations to isolate the variable. Often, the first step is to add or subtract to isolate the term that contains the variable. The next step is to multiply or divide to isolate the variable.

## Examples

**1** **Solve:** $6x - 16 = 8$

$6x - 16 + 16 = 8 + 16$ ← Use the Addition Property of Equality.

$6x = 24$ ← Simplify.

$6x \div 6 = 24 \div 6$ ← Use the Division Property of Equality.

$x = 4$

**Check:** $6x - 16 = 8$

$6(4) - 16 \stackrel{?}{=} 8$ ← Substitute 4 for $x$.

$24 - 16 \stackrel{?}{=} 8$ ← Simplify.

$8 = 8$ True

**2** **Solve:** $3 - \frac{x}{9} = 1$

$3 - 3 - \frac{x}{9} = 1 - 3$ ← Use the Subtraction Property of Equality.

$\frac{x}{-9} = -2$ $\boxed{-\frac{x}{9} = \frac{-x}{9} = \frac{x}{-9}}$

$(\cancel{-9}^{1})\frac{x}{\cancel{-9}_{1}} = (-9)(-2)$ ← Use the Multiplication Property of Equality.

$x = 18$

**Check:** $3 - \frac{x}{9} = 1$

$3 - \frac{18}{9} \stackrel{?}{=} 1$ ← Substitute 18 for $x$.

$3 - 2 \stackrel{?}{=} 1$ ← Simplify.

$1 = 1$ True

**3** **Solve:** $\frac{54}{x} = 6$

$x\left(\frac{54}{x}\right) = 6x$ ← Multiply both sides by $x$ so the variable will not be below the division bar.

$54 = 6x$ ← Simplify.

$\frac{54}{6} = \frac{6 \bullet x}{6}$ ← Divide both sides by 6 to isolate $x$.

$9 = x$

**Check:** $\frac{54}{x} = 6$

$\frac{54}{9} \stackrel{?}{=} 6$ ← Substitute 9 for $x$.

$6 = 6$ True

## Try These

**Solve and check.**

1. $-7x - 12 = 16$
2. $\frac{n}{3} + 9 = 12$
3. $\frac{63}{x} = 9$
4. $21 = 8x + 5$
5. $34 = 10 + 4c$
6. $\frac{t}{-2} - 6 = -1$
7. **Discuss and Write** Are these two equations the same: $\frac{54}{x} = 6$ and $54\left(\frac{1}{x}\right) = 6$? Explain your answer.

Go to PRACTICE BOOK Lesson 2-8 for exercise sets.

Algebra 2-9

# Formulas

**Objective** To find missing values in problems involving formulas

A terrarium is 60 centimeters long, 30 centimeters wide, and 40 centimeters tall. What is the volume, in cubic centimeters, of the terrarium?

To find how many cubic centimeters of space is inside the terrarium, use the formula for the volume of a rectangular prism.

► A formula is an equation, or rule, that shows a mathematical relationship between two or more quantities.

Each variable in the formula represents a part of the problem that can change. Each constant represents a number that does not change.

Use the formula for the volume of a rectangular prism.

Volume = length × width × height

$$V = \ell \bullet w \bullet h$$

$$= 60 \text{ cm} \bullet 30 \text{ cm} \bullet 40 \text{ cm}$$ ← Substitute the numbers given. Then solve.

$$= 72{,}000 \text{ cubic centimeters}$$

The volume of the terrarium is 72,000 cubic centimeters.

► You can use formulas to find missing dimensions of figures.

## Examples

For Examples 1 and 2, use the formula for the perimeter of a rectangle.

**1** Find $w$ when $P = 44$ cm and $\ell = 13$ cm.

**Solve:** $P = 2\ell + 2w$

$44 = 2(13) + 2w$ ← Substitute the known values.

$44 = 26 + 2w$ ← Simplify.

$44 - 26 = 26 - 26 + 2w$ ← Use the Subtraction Property of Equality.

$18 = 2w$ ← Simplify.

$18 \div 2 = 2w \div 2$ ← Use the Division Property of Equality.

$9 = w$

**Check:** $44 = 2(13) + 2w$

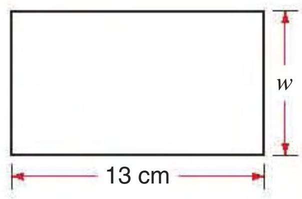
$44 \stackrel{?}{=} 2 \bullet 13 + 2 \bullet 9$ ← Substitute 9 for $w$.

$44 \stackrel{?}{=} 26 + 18$

$44 = 44$ True

So the width is 9 cm.

**Remember:**
**Perimeter of a Rectangle:**
$P = 2\ell + 2w$
or
$P = 2(\ell + w)$



**2** Find $\ell$ when $P = 28$ in. and $w = 8$ in.

**Solve:** $P = 2\ell + 2w$

$28 = 2\ell + 2(8)$ ← Substitute the known values.

$28 = 2\ell + 16$ ← Simplify.

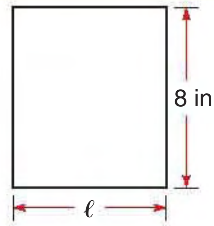
$28 - 16 = 2\ell + 16 - 16$ ← Use the Subtraction Property of Equality.

$12 = 2\ell$

$12 \div 2 = 2\ell \div 2$ ← Use the Division Property of Equality.

$6 = \ell$

So the length is 6 in.




**Check:** $28 = 2\ell + 2 \bullet 8$

$28 \stackrel{?}{=} 2 \bullet 6 + 2 \bullet 8$ ← Substitute 6 for $\ell$.

$28 \stackrel{?}{=} 12 + 16$

$28 = 28$ True

For Examples 3 and 4, use the formula for the area of a rectangle.

**3** Find $w$ when $A = 55\text{ mm}^2$ and $\ell = 11$ mm.

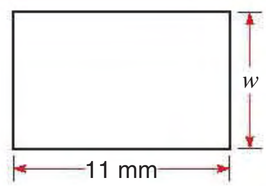
**Solve:** $A = \ell w$

$55 = 11w$ ← Substitute the known values.

$\frac{55}{11} = \frac{11w}{11}$ ← Use the Division Property of Equality.

$5 = w$

So the width is 5 mm.




**Remember: Area of a Rectangle:** $A = \ell w$

**Check:** $55 = 11 \bullet w$

$55 \stackrel{?}{=} 11 \bullet 5$ ← Substitute 5 for $w$.

$55 = 55$ True

**4** Find $\ell$ when $A = 42\text{ ft}^2$ and $w = 3$ ft.

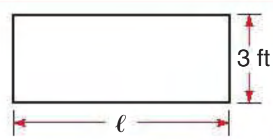
**Solve:** $A = \ell w$

$42 = \ell \bullet 3$ ← Substitute the known values.

$42 \div 3 = 3\ell \div 3$ ← Use the Division Property of Equality.

$14 = \ell$

So the length is 14 ft.




**Check:** $42 = \ell \bullet 3$

$42 \stackrel{?}{=} 14 \bullet 3$ ← Substitute 14 for $\ell$.

$42 = 42$ True

## Try These

**Solve for the missing value. Use the volume, perimeter, and area formulas.**

**1.** Find $V$ when $\ell = 6$ ft, $w = 3$ ft, and $h = 9$ ft.

**2.** Find $w$ when $A = 117\text{ in.}^2$ and $\ell = 9$ in.

**3.** Find $\ell$ when $P = 48$ cm and $w = 6$ cm.

**4.** **Discuss and Write** Given $V = \ell wh$, describe the steps for finding the height if you know the values for $V$, $\ell$, and $w$. Show the steps to justify your answer.

Go to PRACTICE BOOK Lesson 2-9 for exercise sets.

# 2-10 Problem-Solving Strategy: Organize Data

**Objective** To solve problems using the strategy *Organize Data*

**Problem 1:** Alice, Bob, Colleen, and Derek each have a different job. Their jobs are zookeeper, trailer-truck driver, math teacher, and cab driver. Use the following clues to determine each person's job.
**Clue 1:** Alice does not have a driver's license.
**Clue 2:** Derek spends most of his workday standing up.
**Clue 3:** Derek is allergic to animals.
**Clue 4:** Colleen has never driven a vehicle with more than four wheels.

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Four people have four different jobs.
Clues about the people and jobs are given.

**Question:** What is each person's job?

**Problem-Solving Strategies**
1. Guess and Test
2. **Organize Data**
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**
You can use the strategy *Organize Data*.
Make a chart showing all the possibilities.
Then use the process of elimination to find the answer.

**Solve** **Apply the strategy.**
Make a chart like the ones below.

Using Clue 1, you know that Alice is not the trailer-truck driver or the cab driver. Write *N*s (for *No*) in the corresponding cells.

Clues 2 and 3 tell you that Derek is not the trailer-truck driver, zookeeper, or cab driver. Record this information. So Derek must be the math teacher. Write a *Y* (for *Yes*) in the appropriate cell and *N*s in the rest of the cells in that column.

| | Zookeeper | Trailer-truck Driver | Math Teacher | Cab Driver |
|---|---|---|---|---|
| Alice | | N | N | N |
| Bob | | | N | |
| Colleen | | | N | |
| Derek | N | N | Y | N |

Record *Y* in the empty cell in Alice's row and *N*s in the rest of the cells in the zookeeper column. Colleen is not the trailer-truck driver (clue 4). Record this information.

You can now see that Colleen must be the cab driver and Bob must be the trailer-truck driver.

| | Zookeeper | Trailer-truck Driver | Math Teacher | Cab Driver |
|---|---|---|---|---|
| Alice | Y | N | N | N |
| Bob | N | Y | N | N |
| Colleen | N | N | N | Y |
| Derek | N | N | Y | N |

**Check** **Check to make sure your answer makes sense.**
Compare the results in your table to the clues.
**Clue 1:** Alice is a zookeeper, so she does not need a driver's license.
**Clue 2:** Derek is a math teacher, so he probably spends much of his day standing.
**Clue 3:** Derek does not have to work with animals in his job as a math teacher.
**Clue 4:** Colleen drives a cab, which has only four wheels.

**Problem 2:** Quinn and Sherrod are competing in a checkers tournament. The first player to win either two games in a row or a total of three games wins the match. In how many different ways can the match be played?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Quinn and Sherrod are competing in checkers. The first person to win either two consecutive games or a total of three games wins the match.

**Question:** In how many different ways can their match be played?

**Plan** **Select a strategy.**
You can use the strategy *Organize Data* to make a chart showing all the possibilities.

**Solve** **Apply the strategy.**

• First list all the possibilities in which Quinn wins the first game.

| 1st game | 2nd game | 3rd game | 4th game | 5th game | |
|---|---|---|---|---|---|
| Quinn | Quinn | | | | Quinn wins the match! |
| Quinn | Sherrod | Sherrod | | | Sherrod wins the match! |
| Quinn | Sherrod | Quinn | Quinn | | Quinn wins the match! |
| Quinn | Sherrod | Quinn | Sherrod | Quinn | Quinn wins the match! |
| Quinn | Sherrod | Quinn | Sherrod | Sherrod | Sherrod wins the match! |

• Next list all the possibilities in which Sherrod wins first.

| 1st game | 2nd game | 3rd game | 4th game | 5th game | |
|---|---|---|---|---|---|
| Sherrod | Sherrod | | | | Sherrod wins the match! |
| Sherrod | Quinn | Quinn | | | Quinn wins the match! |
| Sherrod | Quinn | Sherrod | Sherrod | | Sherrod wins the match! |
| Sherrod | Quinn | Sherrod | Quinn | Sherrod | Sherrod wins the match! |
| Sherrod | Quinn | Sherrod | Quinn | Quinn | Quinn wins the match! |

So there are 10 possible ways in which the match can be played.

**Check** **Check to make sure your answer makes sense.**
The least number of games possible in the match is 2, and the greatest number is 5.
Check your list. Did you list all the possibilities for the matches? Yes ✓
How many matches are settled after 2 games? 2 After 3 games? 2
After 4 games? 2 After 5 games? 4
Are there 10 games in all? Yes, $2 + 2 + 2 + 4 = 10$ ✓

Go to PRACTICE BOOK Lesson 2-10 for exercise sets.

# Enrichment:
## Be a Math Magician

**Objective** **To use expressions and equations • To explain a number trick algebraically**

Some magic tricks have more to do with math than with magic. A magician told Mr. Green that she would read his mind if he followed her instructions carefully.

► "Choose a number from 2 through 9. Choose 2, choose 9, or anything in between. Are you ready? Good. Multiply your number by the sum of 6 plus 3. This gives you a new number. Add its digits. Subtract 3 from that sum to find your key number."

"Take your key number, and change it to a letter of the alphabet this way. For 1, the letter is A; for 2, it is B, and so on. Find your letter and think of two names: the name of a state that begins with that letter and the name of a large animal that begins with the last letter of the state's name. Think hard, so I can read your mind."

"Mr. Green, you are thinking of an alligator in Florida, aren't you?" Mr. Green couldn't believe it! The magician was right. Did she really read his mind?

► If you analyze the trick carefully, you will find that it is really very simple.

- The sum of $6 + 3$ is 9. The product of 9 and any number from 2–9 is a two-digit number whose digit sum is 9.
- Subtracting 3 from 9 gives 6. The sixth letter of the alphabet is *F.* Only one state begins with the letter *F*—Florida.
- Most people who have Florida in mind will think of an alligator when they are asked about a large animal whose name begins with the letter *A*.

This number trick relies on arithmetic and common knowledge. In this lesson, you will try some more tricks and then analyze them to figure out how they work.

► Try the trick below with several different starting numbers.

Steps of Trick 1:
**a.** Pick a number.
**b.** Multiply it by 3.
**c.** Add 6 to the product.
**d.** Divide the sum by 3.
**e.** Subtract the original number.

Did you get 2 every time? To see how this works, you need to use a little algebra. The table below gives the steps for the number 246 and for "any number" $n$. The right column shows why the result will always be 2.

| Steps of Trick | With 246 | With $n$ |
|---|---|---|
| **a.** Pick a number. | 246 | $n$ |
| **b.** Multiply it by 3. | 738 | $3n$ |
| **c.** Add 6 to the product. | 744 | $3n + 6$ |
| **d.** Divide the sum by 3. | 248 | $n + 2$ |
| **e.** Subtract the original number. | 2 | 2 |

► Here is another trick. Try it and see what result you get.

Steps of Trick 2:
**a.** Start with your age in years.
**b.** Multiply it by 2.
**c.** Add 10.
**d.** Multiply the sum by 5.
**e.** Add the number of siblings you have.
**f.** Subtract 50.

You now have a three-digit number. The first two digits are your age, and the last digit tells how many siblings you have.

To understand how this trick works, you first have to know that any one- or two-digit number can be written as $10t + u$, where $t$ is the tens digit and $u$ is the units digit. For example, 34 can be written as $10 \cdot 3 + 4$, and 8 can be written as $10 \cdot 0 + 8$.

| Steps of Trick | With Age 12 and 2 Siblings | With Any Age and $s$ Siblings |
|---|---|---|
| **a.** Start with your age in years. | 12 | $10t + u$, where $t$ is the tens digit and $u$ is the ones digit |
| **b.** Multiply it by 2. | 24 | $2(10t + u) = 20t + 2u$ |
| **c.** Add 10. | 34 | $20t + 2u + 10$ |
| **d.** Multiply the sum by 5. | 170 | $5(20t + 2u + 10) =$ $100t + 10u + 50$ |
| **e.** Add the number of siblings. | 172 | $100t + 10u + 50 + s$, where $s$ is the number of siblings |
| **f.** Subtract 50. | 122 | $100t + 10u + 50 + s - 50 =$ $100t + 10u + s$ |

No matter what age you start with, the number you end with will have the tens digit, $t$, of your age in the hundreds place, the units digit, $u$, of your age in the tens place, and the number of siblings, $s$, in the ones place (assuming you have fewer than 10 siblings).

## Try These

**Use different starting numbers. Describe the results. Then use algebra to explain how the trick works.**

**1.** **a.** Choose any whole number.
**b.** Add the next greater whole number to it.
**c.** Add 9 to that sum.
**d.** Divide the new sum by 2.
**e.** Subtract the number you chose.

**2.** **Discuss and Write** Make up a magic number trick of your own. Test it to see that it works. Then ask a friend to try it.

Go to PRACTICE BOOK pages 53–54 for exercise sets.

# Test Prep: Multiple-Choice Questions

## Strategy: Try All the Answers

Multiple-choice questions provide several answer choices, but only one of them is the correct choice. If you are not sure about how to solve a problem directly, you might be able to *work backward* by testing each answer choice to see which one works in the problem.

Look at the sample test item.

**Sample Test Item**

What value of $x$ makes the equation below true?

$-6x - 3 = 21$

**F.** 4
**G.** 3
**H.** $-3$
**J.** $-4$

**Read the whole test item, including the answer choices.**

- Underline important words.
  What value of $x$ makes $-6x - 3 = 21$ true?
  The *value of x* is the number that replaces the variable.
- Restate the question in your own words.
  What number can you use to replace $x$ so that the equation is true?

**Solve the problem.**

- Work backward from the answer choices.
  Replace $x$ in the expression $-6x - 3$ with each answer choice to find the one that equals 21.
- Use rules for signed numbers and follow the order of operations to evaluate the expressions.

| | | | |
|---|---|---|---|
| $-6(4) - 3 = ?$ | $-6(3) - 3 = ?$ | $-6(-3) - 3 = ?$ | $-6(-4) - 3 = ?$ |
| $-24 - 3 = -27$ | $-18 - 3 = -21$ | $18 - 3 = 15$ | $24 - 3 = 21$ |

**Test-Taking Tips**

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices. Watch out for distractors.

**F.** 4 ← $-6(4) - 3 = -27$ and $-27 \neq 21$. Eliminate this choice.

**G.** 3 ← $-6(3) - 3 = -21$ and $-21 \neq 21$. Eliminate this choice.

**H.** $-3$ ← $-6(-3) - 3 = 15$ and $15 \neq 21$. Eliminate this choice.

**J.** $-4$ ← $-6(-4) - 3 = 21$. This is the correct choice!

**Try These**

**Choose the correct answer. Explain how you used strategies.**

**1.** If $xy + 3y = 50$ and $y = 5$, what is the value of $x$?

**A.** $x = -8$
**B.** $x = -7$
**C.** $x = 7$
**D.** $x = 8$

**2.** Which ordered pair lies in Quadrant III?

**F.** $(-2, 1)$
**G.** $(-1, -3)$
**H.** $(3, 2)$
**J.** $(4, -1)$

Go to PRACTICE BOOK page 55 for exercise sets.

CHAPTER 3

# Inequalities

## In This Chapter You Will:

- Use math symbols to express word sentences as inequalities
- Graph inequalities on number lines
- Write inequalities for graphs on number lines
- Solve inequalities using the Addition, Subtraction, Multiplication, and Division Properties of Inequality
- Apply the strategy: *Find a Pattern*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- Mathematical expressions can be numerical expressions or algebraic expressions.
- To evaluate an algebraic expression, substitute given values for the variables, and compute according to the order of operations.
- The absolute value of a number is its distance from 0 on a number line.
- Algebraic equations can be solved by using the Properties of Equality.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 61–82

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

## Critical Thinking

The Thunderbolt roller coaster has 6 cars, and each car has 8 seats. The Thunderbolt completes 10 rides per hour. If $p$ = the number of people who ride the Thunderbolt in 1 hour, which of the following statements about the Thunderbolt cannot be true? Explain your reasoning.

**A.** $p < 500$
**B.** $p > 450$
**C.** $\frac{p}{2} = 250$

Algebra

# 3-1 Inequalities

**Objective** To identify and use inequality symbols • To express inequalities as word sentences • To express word sentences as inequalities • To determine if a value makes an inequality true or false

Tina travels by car across the country. Her car can get as much as 36 miles per gallon of gas on the highway. How can you express the number of miles per gallon of gas Tina's car can get on a highway as an inequality?

► An inequality is a mathematical sentence that compares two expressions using $<$, $>$, $\neq$, $\leq$, or $\geq$.

To represent "as much as 36 miles per gallon," use the inequality symbol for *is less than or equal to*, $\leq$.

Let $x$ = the car's gas mileage.

$x \leq 36$ ← Read as: $x$ *is less than or equal to* 36.

inequality

**Key Concept**

**Inequality Symbols**

$<$ → *is less than*
$>$ → *is greater than*
$\neq$ → *is not equal to*
$\leq$ → *is less than or equal to*
$\geq$ → *is greater than or equal to*

► Inequalities can be written as word sentences.

| Inequality | Word Sentence |
|---|---|
| $c - \lvert -6 \rvert < 7$ | A number, $c$, minus the absolute value of negative 6 is less than 7. |
| $b \geq 8$ | A number, $b$, is greater than or equal to 8. |
| $\frac{c}{2} \neq 6$ | A number, $c$, divided by 2 is not equal to 6. |
| $n \leq 4$ | A number, $n$, is less than or equal to 4. |

► Word sentences can be written as inequalities.

| Word Sentence | Inequality |
|---|---|
| A number, $d$, plus negative three is greater than five. | $d + (-3) > 5$ |
| A number, $n$, squared is less than or equal to four. | $n^2 \leq 4$ |
| Ben runs no more than 6 miles each day. Let $m$ = miles run. | $m \leq 6$ mi |
| Half the cost of a pizza is more than \$5. Let $p$ = cost of pizza. | $\frac{p}{2} > \$5$ |
| Jasmine rides her bike at a speed of at least 15 miles per hour. Let $x$ = Jasmine's speed. | $x \geq 15$ mi/h |

► Inequalities with variables are called open sentences. Open sentences are neither true nor false. When a value is substituted for the variable, the inequality becomes a closed sentence.

Once a value has been substituted for a variable, you can determine whether the inequality is true or false:

- If the closed sentence is true, the inequality is true for that value of the variable.
- If the closed sentence is false, the inequality is false for that value of the variable.

**Remember:** Closed sentences are either true or false.

For an inequality with the symbol $\leq$ or $\geq$, test both parts of the inequality symbol:

- If at least one of the closed sentences is true, the inequality is true for that value of the variable.
- If both closed sentences are false, the inequality is false for that value of the variable.

## Examples

Tell whether the inequality is true or false for the given value.

**1** $x > 2$, when $x = 5$

**Test:** $5 \overset{?}{>} 2$ ← Substitute 5 for $x$.

$5 > 2$ ← The closed sentence is true.

The inequality is true for $x = 5$.

**2** $a < 2$, when $a = 4$

**Test:** $4 \overset{?}{<} 2$ ← Substitute 4 for $a$.

$4 < 2$ ← The closed sentence is false.

The inequality is false for $a = 4$.

**3** $m \leq 9$, when $m = 1$

**Test:** $1 \overset{?}{<} 9$ ← Substitute 1 for $m$. True

**Test:** $1 \overset{?}{=} 9$ ← Substitute 1 for $m$. False

Since one closed sentence is true, the inequality is true for $m = 1$.

**4** $g \geq 11$, when $g = 7$

**Test:** $7 \overset{?}{>} 11$ ← Substitute 7 for $g$. False

**Test:** $7 \overset{?}{=} 11$ ← Substitute 7 for $g$. False

Since both closed sentences are false, the inequality is false for $g = 7$.

► A compound inequality is an inequality that consists of two or more inequalities that are connected. The following inequality expresses *two related statements* joined by the word *and*.

| Word Sentence | Two Inequalities | Compound Inequality |
|---|---|---|
| Kim is older than her 6-year-old sister, *and* Kim is younger than her 15-year-old brother. | Let $k$ = Kim's age.<br>$k > 6$ *and* $k < 15$ | $6 < k < 15$<br>Read as $k$ *is greater than* 6 and *is less than* 15. |

## Try These

**Write each as an inequality.**

1. Tony is at least fourteen years old.
2. Negative eight increased by a number is greater than or equal to twenty.
3. A number, $x$, is greater than or equal to 6 and is less than 12.

**Write each as a word sentence.**

4. $x \geq 8 - 32$
5. $|-7| \leq 9n$
6. $-(-4) \div z > 9$

7. **Discuss and Write** Explain why $x \leq -4$ is false when $x = 0$.

Go to PRACTICE BOOK Lesson 3-1 for exercise sets.

Algebra

# 3-2 Graph Inequalities on a Number Line

**Objective** To use a replacement set to identify the solution set of an inequality • To graph the solution set of an inequality on a number line • To write an inequality that describes a given graph

Pete scored *at least* 92 points out of 100 points on his science test. If test scores are whole numbers, what is the range of possible test scores Pete might have received on his science test?

► To find the range of possible test scores, write an inequality and identify the solution set as a specific part of the replacement set.

Let $x$ = Pete's possible test score.

$x \geq 92$

**Think**
*at least* 92 points means 92 or more points.

A replacement set, $R$, is a set of numbers to be used as possible values for the variable in an equation or inequality. Sometimes you may be told what the replacement set is.

$R = \{0, 1, 2, 3, \ldots 100\}$

The *replacement set* for the test scores is the set of whole numbers from 0 to 100.

A solution set, $S$, contains all the values for the variable that make the equation or inequality true.

$S = \{92, 93, 94, 95, 96, 97, 98, 99, 100\}$

The *solution set* for the test scores is the set of whole numbers from 92 to 100.

So the range of test scores that Pete might have received is 92–100.

► You can graph the solution set of the inequality on a number line.


► The solution set for an inequality can be one, some, all, or none of the values in the replacement set.

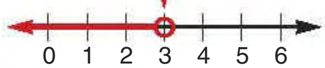
| Inequality | Replacement Set | Solution Set |
|---|---|---|
| $y < 25$ | $\{24, 25, 26, \ldots\}$ | $\{24\}$ |
| $x > 8$ | $\{0, 1, 2, 3, \ldots\}$ | $\{9, 10, 11, 12, \ldots\}$ |
| $x \leq 8$ | $\{4, 5, 6, 7, \ldots\}$ | $\{4, 5, 6, 7, 8\}$ |
| $t \geq 10$ | $\{-3, -2, -1, 0, 1, 2, \ldots\}$ | $\{10, 11, 12, 13, \ldots\}$ |
| $t \leq 10$ | $\{\ldots, 8, 9, 10\}$ | $\{\ldots, 8, 9, 10\}$ |
| $a < 0$ | $\{0, 1, 2, 3, \ldots\}$ | $\{\}$ or $\varnothing$. |

When *no* number from the replacement set makes the inequality true, the solution set is the empty set, { } or $\varnothing$.

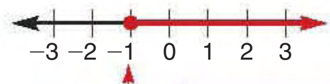
▶ Often the solution set is *not* limited to whole numbers. It may include other number types such as integers, fractions, and decimals. To graph this type of solution set on a number line, a line segment or a ray is used instead of separate dots. This shows that all the numbers *between* whole numbers are included in the solution set.

Graph: $x < 3$

3 is *not* in the solution set.




Graph: $x \geq -1$




−1 *is* in the solution set.

**Key Concept**

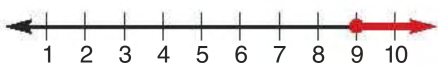
**Graphing Inequalities**

- Use a circle, ○, to show that a number is *not* in the solution set.
- Use a dot, ●, to show that a number *is* in the solution set.

▶ You can write an inequality to describe a given graph in two ways. These are equivalent inequalities—inequalities that have the same solution set.

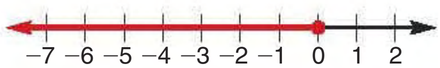

**Inequality:** $x > 4$ or $4 < x$

4 is not in the solution set.



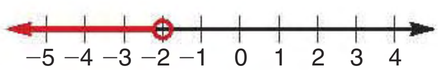

**Inequality:** $x \geq 9$ or $9 \leq x$

9 is in the solution set.




**Inequality:** $x \leq 0$ or $0 \geq x$

0 is in the solution set.




**Inequality:** $x < -2$ or $-2 > x$

−2 is not in the solution set.

## Try These

**Find the solution set for each inequality. Then graph each on a number line.**

**1.** $x < -9$ **2.** $x \geq 2$ **3.** $x > 0$ **4.** $x \leq 11$

**5. Discuss and Write** Explain when you need to use a circle and when you need to use a dot to graph solution sets of inequalities. Give examples to justify your answer.

Go to PRACTICE BOOK Lesson 3-2 for exercise sets.

## Acceleration: Graph Compound Inequalities

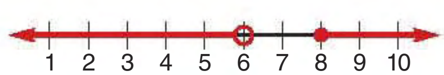
You can also graph solution sets of compound inequalities.

Graph: $1 < x < 5$

All numbers between 1 ***and*** 5 are solutions.


Graph: $d \geq 8$ or $d < 6$

All numbers *less than* 6 ***or*** all numbers *greater than or equal to* 8 are solutions.



Algebra

3-3

# Model Properties of Inequality

**Objective** To use algebra tiles and number lines to model properties of inequality

PROBLEM-SOLVING STRATEGY 3

Tamika is 5 years old and her brother Rashid is 3 years old. The table at the right shows inequalities that compare their:

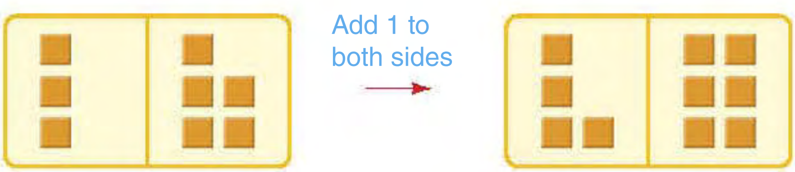
- present ages
- ages one year from now
- ages two years ago

| Ages | | |
|---|---|---|
| Present | One Year From Now | Two Years Ago |
| $5 > 3$ | $5 + 1 > 3 + 1$<br>$6 > 4$ | $5 - 2 > 3 - 2$<br>$3 > 1$ |
| $3 < 5$ | $3 + 1 < 5 + 1$<br>$4 < 6$ | $3 - 2 < 5 - 2$<br>$1 < 3$ |

All inequalities in the table show that Tamika's age is greater than her brother's age. This example illustrates the Addition and Subtraction Properties of Inequality.

► You can use algebra tiles to model the Addition and Subtraction Properties of Inequality.

## Addition Property of Inequality



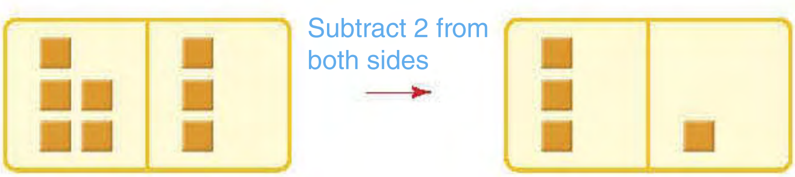

$3 < 5$ $\quad 3 + 1 < 5 + 1 \quad$ $4 < 6$

true statement $\quad$ true statement

**Key Concept**

**Addition Property of Inequality**
When you add the same number to both sides of an inequality, you get a true statement.

## Subtraction Property of Inequality



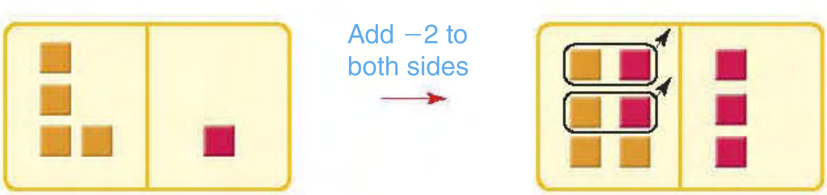

$5 > 3$ $\quad 5 - 2 > 3 - 2 \quad$ $3 > 1$

true statement $\quad$ true statement

**Key Concept**

**Subtraction Property of Inequality**
When you subtract the same number from both sides of an inequality, you get a true statement.

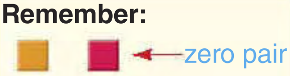
► You can use algebra tiles to model adding a negative number to both sides of an inequality.




$4 > -1$ $\quad 4 + (-2) > -1 + (-2) \quad$ $2 > -3$

true statement $\quad$ true statement

**Remember:**

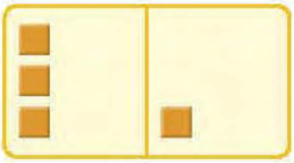



$1 + (-1) = 0$

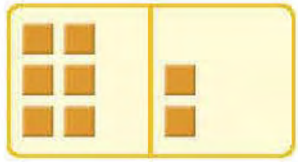
► You can also use algebra tiles and number lines to model *multiplying* and *dividing* both sides of an inequality by a positive or a negative number.

## Multiplication Property of Inequality

**Multiply by a positive number**



Multiply both sides by 2 →

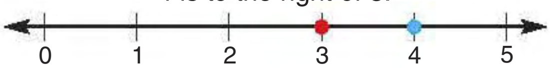


$3 > 1$ true statement $\quad 2(3) > 2(1) \quad$ $6 > 2$ true statement

> **Key Concept**
>
> **Multiplication Property of Inequality**
>
> - When you multiply both sides of an inequality by the same positive number, you get a true statement.
> - When you multiply both sides of an inequality by the same negative number *and* you reverse the inequality symbol, you get a true statement.

**Multiply by a negative number**

4 is to the right of 3.




$4 > 3$

Multiply both sides by −2. →

$-2(4) \; ? \; -2(3)$

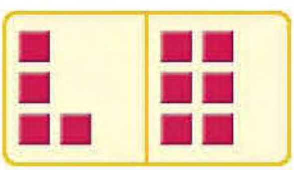
−8 is to the left of −6.


$-8 < -6$

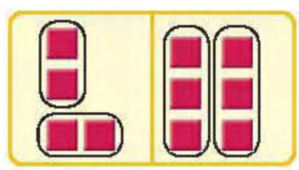
Reverse the inequality symbol.

## Division Property of Inequality

**Divide by a positive number**



Divide both sides by 2 →



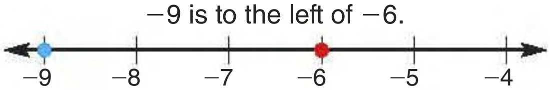
$-4 > -6$ true statement $\quad -4 \div 2 > -6 \div 2 \quad$ $-2 > -3$ true statement

2 equal groups

> **Key Concept**
>
> **Division Property of Inequality**
>
> - When you divide both sides of an inequality by the same positive number, you get a true statement.
> - When you divide both sides of an inequality by the same negative number *and* you reverse the inequality symbol, you get a true statement.

**Divide by a negative number**

−9 is to the left of −6.

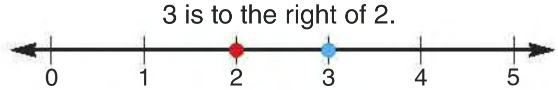



$-9 < -6$

Divide both sides by −3. →

$-9 \div (-3) \; ? \; -6 \div (-3)$

3 is to the right of 2.



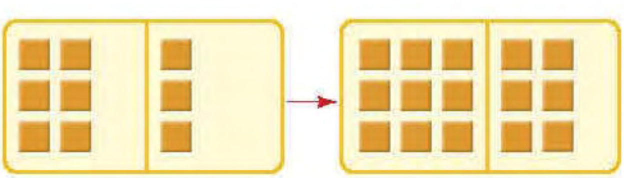

$3 > 2$

Reverse the inequality symbol.

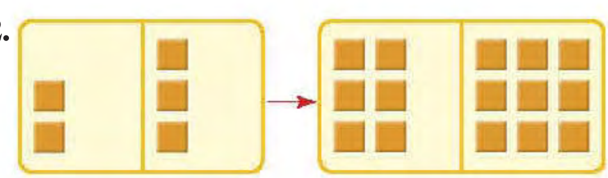
## Try These

**Write an inequality for each model. Then tell which property of inequality is illustrated.**

**1.**



**2.**



**3.** **Discuss and Write** Explain how you would model subtracting 1 from both sides of $-7 < -5$ using algebra tiles.

Go to PRACTICE BOOK Lesson 3-3 for exercise sets.

Algebra 3-4

Update your skills. See page 409 VI.

# Solve Inequalities Using Addition and Subtraction

**Objective** To solve one-step inequalities by applying the Addition and Subtraction Properties of Inequality • To graph the solution set of an inequality

PROBLEM-SOLVING STRATEGY 2

Eddie's little brother has been saving only dollar bills in a piggy bank. On Monday, after he put \$12 into his piggy bank, there was at least \$33 in it. How much money was in Eddie's little brother's piggy bank before Monday?

► To find how much money, write and solve an inequality.
Let $y$ = the dollars in the piggy bank before Monday.

| dollars before Monday | plus | dollars added | is at least | \$33 |
|---|---|---|---|---|
| $y$ | $+$ | $12$ | $\geq$ | $33$ |

Solve inequalities involving addition the same way you solve addition equations.

**Solve:** $y + 12 \geq 33$

$y + 12 - 12 \geq 33 - 12$ ← Subtract 12 from both sides to isolate $y$.

$y \geq 21$ ← Simplify.

**Think**
Will the solution set to this inequality include integers, fractions, and decimals or will it be limited to a particular group of numbers such as whole numbers?

### Key Concept

**Subtraction Property of Inequality**
When you subtract the same number from both sides of an inequality, you get a true statement.

If $a < b$, then $a - c < b - c$.

This statement is also true if $<$ is replaced by $>$, $\leq$, or $\geq$.

Since only dollar bills were put into the bank, the solution set is the whole numbers that are greater than or equal to 21.

**Check:** According to the solution set, 21 is a solution and 18 is not.

$21 + 12 \overset{?}{\geq} 33$ ← Substitute 21 for $y$.

$33 \geq 33$ True

$18 + 12 \overset{?}{\geq} 33$ ← Substitute 18 for $y$.

$30 \geq 33$ False

So, before Monday, Eddie's little brother had \$21 or more in his piggy bank.

► You can also solve inequalities involving subtraction using the Addition Property of Inequality.

**Solve:** $n - 9 < 11$

$n - 9 + 9 < 11 + 9$ ← Add 9 to both sides to isolate $n$.

$n < 20$ ← Simplify.

### Key Concept

**Addition Property of Inequality**
When you add the same number to both sides of an inequality, you get a true statement.

If $a < b$, then $a + c < b + c$.

This statement is also true if $<$ is replaced by $>$, $\leq$, or $\geq$.

So the solution set is numbers that are less than 20.

**Check:** According to the solution set, 19 is a solution and 20 is not.

$19 - 9 \overset{?}{<} 11$ ← Substitute 19 for $n$.

$10 < 11$ True

$20 - 9 \overset{?}{<} 11$ ← Substitute 20 for $n$.

$11 < 11$ False

After you solve an inequality, you can use a number line to graph and check the solution set.

**Solve:** $s + 4 \geq 15$

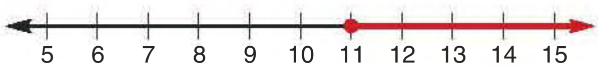
$s + 4 - 4 \geq 15 - 4$ ← Subtract 4 from both sides to isolate $s$.

$s \geq 11$ ← Simplify.

The solution set is all numbers that are greater than or equal to 11.

Since $s \geq 11$, use a dot and a ray extending toward the right to indicate that numbers greater than or equal to 11 are in the solution set.

**Graph:**




**Think**
Integers, fractions, and decimals greater than or equal to 11 are solutions.

**Check:** According to the solution set, both 11 and 13 are solutions.

$11 + 4 \overset{?}{\geq} 15$ ← Substitute 11 for $s$.

$15 \geq 15$ True.

So the dot on the number line is correct.

$13 + 4 \overset{?}{\geq} 15$ ← Substitute 13 for $s$.

$17 \geq 15$ True

So the direction of the ray is correct.

## Example

**1** **Solve:** $-22 > x - 6$

$-22 + 6 > x - 6 + 6$ ← Add 6 to both sides to isolate $x$.

$-16 > x$ ← Simplify.

The solution set is all numbers that are less than $-16$.

**Graph:**


Since $-16 > x$, use a circle and a ray extending toward the left to indicate that numbers less than $-16$, not including $-16$, are in the solution set.

**Think**
Integers, fractions, and decimals less than $-16$ are solutions.

**Check:** According to the solution set, $-17$ is a solution and $-16$ is not.

$-22 \overset{?}{>} -17 - 6$ ← Substitute $-17$ for $n$.

$-22 > -23$ True

So the circle on the number line is correct.

$-22 \overset{?}{>} -16 - 6$ ← Substitute $-16$ for $n$.

$-22 > -22$ False

So the direction of the ray is correct.

## Try These

**Solve and graph each inequality. Check your work to justify your answer.**

**1.** $n + 8 \geq 14$

**2.** $4 < n - 9$

**3.** $d + (-3) > 2$

**4. Discuss and Write** How is solving inequalities involving addition and subtraction like solving addition and subtraction algebraic equations?

Go to PRACTICE BOOK Lesson 3-4 for exercise sets.

Algebra

# 3-5 Solve Inequalities Using Multiplication

Update your skills. See page 412 XII.

**Objective** To solve one-step inequalities by applying the Multiplication Property of Inequality • To graph the solution set of an inequality

Five friends evenly divided the cost of a trip to Yosemite National Park. Each friend paid a whole dollar amount that was at most \$13. What was the total cost of the trip?

► To find the total cost of the trip, write and solve an inequality. Let $c$ = the total cost of the trip.

| total cost | divided by | number of friends | is at most | cost per friend |
|---|---|---|---|---|
| $c$ | $\div$ | 5 | $\le$ | \$13 |

Solve inequalities involving division the same way that you solve division equations.

**Think** $\frac{c}{5}$ means $c \div 5$

**Solve:** $\frac{c}{5} \le 13$

$(5)\frac{c}{5} \le 13(5)$ ← Multiply both sides by 5 to isolate $c$.

$\frac{\cancel{5}^{1}c}{\cancel{5}_{1}} \le 13 \bullet 5$ ← Simplify.

$c \le 65$

So the solution set is the whole numbers less than 65.

**Check:** According to the solution set, 65 is a solution and 70 is not.

$\frac{65}{5} \overset{?}{\le} 13$ ← Substitute 65 for $c$.

$13 \le 13$ True

$\frac{70}{5} \overset{?}{\le} 13$ ← Substitute 70 for $c$.

$14 \le 13$ False

The total cost of the trip was whole dollar amounts of \$65 or less.

**Key Concept**

**Multiplication Property of Inequality (multiply by a positive number)**

When you multiply both sides of an inequality by the same positive number, you get a true statement.

If $a < b$ and $c$ is positive, then $ac < bc$.

A similar statement can be written for $a > b$, $a \le b$, and $a \ge b$.

► When an inequality is multiplied by a *negative number*, the symbol must be reversed to form a true statement.

**Solve:** $\frac{p}{-3} < 12$

$(-3)\frac{p}{-3} > 12(-3)$ ← Multiply both sides by $-3$ to isolate $p$ and reverse the inequality symbol.

$p > -36$ ← Simplify.

So the solution set is all numbers greater than $-36$.

**Check:** According to the solution set, $-33$ is a solution and $-36$ is not.

$\frac{-33}{-3} \overset{?}{<} 12$ ← Substitute $-33$ for $p$.

$11 < 12$ True

$\frac{-36}{-3} \overset{?}{<} 12$ ← Substitute $-36$ for $p$.

$12 < 12$ False

**Key Concept**

**Multiplication Property of Inequality (multiply by a negative number)**

When you multiply both sides of an inequality by the same negative number *and* you reverse the inequality symbol, you get a true statement.

If $a < b$ and $c$ is negative, then $ac > bc$.

A similar statement can be written for $a > b$, $a \le b$, and $a \ge b$.

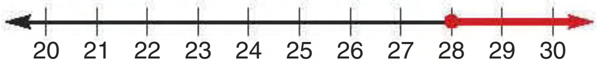
▶ After you solve an inequality, you can use a number line to graph and check the solution set.

**Solve:** $\frac{a}{7} \geq 4$

$(7)\frac{a}{7} \geq 4(7)$ ← Multiply both sides by 7 to isolate $a$.

$a \geq 28$ ← Simplify.

**Graph:**




**Think**
All integers, fractions, and decimals greater than or equal to 28 are solutions.

The solution set is all numbers that are greater than or equal to 28.

**Check:** According to the solution set, both 28 and 35 are solutions.

$28 \div 7 \overset{?}{\geq} 4$ ← Substitute 28 for $a$.

$4 \geq 4$ True

So the dot on the number line is correct.

$35 \div 7 \overset{?}{\geq} 4$ ← Substitute 35 for $a$.

$5 \geq 4$ True

So the direction of the ray is correct.

## Example

**1** **Solve:** $4 \leq \frac{r}{-6}$

$(-6)4 \geq \frac{r}{-6}(-6)$ ← Multiply both sides by $-6$ to isolate $r$ and reverse the inequality symbol.

$-24 \geq r$ ← Simplify.

The solution set is all numbers that are less than or equal to $-24$.

**Graph:**


**Think**
All integers, fractions, and decimals less than or equal to $-24$ are solutions.

**Check:** According to the solution set, $-24$ is a solution and $-18$ is not.

$4 \overset{?}{\leq} \frac{-24}{-6}$ ← Substitute $-24$ for $r$.

$4 \leq 4$ True

So the dot on the number line is correct.

$4 \overset{?}{\leq} \frac{-18}{-6}$ ← Substitute $-18$ for $r$.

$4 \leq 3$ False

So the direction of the ray is correct.

## Try These

**Solve and graph each inequality. Check your work to justify your answer.**

**1.** $\frac{y}{4} > 11$

**2.** $p \div (2) \leq 12$

**3.** $25 < \frac{n}{-5}$

**4. Discuss and Write** Explain the difference between solving $\frac{x}{-2} > 6$ and $\frac{x}{2} > 6$. Solve each inequality to support your answer.

Go to PRACTICE BOOK Lesson 3-5 for exercise sets.

Algebra 3-6

Update your skills. See page 411 XI.

# Solve Inequalities Using Division

**Objective** To solve one-step inequalities by applying the Division Property of Inequality
• To graph the solution set of an inequality

PROBLEM-SOLVING STRATEGY 1

Ben found some whole shells on the beach. Emma found three times as many whole shells as Ben. Emma found at least 15 whole shells. How many whole shells did Ben find?

► To find how many whole shells Ben found, write and solve an inequality.

Let $y$ = the number of whole shells Ben found.

3 times the number that Ben found → $3y$

is greater than or equal to 15 → $\geq 15$

Solve inequalities involving multiplication the same way you solve multiplication equations.

**Solve:** $3y \geq 15$

$\frac{3y}{3} \geq \frac{15}{3}$ ← Divide both sides by 3 to isolate $y$.

$\frac{\cancel{3}^{1}y}{\cancel{3}_{1}} \geq \frac{\cancel{15}^{5}}{\cancel{3}_{1}}$ ← Simplify.

$y \geq 5$

**Key Concept**

**Division Property of Inequality (divide by a positive number)**

When you divide both sides of an inequality by the same positive number, you get a true statement.

If $a < b$ and $c$ is positive, then $\frac{a}{c} < \frac{b}{c}$.

A similar statement can be written for $a > b$, $a \leq b$, and $a \geq b$.

So the solution set is the whole numbers greater than or equal to 5.

**Check:** According to the solution set, 10 is a solution and 4 is not.

$3(10) \overset{?}{>} 15$ ← Substitute 10 for $y$.

$30 > 15$ True

$3(4) \overset{?}{>} 15$ ← Substitute 4 for $y$.

$12 > 15$ False

So Ben found 5 or more whole shells.

► When an inequality is divided by a *negative number*, the symbol must be reversed to form a true statement.

**Solve:** $6 \geq -2b$

$6 \div (-2) \leq -2b \div (-2)$ ← Divide both sides by $-2$ to isolate $b$ and reverse the inequality symbol.

$-3 \leq b$ ← Simplify.

**Key Concept**

**Division Property of Inequality (divide by a negative number)**

When you divide both sides of an inequality by the same negative number *and* you reverse the inequality symbol, you get a true statement.

If $a < b$ and $c$ is negative, then $\frac{a}{c} > \frac{b}{c}$.

A similar statement can be written for $a > b$, $a \leq b$, and $a \geq b$.

The solution set is the numbers greater than or equal to $-3$.

**Check:** According to the solution set, $-3$ is a solution and $-10$ is not.

$6 \overset{?}{\geq} -2(-3)$ ← Substitute $-3$ for $b$.

$6 \geq 6$ True

$6 \overset{?}{\geq} -2(-10)$ ← Substitute $-10$ for $b$.

$6 \geq 20$ False

▶ After you solve an inequality, you can use a number line to graph and check the solution set.

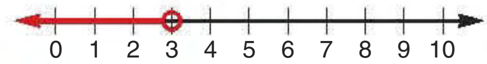
**Solve:** $8x < 24$

$$\frac{8x}{8} < \frac{24}{8} \quad \leftarrow \text{Divide both sides by 8 to isolate } x.$$

$$\frac{\cancel{8}^{1}x}{\cancel{8}_{1}} < \frac{\cancel{24}^{3}}{\cancel{8}_{1}} \quad \leftarrow \text{Simplify.}$$

$$x < 3$$

**Graph:**




**Think**
All integers, fractions, and decimals less than 3 are solutions.

The solution set is the numbers less than 3.

**Check:** According to the solution set, 0 is a solution and 3 is not.

$8 \bullet 3 \overset{?}{<} 24$ ← Substitute 3 for $x$.

$24 < 24$ False

So the circle on the number line is correct.

$8 \bullet 0 \overset{?}{<} 24$ ← Substitute 0 for $x$.

$0 < 24$ True

So the direction of the ray is correct.

## Try These

**Solve and graph each inequality. Check your work to justify your answer.**

**1.** $7n > 21$ **2.** $9 \leq 3x$ **3.** $-12w < 96$

**4. Discuss and Write** Is $x > -4$ the solution to the inequality $3x < -12$? Justify your answer.

Go to PRACTICE BOOK Lesson 3-6 for exercise sets.

## Acceleration: Two-Step Inequalities

Solving a two-step inequality is similar to solving a two-step equation.

- First, apply the Addition or Subtraction Property of Inequality to isolate the term that contains the variable.
- Then apply the Multiplication or Division Property of Inequality to get the variable alone on one side.

**Solve:** $4k - 6 > 14$

$$4k - 6 + 6 > 14 + 6 \quad \leftarrow \text{Add 6 to both sides.}$$

$$4k > 20 \quad \leftarrow \text{Simplify.}$$

$$\frac{\cancel{4}^{1}k}{\cancel{4}_{1}} > \frac{\cancel{20}^{5}}{\cancel{4}_{1}} \quad \leftarrow \text{Divide both sides by 4 and simplify.}$$

$$k > 5$$

The solution set is all numbers greater than 5.

**Check:** According to the solution set, 6 is a solution and 5 is not.

$(4 \bullet 6) - 6 \overset{?}{>} 14$ ← Substitute 6 for $k$.

$24 - 6 \overset{?}{>} 14$ ← Simplify.

$18 > 14$ True

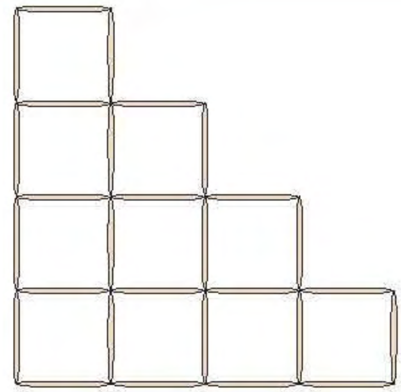
$(4 \bullet 5) - 6 \overset{?}{>} 14$ ← Substitute 5 for $k$.

$20 - 6 \overset{?}{>} 14$ ← Simplify.

$14 > 14$ False

# 3-7 Problem-Solving Strategy: Find a Pattern

**Objective** To solve problems using the strategy *Find a Pattern*



**Problem 1:** The stair-step design at the right, which is made out of toothpicks, is said to be "4 rows deep." A stair-step design that is 10 rows deep would require how many toothpicks?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** You are given a stair-step arrangement of squares made from toothpicks.
The pattern is 4 rows deep and uses 28 toothpicks.

**Question:** How many toothpicks are used to make a stair-step pattern that is 10 rows deep?

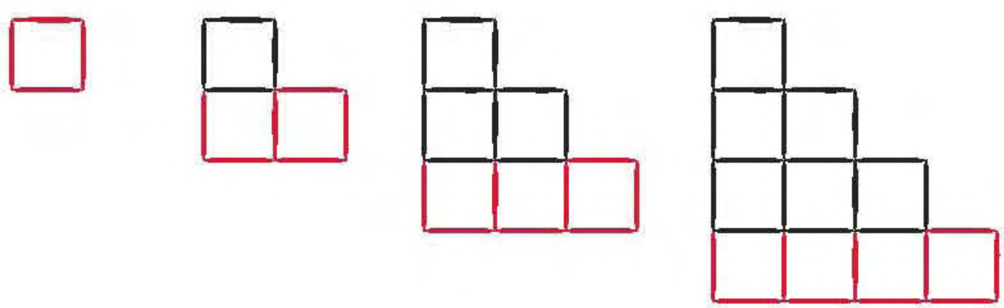
**Plan** **Select a strategy.**
You can use the strategy *Find a Pattern*. Examine the designs that are 1, 2, 3, and 4 rows deep, and find a pattern that you can use to extend to 10 rows.

**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Solve** **Apply the strategy.**
A design with 1 row is just a single small square of 4 toothpicks.
To create row 2, add 6 more toothpicks, for a total of 10 toothpicks.
To create row 3, add 8 more toothpicks, for a total of 18 toothpicks.
For row 4, add 10 more toothpicks, for a total of 28 toothpicks.

**4 toothpicks** **Add 6 more** **Add 8 more** **Add 10 more**



So each new row was created by adding two more toothpicks than were added to create the previous row. To find the number of toothpicks in a design with 10 rows, compute this sum:

$4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22$

You can find this sum easily by forming 5 sums of 26, as shown below.
So 5 • 26, or 130, toothpicks are required.

$4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22$

**Check** **Check to make sure your answer makes sense.**

You can check the result by finding a different pattern.

A 1-row design has $1 + 1$ horizontal toothpicks.
A 2-row design has $1 + 2 + 2$ horizontal toothpicks.
A 3-row design has $1 + 2 + 3 + 3$ horizontal toothpicks; and so on.

So the number of horizontal toothpicks in a 10-row design will be $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 10$, or 65 in all.

The number of vertical toothpicks is the same as the number of horizontal toothpicks.

So the total number of toothpicks for a 10-row design is $2 \bullet 65$, or 130 toothpicks.

**Problem 2:** Use the fact that $3^8$ is 6561 to find the following sum without using a calculator: $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \frac{1}{3^6} + \frac{1}{3^7} + \frac{1}{3^8}$

**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** $3^8 = 6561$

**Question:** Find $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \frac{1}{3^6} + \frac{1}{3^7} + \frac{1}{3^8}$ without a calculator.

**Plan** **Select a strategy.**

Examine sums of the first term, the first *two* terms, the first *three* terms, and so on. Use the strategy *Find a Pattern* that would help find the sum of all *eight* terms.

**Solve** **Apply the strategy.**

The "sum" of the first term is just $\frac{1}{3}$.

The sum of the first *two* terms is $\frac{1}{3} + \frac{1}{3^2} = \frac{1}{3} + \frac{1}{9} = \frac{3}{9} + \frac{1}{9} = \frac{4}{9}$.

The sum of the first *three* terms is
$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} = \frac{9}{27} + \frac{3}{27} + \frac{1}{27} = \frac{13}{27}$.

The sum of the first *four* terms is
$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} = \frac{27}{81} + \frac{9}{81} + \frac{3}{81} + \frac{1}{81} = \frac{40}{81}$.

In each case, the denominator is $3^n$, where $n$ is the number of terms, and the numerator is found by subtracting 1 from the denominator and then dividing by 2.

If this pattern continues, then the denominator of the sum of all *eight* terms will be $3^8$, which is given as 6561; and the numerator will be $(6561 - 1) \div 2$, or 3280.

So $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \frac{1}{3^6} + \frac{1}{3^7} + \frac{1}{3^8} = \frac{3280}{6561}$.

**Check** **Check to make sure your answer makes sense.**

Add the terms with your calculator. The result will be 0.4999237921, which is equivalent to $\frac{3280}{6561}$.

Go to PRACTICE BOOK Lesson 3-7 for exercise sets.

ONLINE www.progressinmathematics.com
More Enrichment Topics

# Enrichment:
## Define, Substitute, and Compute

**Objective** To invent arithmetic operations • To explore properties of invented operations

Each of the four basic mathematical operations combines two numbers to get another number. For example, each basic operation below is combining 12 and 3:

$12 + 3 = 15$ $12 - 3 = 9$ $12 \bullet 3 = 36$ $12 \div 3 = 4$

► You can invent your own operations based on the four basic operations.

For example, you could define an operation so that for any real numbers $a$ and $b$, $a \star b = \frac{a+b}{2}$. So $5 \star 2 = \frac{5+2}{2} = \frac{7}{2} = 3.5$.

The table at the right shows the values of $a \star b$ for more values of $a$ and $b$. Note the following about the operation $\star$.

| $a$ | $b$ | $a \star b$ |
|---|---|---|
| 0 | 2 | 1 |
| 1 | 3 | 2 |
| 2 | 2 | 2 |
| −3 | 6 | 1.5 |
| −25 | −33 | −29 |
| 6.4 | 2.2 | 4.3 |

- The operation $\star$ is commutative.
  For any numbers $a$ and $b$, $a \star b = \frac{a+b}{2} = \frac{b+a}{2} = b \star a$.
- For any number $a$, $a \star a = \frac{a+a}{2} = \frac{2a}{2} = a$.

► Consider an operation ❖ that is defined for *only* five whole numbers: 0, 1, 2, 3, and 4. The result of $a$ ❖ $b$ is the *remainder* when $a + b$ is divided by 5.

For example, 4 ❖ 3 = 2, because $4 + 3 = 7$ and the remainder when 7 is divided by 5 is 2.

You can use a table to show the result of every possible combination of numbers. The table shows:

- The operation ❖ is commutative.
  For example, 2 ❖ 4 = 4 ❖ 2 = 6.
- 0 is the identity element because $a$ ❖ 0 equals $a$.
- The inverse of a number $a$ is $5 - a$. That is, $a$ ❖ $(5 - a) = 0$.
  For example, the inverse of 3 is 2, and the inverse of 1 is 4.

| ❖ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **0** | 0 | 1 | 2 | 3 | 4 |
| **1** | 1 | 2 | 3 | 4 | 0 |
| **2** | 2 | 3 | 4 | 0 | 1 |
| **3** | 3 | 4 | 0 | 1 | 2 |
| **4** | 4 | 0 | 1 | 2 | 3 |

### Try These

1. For all real numbers $a$ and $b$, $a @ b = (a - b)^2$.
   **a.** Find $a @ b$ for at least five pairs of numbers.
   **b.** Is @ commutative? If so, explain how you know. If not, give a counterexample.
2. For all integers $a$ and $b$, $a \# b = 0$ if $a + b$ is even and $a \# b = 1$ if $a + b$ is odd.
   **a.** Find $a \# b$ for at least five pairs of integers.
   **b.** Is # commutative? If so, explain how you know. If not, give a counterexample.
3. **Discuss and Write** Define an operation that applies to all real numbers. Make a table showing the result of your operation for at least five pairs of numbers.

Go to PRACTICE BOOK page 75 for exercise sets.

# Test Prep: Short-Answer Questions

## Strategy: Show All Your Work

Short-answer questions usually have one question to answer and ask that you show the steps you take to reach a solution. Showing all your work is important since partial credit might be given for the process even if your final answer is incorrect. When you need to write an equation or inequality to solve the problem, you may find it helpful to *write a verbal model* first.

### Sample Test Item

When empty, Casey's suitcase weighs 4 pounds. The maximum allowed weight for a carry-on bag on her flight is 18 pounds. What amount of weight can Casey pack in her suitcase and still be allowed to bring it on the plane?

***Show all your work.***

Look at the sample test item.

**Read the whole test item carefully.**

- Reread the test item carefully and ask yourself questions to clarify the meaning of the text.
  A maximum weight of 18 pounds means that the total weight must be less than or equal to 18 pounds.
- Write a verbal model to describe the problem.
  weight of suitcase + weight of what is packed ≤ allowed weight

### Test-Taking Tips

- Reread the test item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply an appropriate strategy.
  Translate the verbal model into an algebraic inequality and then solve.

  weight of suitcase + weight of what is packed ≤ allowed weight

  Let $p$ = weight Casey can pack.

  $4 + p \leq 18$ ← Substitute 4 for the suitcase weight and 18 for the allowed weight.

  $4 - 4 + p \leq 18 - 4$ ← Use the Subtraction Property of Inequality.

  $p \leq 14$

  **Answer:** Casey can pack 14 pounds or less in her suitcase and still be allowed to bring it on the plane.

### Item Analysis

**Check to make sure you show all your work.**

- Analyze your answer. Does it make sense?
  Substitute values for $p$ in the inequality. Do you get true statements?

  Let $p$ = 10 pounds.

  $4 + p \leq 18$

  $4 + 10 \stackrel{?}{\leq} 18$

  $14 \leq 18$ ✓

  Let $p$ = 14 pounds.

  $4 + p \leq 18$

  $4 + 14 \stackrel{?}{\leq} 18$

  $18 \leq 18$ ✓

**Try These** Item 1 is partially worked out for you.

**Solve. Use a verbal model to help write an algebraic expression or sentence.**

**1.** Tamara ordered 3 CDs. Each CD cost the same amount. With a shipping charge of \$5, the total cost was \$41. How much did each CD cost?
***Show all your work.***

**Read the test item for a general idea of the problem.**

- Reread the test item carefully and ask yourself questions to clarify the meaning of the text. Write an equation to relate the cost of the CDs plus shipping to the total cost.
- Write a verbal model to describe the problem.

  3 times the cost of each CD + shipping = total cost

**Solve the problem.**

- Translate the verbal model into an algebraic equation and then solve.

  Let $c$ = the cost of each CD.

  To solve the equation, remember to isolate the variable on one side.

## Test-Taking Tips

- Reread the test item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Item Analysis**

**Check your work. Make sure you show all your work.**

- Analyze your answers. Do they make sense? Substitute the value for each CD back into the equation. Is the result a true statement?

**2.** Will needs to survey at least 20 people for a science project. He has surveyed 16 people so far. How many more people does Will need to survey?

**3.** The cost of a gallon container of apple juice is \$3.50. What is the maximum number of containers you can buy for \$15?

**4.** The temperature at a ski resort in the early morning is $-3°$ Fahrenheit. By mid-morning, the temperature rises to 10° Fahrenheit. By how many degrees Fahrenheit does the temperature increase?

**5.** This month Clara worked 4 hours less than twice the number of hours she worked last month. Write an expression for the number of hours Clara worked this month.

**6.** It costs \$3 per trip to ride the bus. You can buy an unlimited bus pass for \$125 that pays for all the rides you take during a 30-day period. What is the least number of rides you would have to take in a 30-day period to save money by buying the unlimited pass? What inequality can you solve to help you answer the question?

Go to PRACTICE BOOK pages 77–78 for exercise sets.

# Rational Numbers: Decimals

CHAPTER 4

## In This Chapter You Will:

- Compare and order decimals
- Estimate solutions of decimal operations using rounding, front-end estimation, and compatible numbers
- Solve equations by adding, subtracting, multiplying, and dividing decimals
- Express decimal numbers in scientific notation
- Multiply and divide numbers with positive and negative exponents
- Rename metric units of measure
- Review problem-solving strategies
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- An inequality is a mathematical sentence that uses a comparison symbol to indicate that two quantities are *not* equal.
- Open sentences have variables.
- A replacement set is a set of numbers to be used as possible values for the variable in an equation or an inequality.
- A solution set contains the values for the variable that make an equation or an inequality true.

**For Practice Exercises:**

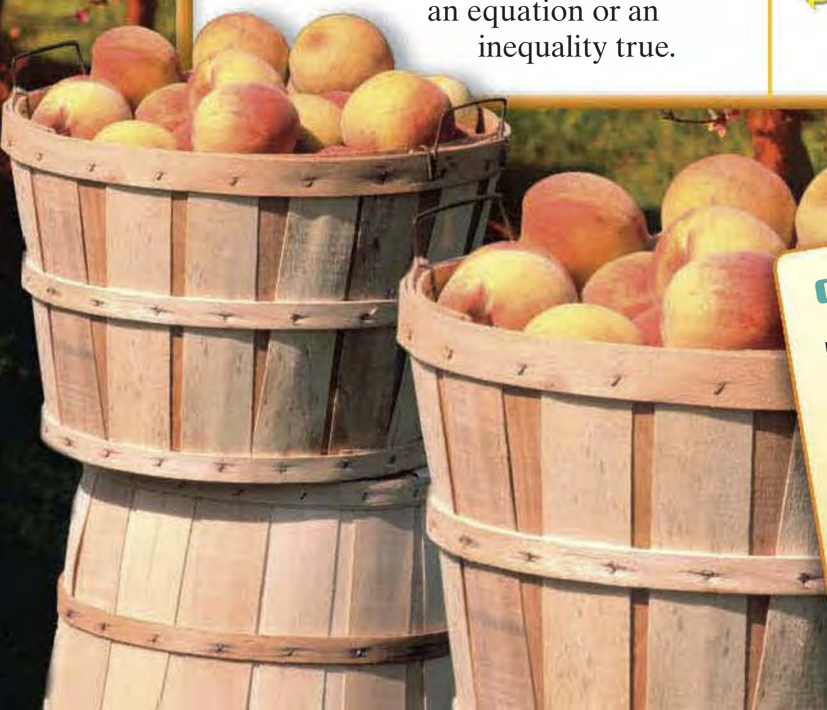
Go to PRACTICE BOOK, pp. 83–122

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES



## Critical Thinking

Daniela and Colin fill small, medium, and large baskets of peaches at the farmers' market. The small baskets hold half as many peaches as the medium baskets, and the medium baskets hold half as many peaches as the large baskets. What fractional part of a large basket does a small basket hold? What is that fractional part expressed as a decimal?

Algebra

4-1

# Rational Numbers

**Objective** To identify rational numbers • To identify rational numbers as terminating or nonterminating repeating decimals • To locate rational numbers on a number line

Inez is a production manager for a coal mining company. On Monday, she recorded that coal production increased by half a ton. The following day, she recorded that coal production fell three quarters of a ton. How can you use rational numbers to represent the changes in the number of tons of coal produced?

► A rational number is any number that can be written in fractional form, $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$.

- To represent the increase in production, use a positive rational number.
- To represent the decrease in production, use a negative rational number.

production *rose* half a ton → $\frac{1}{2}$

production *fell* three quarters of a ton → $-\frac{3}{4}$

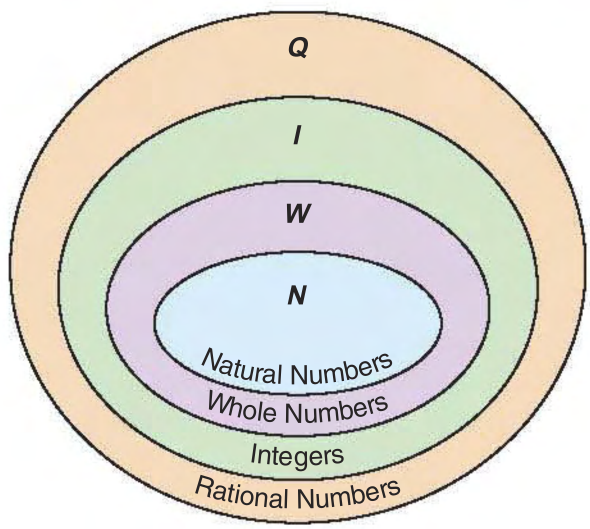
So the rational numbers $\frac{1}{2}$ and $-\frac{3}{4}$ represent the changes in the number of tons of coal produced.

The set of rational numbers, $Q$, contains:

the set of natural numbers, $N$: 1, 2, 3, . . .

the set of whole numbers, $W$: 0, 1, 2, 3, . . .

the set of integers, $I$: . . . , −3, −2, −1, 0, 1, 2, 3, . . .

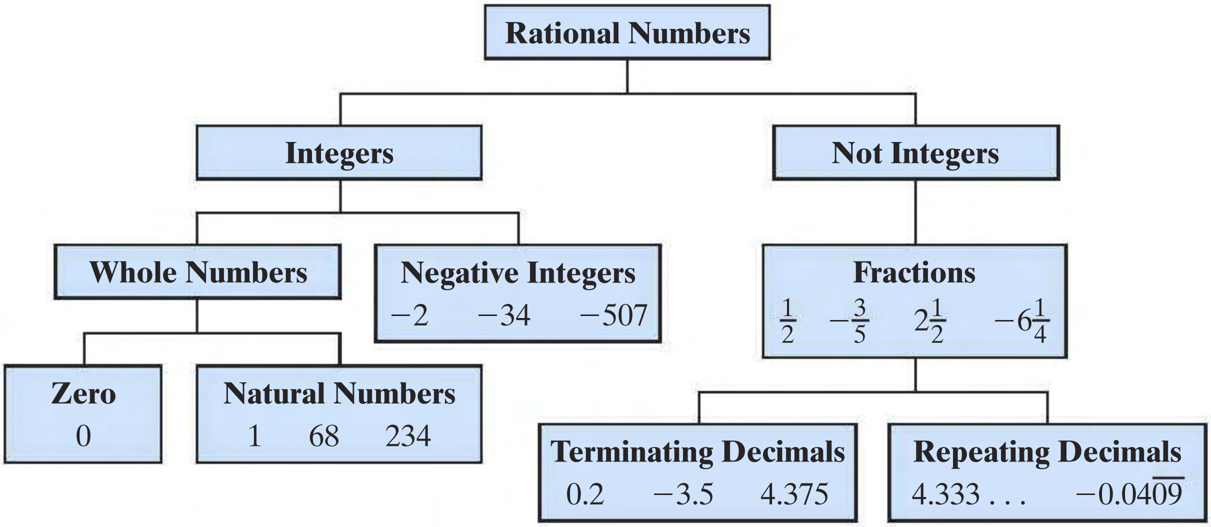



► Integers can be written in fractional form and in decimal form. For example, the integer 0 can be written in fractional form as $\frac{0}{1}$, and the integer −5 can be written as $\frac{-5}{1}$. The integer 4 can be written in decimal form as 4.0, and the integer −78 can be written as −78.0.

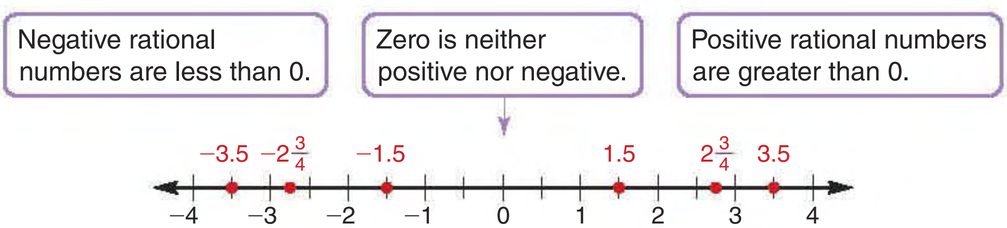
► The set of rational numbers, $Q$, also includes proper fractions, improper fractions, mixed numbers, terminating decimals, repeating decimals, ratios, and percents.

- A terminating decimal is a decimal that has a finite number of decimal places—for example, 0.5. Every terminating decimal can be written as a fraction having a denominator that is a power of 10.
- A repeating decimal is a decimal in which a digit or sequence of digits repeats without end. To show that a digit or digits repeat in the decimal, use bar notation. Place a bar over the digit or digits that repeat—for example, $0.5656\ldots = 0.\overline{56}$, and $8.1090909 = 8.1\overline{09}$.

▶ The diagram below shows the set of rational numbers.




▶ You can graph rational numbers on a number line. Each rational number represents one point on the number line. Every rational number has an opposite, which is also a rational number.




The opposite of 3.5 is −3.5.

$-(3.5) = -3.5$

The opposite of $2\frac{3}{4}$ is $-2\frac{3}{4}$.

$-\left(2\frac{3}{4}\right) = -2\frac{3}{4}$

The opposite of −1.5 is 1.5

$-(-1.5) = 1.5$

## Try These

**Identify each decimal as terminating or repeating.**

**1.** 7.4 **2.** 0.6 **3.** $0.\overline{6}$ **4.** 0.90252525. . .

**5.** 3.0 **6.** 8.88 **7.** 4.333. . . **8.** 9.010

**9.** Plot these points on a number line: 0.5, −1.75, −2.25, $\frac{1}{2}$, $-\frac{3}{4}$, $-1\frac{3}{4}$

**10.** Write the opposite of −6.78 and of $\frac{11}{45}$.

**11. Discuss and Write** Explain how you can tell when a decimal is a rational number.

Go to PRACTICE BOOK Lesson 4-1 for exercise sets.

Algebra
# 4-2

Update your skills. See pages 412 XII, 413 XIV.

# Equivalent Rational Numbers

**Objective** To write rational numbers in equivalent forms

Until the 21st century, stock prices were recorded as fractions. Today, those prices are shown as decimals. For example, a gain of \$2.25 per share used to be recorded as $2\frac{1}{4}$. Today, this is recorded as 2.25. How can a loss of $\frac{5}{8}$ per share be recorded as a decimal?

▶ You can express a rational number in fractional form as a decimal by dividing.

## Examples

**1** $\frac{4}{5} \rightarrow 4 \div 5 \rightarrow$

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ -\,4\,0 \\ \hline 0 \end{array}$$

← Write zeros as needed.

← The remainder is 0.

So $\frac{4}{5} = 0.8$, which is a terminating decimal.

**Remember:**
The fraction bar indicates division.

4.0 is the same as 4. You can write zeros to the right of the decimal point without changing the value.

**2** $\frac{1}{3} \rightarrow 1 \div 3 \rightarrow$

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ -\,9\downarrow\; \\ \hline 10\; \\ -\,9\downarrow \\ \hline 10 \\ -\,9 \\ \hline 1 \end{array}$$

← The digits in the quotient repeat.

← Each division has the same repeating remainder.

So $\frac{1}{3} = 0.333\ldots = 0.33\overline{3} = 0.\overline{3}$, which is a repeating decimal.

### Key Concept

**Terminating and Repeating Decimals**

When you divide the numerator of a fraction by its denominator and the division ends, or terminates, because the remainder is zero, the decimal is a terminating decimal.

When the digits in the quotient repeat because the remainder repeats, the decimal is a repeating decimal.

▶ Now write a loss of $\frac{5}{8}$ per share as a decimal.

$-\frac{5}{8}$ ← Represents the loss of $\frac{5}{8}$ per share.

$$\begin{array}{r} -0.\;6\;\;2\;\;5 \\ 8\overline{)-5.{}^{5}0{}^{2}0{}^{4}0} \end{array}$$

← Divide. Write zeros as needed.

$-\frac{5}{8} = -0.625$ ← terminating decimal

So the loss of $\frac{5}{8}$ per share can be recorded as the decimal $-0.625$.

Negative fractional numbers can be written in any of these three equivalent forms:
$-\frac{5}{8} = \frac{-5}{8} = \frac{5}{-8}$

You can rewrite $-\frac{5}{8}$ as $\frac{-5}{8}$.
So $-\frac{5}{8} = 8\overline{)-5}$.

## Examples

Write each fractional number as an equivalent decimal.

**1** $\frac{7}{2} \rightarrow 2\overline{)7.^{1}0} = 3.5 \rightarrow 3.5$

$\frac{7}{2} = 3.5$

**2** $\frac{9}{11} \rightarrow 11\overline{)9.0^{2}0^{9}0^{2}0} = 0.8181 \rightarrow 0.\overline{81}$

$\frac{9}{11} = 0.8181\ldots = 0.\overline{81}$

**Remember:** Write a bar above the decimal digits that repeat.

► To write a *mixed number* as an equivalent decimal:

**Remember:**
$2\frac{3}{8}$ means $2 + \frac{3}{8}$.
2.25 means 2 + 0.25.

1. Write the mixed number as the sum of an integer and a fraction. $2\frac{3}{8} = 2 + \frac{3}{8}$
2. Divide to rename the fraction as a decimal. $2 + 8\overline{)3.000} = 0.375$
3. Add the integer to the decimal. So $2\frac{3}{8} = 2 + 0.375 = 2.375$

► You can also write a *terminating decimal* as an equivalent fraction.

When the decimal is *less than* 1:

1. Read the decimal. $0.125 \rightarrow 125$ thousandths
2. Use the place-value name to determine the denominator. So $0.125 = \frac{125}{1000}$.

When the decimal is *greater than* 1:

1. Read the decimal. $2.25 \rightarrow 2$ and 25 hundredths
2. Write the decimal as the sum of an integer and a decimal. $2 + 0.25$
3. Use the place-value name to determine the denominator. $2 + \frac{25}{100}$
4. Write the sum as a mixed number. So $2.25 = 2 + \frac{25}{100} = 2\frac{25}{100}$

## Try These

**Write each fraction or mixed number as an equivalent decimal. Identify the decimal as *terminating* or *repeating*.**

**1.** $\frac{-1}{8}$ **2.** $\frac{5}{16}$ **3.** $2\frac{4}{5}$ **4.** $-1\frac{13}{20}$ **5.** $\frac{-12}{5}$ **6.** $-4\frac{2}{3}$

**Write each decimal as an equivalent fraction or mixed number.**

**7.** 1.25 **8.** −0.875 **9.** −1.6 **10.** 3.09 **11.** −0.7 **12.** −4.45

**13.** **Discuss and Write** Write $\frac{22}{7}$ as a decimal. Is the decimal a rational number? Explain why or why not.

Go to PRACTICE BOOK Lesson 4-2 for exercise sets.

Algebra

# 4-3

Update your skills. See pages 407, 408 IV.

# Compare and Order Decimals

**Objective** To compare decimals • To order decimals

A planet's solar illumination can be measured in units of mean illumination. When Saturn is closest to the Sun, its mean illumination is 0.012 units. When farthest from the Sun, Saturn's mean illumination is 0.0098 units. A greater number indicates a brighter illumination. Which number indicates a brighter illumination?

► To find the number for the brighter illumination, use place value to compare 0.0098 and 0.012.

Compare: 0.0098 _?_ 0.012

0.0098
0.0120 ← $\boxed{0.012 = 0.0120}$

$0 < 1$ ← The digits in the hundredths place are not the same. Compare the hundredths digits.

$0.012 > 0.0098$

So the number 0.012 indicates a brighter illumination.

### Key Concept

**Compare Decimals Using Place Value**

- Write one number under the other with the place values lined up.
- For terminating decimals, write zeros as needed so the numbers have the same number of places. For repeating decimals, write the repeating digits to equalize the number of decimal places.
- Start with the greatest place value. Compare the digits in each place-value position, moving right. Stop at the first place where the digits are not the same. Compare those digits. The decimal that has the digit with the greater place value is the greater decimal.

## Examples

**1** Compare: $94.36\overline{5}$ _?_ 94.3653

94.3655 ← $94.36\overline{5} = 94.3655\ldots$
94.3653

$5 > 3$ ← The digits in the ten-thousandths place are not the same. Compare. $0.0005\ldots > 0.0003$

So $94.36\overline{5} > 94.3653$.

**2** Compare: $-8.20309$ _?_ $-8.204$

$-8.20309$
$-8.20400$ ← $-8.204 = -8.20400$

$-3 > -4$ ← The digits in the thousandths place are not the same. Compare.

So $-8.20309 > -8.204$.

► To compare a fraction and a decimal, rename the fraction as an equivalent decimal. Then you can compare the decimals.

Compare: $-6.2$ _?_ $-6\frac{3}{8}$ ← unlike forms

$-6\frac{3}{8} = -6 + \left(-\frac{3}{8}\right)$ ← $-6$ plus $8\overline{)-3.000}$ $= -0.375$

$= -6.375$ ← $-6 + (-0.375)$

$-6.200$ _?_ $-6.375$ ← The digits in the tenths place are not the same. Compare the tenths digits.

$-2 > -3$

So $-6.2 > -6\frac{3}{8}$.

**Remember:** Negative fractional numbers can be written in any of these three equivalent forms: $-\frac{3}{8} = \frac{-3}{8} = \frac{3}{-8}$.

► When ordering numbers, write the numbers under each other, and align the place values. Moving from left to right, compare the digits in each place-value position.

- Order from *greatest to least*: 20.3463; 20.3782; 21.005; 20.6004

20.3463
20.3782
21.0050
20.6004

$1 > 0$ ← ones

So 21.005 is greatest.

The order is:
21.005; ?; ?; ?

20.3463
20.3782
20.6004

$6 > 3$ ← tenths

So 20.6004 is greatest.

The order is:
21.005; 20.6004; ?; ?

20.3463
20.3782

$7 > 4$ ← hundredths

So $20.3782 > 20.3463$.

The order from greatest to least: 21.005; 20.6004; 20.3782; 20.3463

- Order from *least to greatest*: −17.457; −17.419; −17.415; −17.451

−17.457
−17.419
−17.415
−17.451

$-5 < -1$ ← hundredths

So −17.457 and −17.451 are less than −17.419 and −17.415.

−17.457
−17.451

$-7 < -1$ ← thousandths

So $-17.457 < -17.451$.

The order is:
−17.457; −17.451; ?; ?

−17.419
−17.415

$-9 < -5$ ← thousandths

So $-17.419 < -17.415$.

The order from least to greatest:
−17.457; −17.451; −17.419; −17.415

- Order from *least to greatest*: 48.3463; −48.005; 48.3782; −48.05; 48.6004

Negative decimals are less than positive decimals. When ordering from least to greatest, first compare the negative decimals.

−48.005
−48.050

$-5 < 0$

So $-48.05 < -48.005$

The order from least to greatest:
−48.05; −48.005; ?; ?; ?

Then compare the positive decimals.

48.6004
48.3782
48.3463

48.3782
48.3463

$3 < 6$ $4 < 7$

So $48.3463 < 48.6004$; $48.3782 < 48.6004$; and $48.3463 < 48.3782$.

The order from least to greatest:
−48.05; −48.005; 48.3463; 48.3782; 48.6004

## Try These

**Compare these numbers. Write <, =, or >.**

**1.** −0.04 ? −0.005 **2.** 0.007 ? 0.0022 **3.** −0.501 ? −1.105 **4.** −0.724 ? $-\frac{3}{4}$

**Order from greatest to least.**

**5.** 0.04; 0.027; 0.2; 0.098 **6.** −4.141; −4.114; −4.441; −4.4 **7.** 0.719; −0.79; 0.79; −0.709

**Order from least to greatest.**

**8.** 1.662; 1.62; 0.62; 0.662 **9.** −0.5763; −0.58; −0.5367; −0.0875 **10.** −0.06; −0.062; −0.061; 0.006

**11.** **Discuss and Write** Compare $0.\overline{35}$ and 0.35. Is one greater? Explain why or why not.

Go to PRACTICE BOOK Lesson 4-3 for exercise sets.

Algebra

4-4

Update your skills. See pages 408 III, 409 V.

# Estimate Decimal Sums and Differences

**Objective** **To estimate decimal sums and differences using rounding, front-end, or clustering techniques**

The seventh grade class at Park School collects aluminum for recycling. The students collected 14.836 kg of aluminum the first month, 8.541 kg the second month, and 26.178 kg the third month. About how many kg of aluminum did the seventh grade class collect over the three-month period?

► Estimation can provide a quick answer when an exact answer is not necessary. It is also a way to check whether or not an exact answer is reasonable.

**Key Concept**

**Rounding Rules**

- Find the place to which you are rounding.
- Look at the digit to the right of that place. If the digit is *less than 5*, the digit being rounded stays the same. If the digit is *5 or greater*, the digit being rounded increases by 1.

**Method 1** Round each addend to the nearest whole number.

14.836 ← 8 > 5, so 4 rounds to 5 ones.
8.541 ← 5 ≥ 5, so 8 rounds to 9 ones.
+ 26.178 ← 1 < 5, so 6 stays as 6 ones.

$$\begin{array}{rcr} 14.836 & \rightarrow & 15 \\ 8.541 & \rightarrow & 9 \\ 26.178 & \rightarrow & +\ 26 \\ \hline & & 50 \end{array}$$

**Method 2** Round each addend to a specific decimal place value—for example, to the nearest tenth.

14.836 ← 3 < 5, so 8 stays as 8 tenths.
8.541 ← 4 < 5, so 5 stays as 5 tenths.
+ 26.178 ← 7 > 5, so 1 rounds to 2 tenths.

$$\begin{array}{rcr} 14.836 & \rightarrow & 14.8 \\ 8.541 & \rightarrow & 8.5 \\ 26.178 & \rightarrow & +\ 26.2 \\ \hline & & 49.5 \end{array}$$

So the seventh grade class collected about 49.5 kg or about 50 kg of aluminum for recycling.

► Estimates get closer to the exact answer as the place value of the digit being rounded decreases. So rounding to the nearest tenth gives an estimate closer to the exact answer than rounding to the ones place. Either estimate could be helpful, however, depending on its purpose.

Estimate the difference: 589.483 − 24.678

**Method 1** Round to the Nearest Ten

589.483 ← 9 > 5, so 8 rounds to 9 tens.
− 24.678 ← 4 < 5, so 2 stays as 2 tens.

$$\begin{array}{rcr} 589.483 & \rightarrow & 590 \\ 24.678 & \rightarrow & -\ 20 \\ \hline & & 570 \end{array}$$

**Remember:**
The symbol ≈ means *is approximately equal to.*

So, when each addend is rounded to the nearest ten, $589.483 - 24.678 \approx 570$.

**Method 2** Round to the Nearest Hundredth

When rounding decimals, drop all digits to the right of the digit being rounded.

589.483 ← 3 < 5, so 8 stays as 8 hundredths.
− 24.678 ← 8 > 5, so 7 rounds to 8 hundredths.

589.483 → 589.48
24.678 → − 24.68
564.80

So, when each addend is rounded to the nearest hundredth,
589.483 − 24.678 ≈ 564.80.

► You can use front-end estimation to estimate sums and differences of decimals when the front-end digits of all the numbers have the same place value.

**Key Concept**

**Front-End Estimation with Decimals**
- To estimate the sum or difference, add or subtract the front digits in each number.
- For addition only, adjust the estimate, using the digits in the next place to the right.

Estimate the sum:

5.863 + 1.185 + 6.51 + 0.007 + 4.39

5.863
1.185 } 0.8 + 0.1 is about 1.
6.51
0.007
+ 4.39 } 0.5 + 0.3 is about 1.
16

Estimate: 16 + 1 + 1 = 18

Estimate the difference:

9.482 − 4.189

9.482
− 4.189
5

Estimate: 9 − 4 = 5

► You can also estimate by "clustering." When several addends are close to a certain value, multiply that value by the number of addends.

Estimate by clustering: 9.32 + 8.7 + 9.13 ← All 3 numbers cluster around, or are close to, 9.

So 9.32 + 8.7 + 9.13 is about 3 • 9, or 27.

## Try These

**Estimate the sum or difference by rounding each number to the nearest tenth.**

**1.** 410.3 + 18.01 **2.** 17.6 − 2.98 **3.** 0.87 − 0.44 **4.** 7.3026 + 45.217 + 0.746

**Estimate the sum or difference by using front-end estimation.**

**5.** 8.132 + 7.0802 **6.** 9.2 − 3.7 **7.** −2.0775 + (−1.03391) + (−2.605)

**Estimate the sum by using clustering.**

**8.** 5.32 + 4.98 + 5.01 **9.** 8.06 + 7.77 + 8.45 + 7.9 **10.** 12.986 + 13.003 + 13.299 + 13.196

**11. Discuss and Write** Explain the steps you used to find the answer to exercise 3 above.

Go to PRACTICE BOOK Lesson 4-4 for exercise sets.

Algebra

# 4-5

Update your skills. See page 409 VI.

# Add and Subtract Decimals

**Objective** To add and subtract positive and negative decimals • To add more than two decimals

Over its first six months, a company posted a loss of \$1.05 million. Over its second six months, it posted a gain of \$2.6 million. In millions of dollars, what was the company's net gain or loss for its first year?

Let $n$ equal the net gain or loss.

$n = -1.05 + 2.6$ ← A loss of 1.05 is represented by −1.05, and a gain of 2.6 is represented by 2.6.

▶ You add decimals with *unlike* signs the same way that you add integers with unlike signs.

First estimate by rounding: $n \approx -1 + 3$, or 2

Then add: $n = -1.05 + 2.6$
Subtract the lesser absolute value from the greater absolute value.

$$\begin{array}{r} 2.60 \\ -\ 1.05 \\ \hline 1.55 \end{array}$$

You can write zeros to help align the digits in each place-value column.

$|2.6| > |-1.05|$, so the sum is positive.

$n = 1.55$

So the loss of \$1.05 million and the gain of \$2.6 million created a net gain of \$1.55 million.

1.55 is close to the estimate of 2, so the answer is reasonable.

**Check:** Use a calculator to check the answer.

(−) 1.05 + 2.6 ENTER 1.55

**Key Concept**

**Add Decimals with Unlike Signs**
- Find the absolute value of each addend.
- Subtract the lesser absolute value from the greater absolute value.
- Use the sign of the addend with the greater absolute value for the sum.

▶ To add decimals with *like* signs, align place values, and add the same way that you add integers.

**Key Concept**

**Add Decimals with Like Signs**
- Add the absolute values.
- Use the sign of the addends for the sum.

## Example

**1** Add: $-3.456 + (-2.3)$
First estimate by rounding: $-3 + (-2) = -5$
Then add.

$$\begin{array}{r} -3.456 \\ +\ (-2.300) \\ \hline \end{array} \rightarrow \begin{array}{r} |-3.456| \\ +\ |-2.300| \\ \hline \end{array} \rightarrow \begin{array}{r} 3.456 \\ +\ 2.300 \\ \hline -5.756 \end{array}$$

Both addends are negative, so the sum is negative.

$-5.756$ is close to the estimate of $-5$, so the answer is reasonable.
So $-3.456 + (-2.3) = -5.756$.

▶ To subtract positive and negative decimals, use the rule for subtracting integers, also called the Subtraction Principle.

**Subtraction Principle**
To subtract a rational number, add its opposite.
$a - b = a + (-b)$

## Examples

**1** Subtract: $-2.03 - (-5.7)$
First estimate by rounding: $-2 - (-6) = -2 + 6 = 4$
Then add: $-2.03 + 5.7$

Add the opposite of the number being subtracted.

Subtract the lesser absolute value from the greater absolute value.

$|5.7|$ is greater than $|-2.03|$, so the result is positive.

$$\begin{array}{r} -2.03 \\ \underline{-\ (-5.70)} \end{array} \rightarrow \begin{array}{r} -2.03 \\ \underline{+\ \ 5.70} \end{array} \rightarrow \begin{array}{r} |5.70| \\ \underline{-\ |-2.03|} \end{array} \rightarrow \begin{array}{r} 5.70 \\ \underline{-\ 2.03} \\ 3.67 \end{array}$$

3.67 is close to the estimate of 4, so the answer is reasonable.
So $-2.03 - (-5.7) = 3.67$.

**2** Subtract: $-6 - 2.08$
$-6 + (-2.08)$ ← Subtraction Principle

Estimate using rounding: $-6 + (-2) = -8$
Then add: $-6 + (-2.08)$

$$\begin{array}{r} -6.00 \\ \underline{+\ (-2.08)} \\ -8.08 \end{array}$$

← Both addends are negative, so the sum is negative.

So $-6 - 2.08 = -8.08$

**Check:** The estimate of $-8$ is close to $-8.08$, so the answer is reasonable.

**3** Subtract: $12.4 - (-0.009)$
$12.4 + 0.009$ ← Subtraction Principle

Estimate using rounding: $12 + 0 = 12$
Then add: $12.4 + 0.009$

$$\begin{array}{r} 12.400 \\ \underline{+\ 0.009} \\ 12.409 \end{array}$$

← Both addends are positive, so the sum is positive.

So $12.4 - (-0.009) = 12.409$.

**Check:** The estimate of 12 is close to 12.409, so the answer is reasonable.

► To add more than two rational numbers, follow the rules for the order of operations, and apply appropriate properties to evaluate the expression.

Evaluate $a + b + c$ when $a = -3.5$, $b = 8.1$, and $c = 3.5$.
$-3.5 + 8.1 + 3.5$ ← Substitute $-3.5$ for $a$, 8.1 for $b$, and 3.5 for $c$.
$-3.5 + 3.5 + 8.1$ ← Use the Commutative Property of Addition.
$(-3.5 + 3.5) + 8.1$ ← Use the Associative and the Inverse Properties of Addition.
$0 + 8.1$ ← Use the Identity Property of Addition.
$8.1$

## Try These

**Find the sum or difference.**

**1.** $11.7 + 3.3458$ **2.** $0.7 + (-2)$ **3.** $-7 - 6.1$ **4.** $5.8 - (-3.07)$

**5.** $8 - 0.0909$ **6.** $-0.0123 + (-4.1)$ **7.** $-2.4 - (-5.6)$ **8.** $-3.992 + 7.08$

**9.** Evaluate $a + b + c$ when $a = 4.035$, $b = -17$, and $c = 12.3$.

**10. Discuss and Write** For $a = -3.2$, $b = 7.1$, and $c = -4.5$, which is greater: $a + b + c$ or $a + b - c$? Explain your answer.

Go to PRACTICE BOOK Lesson 4-5 for exercise sets.

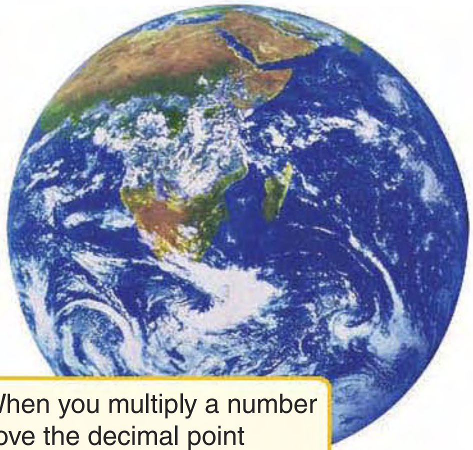
Algebra 4-6

Update your skills. See pages 411 IX, 412 XIII.

# Multiply Decimals

**Objective** To multiply positive and negative decimals • To multiply more than two decimal factors

James, who has started a healthy diet, wants to move to Mars. That is because on Mars, a person's weight is 0.38 as great as it is on Earth. If James weighs 122.5 pounds on his scale in San Diego, what would he weigh on Mars?



► To find out his weight on Mars, multiply:
0.38 • 122.5

First estimate by rounding:

$$\begin{array}{ccc} 0.38 & \bullet & 122.5 \\ \downarrow & & \downarrow \\ 0.4 & \bullet & 100 = 0.40 = 40 \end{array}$$

Round each factor to the greatest place that does not have a zero in it.

**Remember:** When you multiply a number by 100, you move the decimal point in the number 2 places to the right.

Then multiply the original numbers to find the actual product.

$$\begin{array}{rl} 122.5 & \leftarrow 1 \text{ decimal place} \\ \times\ \ 0.38 & \leftarrow 2 \text{ decimal places} \\ \hline 9800 & \\ +\ 3675 & \\ \hline 46.550 & \leftarrow 3 \text{ decimal places} \end{array}$$

James would weigh 46.55 pounds on Mars.

**Key Concept**

**Multiplying Decimals**
- Multiply as you would with integers.
- Count the *total number* of decimal places in both factors.
- Place the decimal point in the answer. The product has the same number of decimal places as the combined number of decimal places in both factors.

**Check:** 46.55 is close to the estimate of 40, so the answer is reasonable.

## Examples

**1** Multiply: 0.5 • (−0.18)

- First estimate: 0.5 • (−0.2) = −0.1
- Then multiply the original numbers to find the actual product.

**Think**
−0.18 is close to −0.2, and half of −0.2 is −0.1.

$$\begin{array}{rl} -0.18 & \leftarrow 2 \text{ decimal places} \\ \times\ \ \ 0.5 & \leftarrow 1 \text{ decimal place} \\ \hline -0.090 & \leftarrow 3 \text{ decimal places} \end{array}$$

Write a placeholder zero to fill the decimal place.

The product of two rational numbers with *unlike* signs is *negative*.

So 0.5 • (−0.18) = −0.090, and −0.090 = −0.09.

**Check:** −0.09 is close to the estimate of −0.1, so the answer is reasonable.

**2** Multiply: $-1.3 \bullet (-0.07)$

- First estimate by rounding: $-1 \bullet (-0.1) = 0.1$
- Then multiply the original numbers to find the actual product.

$$\begin{array}{r} -0.07 \\ \times \ -1.3 \\ \hline 21 \\ + \ \ 70 \\ \hline 0.091 \end{array}$$

$-0.07$ ← 2 decimal places
$-1.3$ ← 1 decimal place
$0.091$ ← 3 decimal places

Write a placeholder zero to fill the decimal place.

The product of two rational numbers with *like* signs is *positive*.

So $-1.3 \bullet (-0.07) = 0.091$.

**Check:** 0.091 is close to the estimate of 0.1, so the answer is reasonable.

When you multiply more than two factors, you can use the Commutative and Associative Properties of Multiplication to change the order and grouping of factors. These properties can help you use mental math.

Multiply: $4 \bullet 2.07 \bullet (-0.25)$

$4 \bullet (-0.25) \bullet 2.07$ ← Use the Commutative Property.

$[4 \bullet (-0.25)] \bullet 2.07$ ← Use the Associative Property.

$-1 \quad \bullet 2.07$ ← Simplify.

$-2.07$

**Remember:**
- When the number of negative factors is *even*, the product is *positive*.
- When the number of negative factors is *odd*, the product is *negative*.

You can also apply the Distributive Property when multiplying decimals.

**Distributive Property of Multiplication over Addition:**
$a(b + c) = (ab) + (ac)$

Evaluate $a(b + c)$ when $a = -2.5$, $b = -0.4$, and $c = 0.8$.

$a(b + c) \rightarrow -2.5(-0.4 + 0.8)$ ← Substitute $-2.5$ for $a$, $-0.4$ for $b$, and 0.8 for $c$.

$[(-2.5)(-0.4)] + [(-2.5)(0.8)]$ ← Use the Distributive Property of Multiplication over Addition.

$1.00 \quad + \quad (-2.00)$

$-1.00 \rightarrow -1$

## Try These

**Find the product.**

**1.** $2.6(0.013)$ **2.** $(-0.5)(-0.08)$ **3.** $-0.45 \bullet 5.02$ **4.** $(0.0375)(-2)$ **5.** $(-2.5)(4.1)(-2)$

**6.** $7.2(0.6 + 0.4)$ **7.** $-3.2(0.2 + 0.5)$ **8.** $-6.15(0.45)$ **9.** $(0.205)(-2)$ **10.** $(-5.01)(3)$

**11. Evaluate:** $(a + b)c$, when $a = 0.2$, $b = 0.6$, and $c = -1.2$

**12. Discuss and Write** Explain how using the properties of multiplication can help you multiply decimals more easily. Show examples to justify your answer.

Go to PRACTICE BOOK Lesson 4-6 for exercise sets.

Algebra

# 4-7 Estimate Decimal Products and Quotients

Update your skills. See page 411 X.

**Objective** To estimate decimal products and quotients by using rounding, compatible numbers, and powers of 10

A scientist in a pharmaceutical lab wants to change a dilution of 0.0057 of a certain chemical to one that is about 0.08 times as strong. What new dilution of the chemical will approximate the dilution she wants?

► You can round to estimate the product of two decimals that are each less than 1.

Estimate: $0.0057 \cdot 0.08$

$0.0057 \cdot 0.08$ ← Round each factor to its greatest nonzero place.

$0.0057 \rightarrow 0.006$ ← $7 > 5$, so 0.005 rounds to 6 thousandths.

$0.08 \rightarrow 0.08$ ← 0.08 stays as 8 hundredths.

Multiply: $0.006 \cdot 0.08 = 0.00048$ ← You need the same number of decimal places in the product as there are in the two factors combined.

So a dilution of 0.00048 will approximate the desired dilution.

**Think** The greatest nonzero place in a number is the place farthest to the left that has a digit that is not zero.

► You can also round to estimate the product of decimals when one factor is greater than 1 and one factor is less than 1.

Estimate: $2500.35 \cdot 0.036$

$2500.35 \cdot 0.036$ ← Round each factor to its greatest nonzero place.

$2500.35 \rightarrow 3000$ ← $5 = 5$, so 2000 rounds to 3 thousands.

$0.036 \rightarrow 0.04$ ← $6 > 5$, so 0.03 rounds to 4 hundredths.

Multiply: $3000 \cdot 0.04 = 120.00$ ← Count the decimal places in both factors. Show the same number of decimal places in the product as there are in the two factors combined.

So $2500.35 \cdot 0.036 \approx 120$.

► You can use *compatible numbers* to estimate the quotient of two decimals that are each greater than 1.

Compatible numbers are numbers that are easy to compute mentally.

Estimate: $310.2 \div 4.19$

Write the nearest compatible whole numbers for the dividend and divisor.

$310.2 \rightarrow 320$
$4.19 \rightarrow 4$

Divide: $320 \div 4 = 80$

So $310.2 \div 4.19 \approx 80$.

**Think** Use a division fact: $32 \div 4 = 8$
Even though 310.2 could round to 310, use 320 because it can be divided evenly by 4.

► Use compatible numbers to estimate the quotient of two decimals that are each less than 1.

- Estimate: $0.078 \div 0.048$
  You can use the division fact $8 \div 4 = 2$.
  8 hundredths ÷ 4 hundredths → $8 \div 4 = 2$
  So $0.078 \div 0.048 \approx 2$.

**Think**
$0.078 \rightarrow 0.08$
$0.048 \rightarrow 0.04$

- Estimate: $0.1905 \div 0.00617$
  You can use the division fact $18 \div 6 = 3$.
  180 thousandths ÷ 6 thousandths → $180 \div 6 = 30$
  So $0.1905 \div 0.00617 \approx 30$.

**Think**
$0.1905 \rightarrow 0.180$
$0.00617 \rightarrow 0.006$

► To estimate the quotient of a decimal less than 1 and a decimal greater than 1, find compatible numbers. For a divisor less than 1, multiply it by the least power of 10 that will make it a whole number. Multiply the dividend by that same power of 10.

**Remember:**
To multiply a number by a power of 10, move the decimal point to the *right* the same number of places as the number of zeros in the standard form of the power.

- To estimate $673.8 \div 0.084$, use the division fact $64 \div 8 = 8$.
  A dividend close to 673.8 is 640, and a divisor close to 0.084 is 0.08. Multiply both the dividend and the divisor by 100 to make the divisor a whole number.
  $[(100)(640)] \div [(100)(0.08)] = 640.00 \div 0.08 = 64{,}000 \div 8 = 8000$

  So $673.8 \div 0.084 \approx 8000$.

**Think**
$0.08(100) = 8$
$640(100) = 64{,}000$

- Estimate: $0.026 \div 41.99$
  Use the division fact $28 \div 4 = 7$.
  $0.028 \div 40 \rightarrow 40\overline{)0.0280}$ with quotient $0.0007$
  So $0.026 \div 41.99 \approx 0.0007$.

► There is more than one way to estimate a quotient. You can use different pairs of compatible numbers.
Estimate: $24.32 \div 4.52$

$24.32 \div 4.52 \rightarrow 20 \div 5 = 4$ | $24.32 \div 4.52 \rightarrow 25 \div 5 = 5$ | $24.32 \div 4.52 \rightarrow 24 \div 4 = 6$

## Try These

**Estimate each product by rounding.**

**1.** 0.93 • 0.52 **2.** 26.1 • 0.537 **3.** 0.016 • 0.135 **4.** 17.3 • 0.0003 **5.** 34.872 • 8.402

**Estimate each quotient by using compatible numbers.**

**6.** $31.62 \div 9.57$ **7.** $46.332 \div 836.4$ **8.** $0.075 \div 0.23$ **9.** $6.19 \div 0.035$ **10.** $0.094 \div 11.4$

**11. Discuss and Write** Explain how estimating products and quotients with decimal numbers is the same as, or different from, the way you estimate products and quotients with integers.

Go to PRACTICE BOOK Lesson 4-7 for exercise sets.

Algebra

# 4-8 Divide Decimals

Update your skills. See page 413 XIV.

**Objective** To divide positive and negative decimals • To evaluate division expressions containing decimals

The town of Holly Hills holds an annual 10-km race to raise money for charity. If a mile is about 1.6 kilometers, about how many miles is the Holly Hills 10-km race?

► To find about how many miles are in a 10-km race, divide: 10 km ÷ 1.6 km

First estimate by using compatible numbers: 10 ÷ 2 = 5
Then divide the actual numbers. Write placeholder zeros in the dividend to complete the division.

1.6(10) = 16 → $1.6\overline{)10.000}$ ← 10(10) = 100

$$\begin{array}{r} 6.25 \\ 16\overline{)100.00} \\ -\,96\phantom{.00} \\ \hline 4\,0\phantom{0} \\ -\,3\,2\phantom{0} \\ \hline 8\,0 \\ -\,8\,0 \\ \hline 0 \end{array}$$

**Key Concept**

**Dividing with Decimals**

- Multiply both the dividend and the divisor by the least power of 10 to form an integer divisor.
- Align the decimal point in the quotient with the decimal point in the dividend.
- Divide as with integers.
- Check by multiplying.

**Three ways to check:**

- Multiply the quotient by the original divisor, 1.6.

$$\begin{array}{r} 6.25 \\ \times\ 1.6 \\ \hline 3750 \\ +\ \ 625 \\ \hline 10.000 \end{array}$$

6.25 ← quotient
× 1.6 ← divisor
10.000 ← original dividend

- Use a calculator to check.

  Press 1.6 × 6.25 ENTER

  10. ← original dividend

- You can also compare the actual answer to the estimate. The actual answer, 6.25, is close to the estimate of 5, so the answer is reasonable.

So 10 ÷ 1.6 = 6.25. The race is about 6.25 miles long.

► You can find the quotient of two rational numbers that have unlike signs.

**Divide:** 3 ÷ (−0.075)

−0.075(1000) = −75 → $-0.075\overline{)3.000}$ ← 3(1000) = 3000

$$\begin{array}{r} -4\,0. \\ -75\overline{)3\,0\,0\,0.} \\ -3\,0\,0\phantom{0.} \\ \hline 0\,0\phantom{.} \end{array}$$

**Check:** Multiply (−40)(−0.075)

$$\begin{array}{r} -0.075 \\ \times\ -40 \\ \hline 3.000 \end{array}$$

3.000 ← original dividend

So 3 ÷ (−0.075) = −40.

The quotient of two rational numbers with *unlike* signs is *negative*.

▶ Sometimes you need to write zeros in the quotient as placeholders.

**Divide:** $-0.0168 \div (-5.6)$

$$-5.6.\overline{)-0.0.168} \quad \text{quotient } 0.003; \quad -168 \text{ gives remainder } 0$$

$-5.6(10) = -56$ → $-56\overline{)-0.168}$ ← $-0.0168(10) = -0.168$

The quotient of two rational numbers with *like* signs is *positive*.

**Check:** Multiply $0.003(-5.6)$

$$\begin{array}{r} -5.6 \\ \times\ 0.003 \\ \hline -0.0168 \end{array}$$

← The product is the same as the original dividend.

▶ Sometimes you need to round the quotient to a desired place.

**Divide:** $3.26 \div 0.06$
Round the quotient to the *nearest hundredth*.

**Think**
To round the quotient to the nearest *hundredth*, you will have to find the quotient to the nearest *thousandth*.

$$\begin{array}{r} 54.333... \\ 0.06.\overline{)3.26.000} \\ -30\phantom{000} \\ \hline 26\phantom{00} \\ -24\phantom{00} \\ \hline 20\phantom{0} \\ -18\phantom{0} \\ \hline 20\phantom{0} \\ -18\phantom{0} \\ \hline 20\phantom{0} \end{array}$$

20 ← same remainder
20 ← same remainder

There is a division pattern. The quotient is a repeating decimal.

**Check:**
Use a calculator to multiply.
Enter the repeating digit as many times as it will fit on the screen.

Press 54.33333333 × .06

3.26 ← original dividend

So $3.26 \div 0.06 = 54.333\ldots$
To the nearest hundredth, $54.333\ldots$ rounds to $54.33$.

▶ To evaluate an expression with more than two rational numbers, follow the rules for the order of operations.

Evaluate: $\frac{a}{b} + c$, when $a = 3.4$, $b = -0.2$, and $c = 1.7$

$\frac{3.4}{-0.2} + 1.7$ ← Substitute 3.4 for $a$, $-0.2$ for $b$, and 1.7 for $c$.

$-17 + 1.7$ ← Divide to simplify, then add.

$-15.3$

## Try These

**Find the quotient. Check by multiplying.**

1. $7.47 \div 0.6$
2. $0.1925 \div -7.7$
3. $-5.1 \div -0.03$
4. $a \div b$ when $a = -0.64$ and $b = 0.08$

5. **Discuss and Write** When you divide a positive integer by a decimal *less than 1*, will the quotient be *greater than* or *less than* the dividend? If the divisor is a decimal *greater than 1*, will the quotient be *greater than* or *less than* the dividend? Give examples to justify your answer.

Go to PRACTICE BOOK Lesson 4-8 for exercise sets.

Algebra
4-9

Update your skills. See page 407 I.

# Negative Exponents

**Objective** To express powers with negative exponents as decimals • To express decimals as powers with negative exponents • To simplify expressions with negative exponents

In science class, Gloria measures the length of a microorganism as 0.001 millimeter. She knows that this length is less than 1 millimeter and greater than 0 millimeters. How can she write this number as a power of 10?

►To write 0.001 as a power of 10, rewrite the decimal as a fraction. Apply the Law of Exponents for Division to write a decimal as a power of 10.

0.001 = one thousandth

$0.001 = \frac{1}{1000}$ ← Write the decimal in fraction form.

$= \frac{10^0}{10^3}$ ← Write the numerator and the denominator as powers of 10: $1000 = 10 \cdot 10 \cdot 10 = 10^3$.

$= 10^{0-3}$ ← Apply the Law of Exponents for Division.

$= 10^{-3}$

So the microorganism's length of 0.001 millimeter is $10^{-3}$ mm.

**Remember:** Any nonzero number with zero as an exponent equals 1. $a^0 = 1, a \neq 0$

Negative exponents are used to express decimals that have values between 0 and 1.

►The pattern in a place-value chart can also help you use negative exponents to express any decimal that is a power of 10.

Write 0.00001 as a power of 10 in exponent form.

| Thousands $10^3$ | Hundreds $10^2$ | Tens $10^1$ | Ones $10^0$ | Tenths $10^{-1}$ | Hundredths $10^{-2}$ | Thousandths $10^{-3}$ |
|---|---|---|---|---|---|---|
| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |

Notice that the number in each negative exponent is the same as the number of decimal places to the right of the decimal point. Using this pattern, you can see that 0.00001 can be expressed as $10^{-5}$.

►You can write a decimal for a power of 10 given in exponent form.

Write $10^{-2}$ as a decimal.

$10^{-2} = 0.01$

**Think**

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

The negative exponent indicates that the decimal is between 0 and 1.

The number 2 indicates that the decimal will have 2 decimal places.

►You can evaluate a power that has any nonzero integer as the base and a *negative* exponent.

**Key Concept**

**Negative Exponents**

Any nonzero number to the negative $n$ power can be written as 1 divided by the number to the $n$th power.

For any integer $n$ and any number $a$, $a \neq 0$, $a^{-n} = \frac{1}{a^n}$.

## Examples

Evaluate each expression.

**1** $5^{-3}$

$= \frac{1}{5^3}$ ← Apply the meaning of negative exponents.

$= \frac{1}{5 \bullet 5 \bullet 5}$ ← The exponent 3 means that 5 is used as a factor 3 times.

$= \frac{1}{125}$

So $5^{-3} = \frac{1}{125}$.

**2** $4^{-5}$

$= \frac{1}{4^5}$ ← Apply the meaning of negative exponents.

$= \frac{1}{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4}$ ← The exponent 5 means that 4 is used as a factor 5 times.

$= \frac{1}{1024}$

So $4^{-5} = \frac{1}{1024}$.

► To simplify expressions involving multiplication and division, apply the Laws of Exponents.

## Examples

Simplify each expression.

**1** $2^{-3}(2^{-2})$

$2^{-3+(-2)} = 2^{-5}$ ← Apply the Law of Exponents for Multiplication.

$= \frac{1}{2^5}$ ← the meaning of negative exponents

$= \frac{1}{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}$ ← The exponent 5 means that 2 is used as a factor 5 times.

$= \frac{1}{32}$

**2** $\frac{3^{-8}}{3^{-4}}$

$3^{-8-(-4)} = 3^{-4}$ ← Apply the Law of Exponents for Division.

$= \frac{1}{3^4}$ ← the meaning of negative exponents

$= \frac{1}{3 \bullet 3 \bullet 3 \bullet 3}$ ← The exponent 4 means that 3 is used as a factor 4 times.

$= \frac{1}{81}$

## Try These

**Write as a power of 10.**

**1.** 0.000001 **2.** 0.01

**Write as a decimal.**

**3.** $10^{-7}$ **4.** $10^{-8}$

**Evaluate each expression.**

**5.** $3^{-1}$ **6.** $4^{-3}$ **7.** $2^{-5}(2^{-3})$ **8.** $3^{-4}(3^{-2})$

**9.** $\frac{11^{-7}}{11^{-6}}$ **10.** $\frac{5^{-6}}{5^{-2}}$ **11.** $\frac{3^{-7}}{3^{-3}}$ **12.** $\frac{2^{-10}}{2^{-5}}$

**13. Discuss and Write** Do $\frac{5^6}{5^2}$ and $\frac{5^{-6}}{5^{-2}}$ have the same value? Explain.

Go to PRACTICE BOOK Lesson 4-9 for exercise sets.

Algebra

# 4-10

Update your skills. See page 410.

# Scientific Notation

**Objective** **To write the scientific notation for numbers given in standard form • To write the standard form for numbers given in scientific notation • To compare and order numbers in scientific notation**

PROBLEM-SOLVING STRATEGY 1

The world's smallest bird is the male bee hummingbird. The weight of this bird, which lives in Cuba, is 0.056 ounce. How can you express 0.056 ounce using a power of 10?

► To express the mass of the bird using a power of 10, write the number in scientific notation. A number is written in scientific notation when it is the product of two factors: one factor is a number, $a$, that is greater than or equal to 1 but less than 10; the other factor is a power of 10.

$a \times 10^b$, where $1 \le a < 10$ and $b$ is any integer

To write 0.056 in scientific notation, express it as a product of two factors.

$0.056 = \frac{56}{1000}$ ← Read the decimal, and write it as a fraction.

$= \frac{5.6}{100}$ ← To divide both terms by 10, move the decimal point 1 place to the left.

$= \frac{5.6}{10^2}$ ← Write 100 in exponential form.

$= 5.6 \times 10^{-2}$ ← Apply the meaning of negative exponents.

So 0.056 ounce can be written as $5.6 \times 10^{-2}$ ounce.

**Remember:**
- To multiply by a power of 10, move the decimal point 1 place to the right for each zero in the standard form of the divisor.
- To divide by a power of 10, move the decimal point 1 place to the left for each zero in the standard form of the divisor.

► Numbers greater than 1 can also be written in scientific notation.

$4{,}400{,}000{,}000 = 44 \times 100{,}000{,}000$

$= 4.4 \times 1{,}000{,}000{,}000$ ← Divide the first factor by 10, and multiply the second factor by 10 to adjust the place value.

$= 4.4 \times 10^9$ ← Rename as a power of 10 in exponential form.

$4.4 \ge 1$ and $4.4 < 10$

$1{,}000{,}000{,}000 = 10^9$

So 4,400,000,000 in scientific notation is $4.4 \times 10^9$.

► Use a shortcut to write numbers in scientific notation. Move the decimal point to the left or right to write the first factor as a number greater than or equal to 1 but less than 10. Count the number of decimal places moved to write the power of 10 in exponent form.

$0.056 = 5.6 \times 10^{-2}$ ← Move the decimal point 2 places to the right. The exponent of 10 is −2.

$67{,}800 = 6.78 \times 10^4$ ← Move the decimal point 4 places to the left. The exponent of 10 is 4.

**Key Concept**

**Scientific Notation**
- Move the decimal point to write the first factor as a number greater than or equal to 1 but less than 10. If the original number is less than 1, move the decimal point to the right. If the original number is greater than 10, move the decimal point to the left.
- Count the number of places the decimal point was moved. This number is the exponent for the power of 10.
- If the original number is 10 or greater, the exponent is positive. If the original number is less than 1, the exponent is negative.

► There are two ways to write the standard form for numbers in scientific notation.

- Write the power of 10 in standard form, then multiply the two factors.

$5.8 \times 10^6 = 5.8 \times 1{,}000{,}000 = 5{,}800{,}000$

$2.3 \times 10^{-4} = 2.3 \times 0.0001 = 0.00023$

- To show the multiplication by the power of 10, move the decimal point to the right if the exponent is positive and to the left if the exponent is negative. Use placeholder zeros as needed.

$5.8 \times 10^6$ 5.800000. ← The exponent is 6, so the decimal point moves right 6 places.

$2.3 \times 10^{-4}$ 0.00023 ← The exponent is −4, so the decimal point moves left 4 places.

► You can compare two numbers written in scientific notation.

- Compare: $6.7 \times 10^4$ __?__ $6.8 \times 10^6$ ← Compare the exponents: $10^4 < 10^6$.

So $6.7 \times 10^4 < 6.8 \times 10^6$.

- Compare: $2.47 \times 10^{-3}$ __?__ $1.8 \times 10^{-3}$ ← Compare powers of 10: $10^{-3} = 10^{-3}$ Then compare decimal factors: $2.47 > 1.8$

$2.47 > 1.8$

So $2.47 \times 10^{-3} > 1.8 \times 10^{-3}$

**Key Concept**

**Compare Expressions in Scientific Notation**

To compare two expressions in scientific notation, first compare the powers of 10. An expression with a greater power of 10 is greater. If the powers of 10 are the same, compare the decimal factors.

► To order numbers when some are in scientific notation and some are not, rename numbers as needed so that all are in like form. Then compare and order.

- Order from least to greatest:
$9.48 \times 10^4$; $5.9 \times 10^5$; $4.342 \times 10^4$

$5.9 \times 10^5$ ← greatest power

$4.342 \times 10^4$
$9.48 \times 10^4$ ← same power of 10

$4.342 < 9.48$ ← Compare decimal factors.

So the order from least to greatest is
$4.342 \times 10^4$; $9.48 \times 10^4$; $5.9 \times 10^5$.

- Order from least to greatest:
$7.501 \times 10^8$; 75,100,000; $7.51 \times 10^8$

$75{,}100{,}000 \rightarrow 7.51 \times 10^7$ ← Rename in like form; $10^7$ is least power.

$7.501 \times 10^8$; $7.51 \times 10^8$ ← $10^8 = 10^8$; compare the decimal factors.

$7.501 < 7.51$

So the order from least to greatest is
75,100,000; $7.501 \times 10^8$; $7.51 \times 10^8$.

## Try These

**Write in scientific notation.**

**1.** 4,000,000  **2.** 7,500,000

**Write in standard form.**

**3.** $4 \times 10^4$  **4.** $8.13 \times 10^3$

**Compare. Write <, =, or >.**

**5.** $7.234 \times 10^3$ __?__ $5.6 \times 10^5$  **6.** $3.45 \times 10^6$ __?__ $9.9 \times 10^{-4}$  **7.** $3.45 \times 10^2$ __?__ $9.5 \times 10^0$

**Order from least to greatest.**

**8.** $5.83 \times 10^7$; $2.22 \times 10^6$; $7.94 \times 10^7$

**9.** $1.09 \times 10^5$; $7.93 \times 10^6$; $1.4 \times 10^7$; 450,000

**10. Discuss and Write** Explain why $0.003 \times 10^3$ is not scientific notation.

Go to PRACTICE BOOK Lesson 4-10 for exercise sets.

Algebra 4-11

# Operations with Scientific Notation

**Objective** To multiply and divide numbers in scientific notation by a whole number • To use a calculator to multiply and divide numbers in scientific notation by a whole number

Earth travels at an average speed of $6.7 \times 10^4$ mi/h in its orbit around the Sun. What is the average distance around the Sun that Earth travels in 7 hours?

To find the average distance, multiply: $(6.7 \times 10^4) \times 7$

► To multiply a number expressed in scientific notation and a whole number, multiply the whole number and the decimal factor. Express the product in scientific notation. Then multiply the powers of 10.

**Remember:**
Law of Exponents for Multiplication
$a^m \bullet a^n = a^{m+n}, a \neq 0$

**Multiply:** $(6.7 \times 10^4) \times 7$

$(6.7 \times 7) \times 10^4$ ← Use the Commutative and Associative Properties.

$46.9 \times 10^4$ ← Multiply the whole number and the decimal factor.

$(4.69 \times 10^1) \times 10^4$ ← Express 46.9 in scientific notation.

$4.69 \times (10^1 \times 10^4)$ ← Use the Associative Property, then multiply.

$4.69 \times 10^{1+4}$ ← Apply the Law of Exponents for Multiplication.

$4.69 \times 10^5$

**Check:** $(6.7 \times 10^4) \times 7$

$(6.7 \times 10{,}000) \times 7$ ← Write the power of 10 in standard form.

$67{,}000 \times 7$ ← Multiply $6.7 \times 10{,}000$.

$469{,}000$ ← Multiply $67{,}000 \times 7$.

$4.69 \times 10^5$ ← Write 469,000 in scientific notation.

$4.69 \times 10^5 = 4.69 \times 10^5$ True

So Earth travels 469,000, or $4.69 \times 10^5$, miles around the Sun in 7 hours.

## Example

**1** Simplify. Write the product in scientific notation.

Multiply: $4(3.98 \times 10^{-5})$

$(4 \times 3.98) \times 10^{-5}$ ← Use the Associative Property.

$15.92 \times 10^{-5}$ ← Multiply.

$(1.592 \times 10^1) \times 10^{-5}$ ← Express 15.92 in scientific notation.

$1.592 \times (10^1 \times 10^{-5})$ ← Use the Associative Property.

$1.592 \times 10^{[1+(-5)]}$ ← Apply the Law of Exponents for Multiplication.

$1.592 \times 10^{-4}$

**Check:**

$4(3.98 \times 10^{-5})$

$4(3.98 \times 0.00001)$

$4(0.0000398)$

$0.0001592$

$1.592 \times 10^{-4}$

$1.592 \times 10^{-4} = 1.592 \times 10^{-4}$ True

So $4(3.98 \times 10^{-5}) = 1.592 \times 10^{-4}$.

► To divide a number expressed in scientific notation by a whole number, divide the decimal factor by the whole number. Express the result in scientific notation. Then multiply the powers of 10.

**Divide:** $\frac{2.4 \times 10^6}{3}$

$\frac{2.4 \times 10^6}{3} \rightarrow \frac{2.4}{3} \times 10^6$

$0.8 \times 10^6$

$(8 \times 10^{-1}) \times 10^6$ ← Write 0.8 in scientific notation.

$8 \times (10^{-1} \times 10^6)$ ← Use the Associative Property.

$8 \times 10^{(-1+6)}$ ← Apply the Law of Exponents for Multiplication.

$8 \times 10^5$

**Check:** $\frac{2.4 \times 10^6}{3}$

$\frac{2.4 \times 1{,}000{,}000}{3}$ ← Write the power of 10 in standard form.

$\frac{2{,}400{,}000}{3}$ ← Simplify.

$800{,}000$

$8 \times 10^5$

$8 \times 10^5 = 8 \times 10^5$ True

## Example

**1** Simplify. Write the quotient in scientific notation.

**Divide:** $\frac{3.6 \times 10^{-4}}{120}$

$\frac{3.6}{120} \times 10^{-4}$

$0.03 \times 10^{-4}$ ← Divide the decimal factor by the whole number.

$(3 \times 10^{-2}) \times 10^{-4}$ ← Write 0.03 in scientific notation.

$3 \times (10^{-2} \times 10^{-4})$ ← Use the Associative Property.

$3 \times 10^{[-2+(-4)]}$ ← Apply the Law of Exponents for Multiplication.

$3 \times 10^{-6}$

**Check:**

$\frac{3.6 \times 10^{-4}}{120}$

$\frac{3.6 \times 0.0001}{120}$

$\frac{0.00036}{120}$

$0.000003 = 3 \times 10^{-6}$

$3 \times 10^{-6} = 3 \times 10^{-6}$ True

► On the calculator, the EE $x^{-1}$ key is used to enter the exponent of a power of 10 that is being multiplied or divided.

Multiply: $(1.05 \times 10^9)(8)$

Press 1 . 0 5 2ND EE 9 × 8 ENTER

So $(1.05 \times 10^9)(8) = 8.4 \times 10^9$ or 8,400,000,000.

Divide: $(4.35 \times 10^{10}) \div 3000$

Press 4 . 3 5 2ND EE 10 ÷ 3000 ENTER

So $(4.35 \times 10^{10}) \div 3000 = 1.45 \times 10^7$ or 14,500,000.

## Try These

**Multiply or divide. Write the answer in scientific notation.**

**1.** $2(7.5 \times 10^3)$ **2.** $(6.3 \times 10^{-5})(6)$ **3.** $\frac{9.9 \times 10^8}{11}$ **4.** $\frac{2.15 \times 10^7}{5}$

**5.** $3(1.4 \times 10^6)$ **6.** $(4.1 \times 10^3)(9)$ **7.** $\frac{8.03 \times 10^{-6}}{803}$ **8.** $\frac{6.6 \times 10^{-4}}{600}$

**9. Discuss and Write** List the steps you would use to simplify $4(3.2 \times 10^{-5})$.

Go to PRACTICE BOOK Lesson 4-11 for exercise sets.

Algebra

4-12

Update your skills. See page 409 VI.

# Addition and Subtraction Equations with Decimals

**Objective** To apply the Subtraction Property of Equality to solve addition equations with decimals • To apply the Addition Property of Equality to solve subtraction equations with decimals

PROBLEM-SOLVING STRATEGY 2

A submersible rose 132.5 meters to reach a depth of 367.5 meters below sea level. What was the submersible's original depth?

To find the original depth of the submersible, write and solve an addition equation.

Let $x$ = original depth of the submersible.

$x$ meters + 132.5 meters = −367.5 meters

below sea level is negative

► To solve an addition equation with decimals, use the Subtraction Property of Equality.

**Solve:** $x + 132.5 = -367.5$

$x + 132.5 - 132.5 = -367.5 - 132.5$ ← Subtract 132.5 from both sides to isolate $x$.

$x = -500$

**Remember:**
**Subtraction Property of Equality:**
If $a = b$, then $a - c = b - c$.

**Check:** $x + 132.5 = -367.5$

$-500 + 132.5 \stackrel{?}{=} -367.5$ ← Substitute −500 for $x$, then simplify.

$-367.5 = -367.5$ True

The submersible's original depth was 500 meters below sea level.

## Examples

**1** **Solve:** $c + (-10.8) = -16.25$

$c + (-10.8) - (-10.8) = -16.25 - (-10.8)$

$c + (-10.8) + 10.8 = -16.25 + 10.8$

$c = -5.45$

**Check:** $c + (-10.8) = -16.25$

$-5.45 + (-10.8) \stackrel{?}{=} -16.25$

$-16.25 = -16.25$ True

**2** **Solve:** $4.86 + b = 1.36$

$4.86 - 4.86 + b = 1.36 - 4.86$

$b = -3.5$

**Check:** $4.86 + b = 1.36$

$4.86 + (-3.5) \stackrel{?}{=} 1.36$

$1.36 = 1.36$ True

► To solve a subtraction equation with decimals, use the Addition Property of Equality.

**Solve:** $m - (-9.06) = 8.47$

$m - (-9.06) + (-9.06) = 8.47 + (-9.06)$ ← Add −9.06 to both sides to isolate $m$.

$m = -0.59$

$m - (-9.06) = m + 9.06$ and
$m + 9.06 + (-9.06) = m + 0$

**Remember:**
**Addition Property of Equality:**
If $a = b$, then $a + c = b + c$.

To subtract an integer, add its additive inverse:
$a - b = a + (-b)$

**Check:** $m - (-9.06) = 8.47$

$-0.59 - (-9.06) \stackrel{?}{=} 8.47$ ← Substitute $-0.59$ for $m$.

$(-0.59) + (9.06) \stackrel{?}{=} 8.47$

$8.47 = 8.47$ True

Think
$-0.59 + 9.06 = 9.06 - 0.59$

► To solve a subtraction equation, you can write and solve a related sentence.

Related Sentences
$a - b = c$
so $a - c = b$
and $c + b = a$

**Solve:** $5.2 - k = 6.6$

$6.6 + k = 5.2$ ← Write a related sentence.

$6.6 - 6.6 + k = 5.2 - 6.6$ ← Subtract 6.6 from both sides to isolate $k$.

$k = -1.4$

**Check:** $5.2 - k = 6.6$

$5.2 - (-1.4) \stackrel{?}{=} 6.6$ ← Substitute $-1.4$ for $k$.

$5.2 + 1.4 \stackrel{?}{=} 6.6$

$6.6 = 6.6$ True

► You can combine numerical terms before solving an addition and subtraction equation.

**Solve:** $2.5 + y + (-1.8) - 0.5 = 0.8$

$2.5 + y + (-1.8) + (-0.5) = 0.8$ ← Use the additive inverse to rewrite as addition.

$y + 2.5 + [-1.8 + (-0.5)] = 0.8$ ← Use the Commutative and Associative Properties to group numbers and like signs.

$y + 2.5 + [-1.8 + (-0.5)] = 0.8$ ← Add the numbers with like signs.

$y + 2.5 + (-2.3) = 0.8$ ← Add the numbers with unlike signs.

$y + 0.2 = 0.8$

$y + 0.2 - 0.2 = 0.8 - 0.2$ ← Subtract 0.2 from both sides to isolate $y$.

$y = 0.6$

**Check:** $2.5 + y + (-1.8) - 0.5 = 0.8$

$2.5 + 0.6 + (-1.8) - 0.5 \stackrel{?}{=} 0.8$ ← Substitute 0.6 for $y$.

$2.5 + 0.6 + (-1.8) + (-0.5) \stackrel{?}{=} 0.8$ ← Rewrite subtraction as addition. Add numbers with like signs.

$3.1 + (-2.3) \stackrel{?}{=} 0.8$ ← Add the numbers with unlike signs.

$0.8 = 0.8$ True

## Try These

**Solve and check.**

**1.** $x - 11.7 = 28.9$ **2.** $54 = b + (-14.2)$ **3.** $-7.04 - x = -10.33$

**4.** $-88.3 = n - (-6.8)$ **5.** $w + 49.8 + 13.3 = 59.6$ **6.** $0.513 + x - 0.049 = 17.6$

**7. Discuss and Write** List the steps you would use to solve each equation: $n - 0.6 = 2$, $-(0.6) + n = 2$, and $n - 2 = 0.6$. Explain how the steps are the same and how they are different.

Go to PRACTICE BOOK Lesson 4-12 for exercise sets.

Algebra

4-13

Update your skills. See pages 412 XIII, 413 XIV.

# Multiplication and Division Equations with Decimals

**Objective** To apply the Division Property of Equality to solve multiplication equations with decimals • To apply the Multiplication Property of Equality to solve division equations with decimals

Carl and his friends made a paper-clip chain 8.05-km long, an impressive achievement by any standards. The world record, however, is 32.2 km! How many times greater than the length of Carl's chain is the length of the world's-record chain?

► To find how much longer the world's-record chain is, write a multiplication equation. Then use the Division Property of Equality to solve the equation.

Let $x$ = the number of times longer the world's-record chain is.

Equation: 8.05 km • $x$ = 32.2 km

Estimate: $8x = 32$ and $8(4) = 32$, so $x$ is about 4.

**Solve:** $8.05x = 32.2$

$$\frac{8.05x}{8.05} = \frac{32.2}{8.05} \quad \text{Divide both sides by 8.05 to isolate } x.$$

$$\frac{\cancel{8.05}^{1}x}{\cancel{8.05}_{1}} = \frac{\cancel{32.2}^{4}}{\cancel{8.05}_{1}} \quad \text{Simplify.}$$

$x = 4$ The answer is the same as the estimate, so the answer is reasonable.

**Check:** $8.05x = 32.2$

$8.05(4) \stackrel{?}{=} 32.2$ Substitute 4 for $x$.

$32.2 = 32.2$ True

The length of the world's-record paper-clip chain is 4 times greater than the length of Carl's 8.05-km chain.

**Remember:**
**Division Property of Equality:**
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

► Sometimes you need to combine like terms before solving a multiplication equation.

**Solve:** $y + 2.5y = -1.4$

$(y + 2.5y) = -1.4$ Add like terms, $1y + 2.5y$.

$$\frac{3.5y}{3.5} = \frac{-1.4}{3.5} \quad \text{Divide both sides by 3.5 to isolate } y.$$

$$\frac{\cancel{3.5}^{1}y}{\cancel{3.5}_{1}} = \frac{-1.4}{3.5} \quad \text{Simplify.}$$

$y = -0.4$

**Check:** $y + 2.5y = -1.4$

$-0.4 + 2.5(-0.4) \stackrel{?}{=} -1.4$ Substitute $-0.4$ for $y$.

$-0.4 + (-1) \stackrel{?}{=} -1.4$

$-1.4 = -1.4$ True

**Remember:**

**Rules for Multiplying Two Decimals**
- The product of *like* signs is positive.
- The product of *unlike* signs is negative.

**Rules for Dividing Two Decimals**
- The quotient of *like* signs is positive.
- The quotient of *unlike* signs is negative.

► You can also use the Division Property of Equality to solve an equation containing the opposite of a variable.

**Solve:** $-(z) = 3.2$

$-1(z) = 3.2$ Think: $-(z) = -1 \cdot z$

$\frac{-1z}{-1} = \frac{3.2}{-1}$ ← Divide both sides by $-1$ to isolate $z$.

$\frac{\cancel{-1}^{1}z}{\cancel{-1}_{1}} = \frac{3.2}{-1}$ ← Simplify.

$z = -3.2$

**Check:** $-(z) = 3.2$

$-(-3.2) \stackrel{?}{=} 3.2$ ← Substitute $-3.2$ for $z$.

$3.2 = 3.2$ True

► To solve a division equation, estimate the quotient. Then use the Multiplication Property of Equality to solve the equation.

Solve for $n$: $\frac{n}{-1.2} = 18.75$

Estimate.

$\frac{n}{-1} \approx 20 \rightarrow (-1)\frac{n}{-1} \approx 20(-1) \rightarrow n \approx -20$

**Remember:**
**Multiplication Property of Equality:**
If $a = b$, then $ac = bc$.

**Solve:**

$(-1.2)\frac{n}{-1.2} = 18.75(-1.2)$ ← Multiply both sides by $-1.2$ to isolate $n$.

$n = -22.5$

**Check:** $\frac{n}{-1.2} = 18.75$

$\frac{-22.5}{-1.2} \stackrel{?}{=} 18.75$ ← Substitute $-22.5$ for $n$, then simplify.

$18.75 = 18.75$ True

The answer, $-22.5$, is close to the estimate of 20, so the answer is reasonable.

## Try These

**Solve.**

**1.** $\frac{n}{-2} = 4.5$

**2.** $-2.4 = 3y$

**3.** $\frac{x}{0.2} = 0.4$

**4.** $0.5y + 1.5y = -1.8$

**5.** $\frac{w}{-0.6} = 1.4$

**6.** $n + 2.5n + 1.5n = 35$

**7. Discuss and Write** Explain how you can use estimation to check the solution of the equation $\frac{y}{0.556} = 0.08$. Estimate, solve, and check to justify your answer.

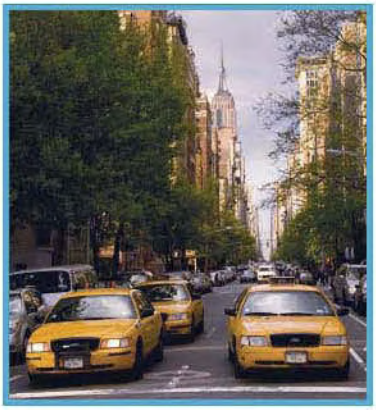
Go to PRACTICE BOOK Lesson 4-13 for exercise sets.

Algebra 4-14

# Solve Two-Step Equations with Decimals

**Objective** To solve two-step algebraic equations containing decimals by applying the appropriate properties of equality

Tom's Taxi Service charged Lisa \$3.50 for tolls and \$1.75 a mile. Lisa's ride in a Tom's Taxi cost \$10.15. How many miles was her taxi cab ride?



► To find the number of miles for Lisa's cab ride, write and solve an equation. When you solve an equation involving more than one operation, apply the appropriate properties of equality using inverse operations to isolate the variable.

Let $m$ = the number of miles for Lisa's cab ride.

Equation: $3.50 + 1.75m = 10.15$

total cost

cost of tolls

number of miles

cost per mile

**Solve:** $3.50 + 1.75m = 10.15$

$3.50 - 3.50 + 1.75m = 10.15 - 3.50$ ← Use the Subtraction Property of Equality to "undo" addition.

$1.75m = 6.65$ ← Simplify.

$\frac{1.75m}{1.75} = \frac{6.65}{1.75}$ ← Use the Division Property of Equality to "undo" multiplication.

$m = 3.8$

**Check:** $3.50 + 1.75m = 10.15$

$3.50 + 1.75(3.8) \stackrel{?}{=} 10.15$ ← Substitute 3.8 for $m$ in the original equation.

$3.50 + 6.65 \stackrel{?}{=} 10.15$ ← Simplify.

$10.15 = 10.15$ True

So Lisa's taxi ride was 3.8 miles long.

**Key Concept**

**Solve Two-Step Equations**

1. First, "undo" addition using the Subtraction Property of Equality, or "undo" subtraction using the Addition Property of Equality.
2. Then "undo" multiplication using the Division Property of Equality, or "undo" division using the Multiplication Property of Equality.

## Examples

**1** **Solve:** $2.7 - 1.5q = -1.8$

$2.7 - 2.7 - 1.5q = -1.8 - 2.7$ ← Subtract 2.7 from both sides to "undo" addition.

$\frac{-1.5q}{-1.5} = \frac{-4.5}{-1.5}$ ← Divide both sides by $-1.5$ to "undo" multiplication.

$q = 3$

**Check:**

$2.7 - 1.5q = -1.8$

$2.7 - 1.5(3) \stackrel{?}{=} -1.8$ ← Substitute 3 for $q$.

$2.7 - 4.5 \stackrel{?}{=} -1.8$ ← Simplify.

$-1.8 = -1.8$ True

**2** **Solve:** $-0.3a + (-3.2) = 4$

Think
$4 - (-3.2) = 4 + 3.2$

$-0.3a + (-3.2) - (-3.2) = 4 - (-3.2)$ ← Subtract $-3.2$ from both sides.

$\dfrac{-0.3a}{-0.3} = \dfrac{7.2}{-0.3}$ ← To "undo" multiplication, divide both sides by $-0.3$.

$a = -24$

**Check:** $-0.3a + (-3.2) = 4$

$-0.3(-24) + (-3.2) \stackrel{?}{=} 4$ ← Substitute $-24$ for $a$.

$7.2 + (-3.2) \stackrel{?}{=} 4$ ← Simplify.

$4 = 4$ True

**3** **Write an equation for the problem situation. Solve and check.**

Eric bought a sweatshirt. Five dollars more than half what he paid is \$23.50. How much did he pay for the sweatshirt? Let $p$ represent the amount Eric paid.

Equation: $\dfrac{p}{2} + 5 = 23.50$

**Solve:** $\dfrac{p}{2} + 5 = 23.50$

$\dfrac{p}{2} + 5 - 5 = 23.50 - 5$ ← Subtract 5 from both sides.

$(2)\dfrac{p}{2} = 18.50(2)$ ← Multiply both sides by 2.

$p = 37$

**Check:** $\dfrac{p}{2} + 5 = 23.50$

$\dfrac{37}{2} + 5 \stackrel{?}{=} 23.50$ ← Substitute 37 for $p$.

$18.50 + 5 \stackrel{?}{=} 23.50$ ← Simplify.

$23.50 = 23.50$ True

So Eric paid \$37 for the sweatshirt.

## Try These

**Solve.**

**1.** $12.6g - 4.8 = 33$
**2.** $\dfrac{w}{9.2} + 7 = 6.5$
**3.** $9k + 4.5 = 2.7$
**4.** $5.1 - 2.3z = 12$
**5.** $2.7h + 4.1 = 12.2$
**6.** $7.8 - 2.4y = 3$
**7.** $\dfrac{28p}{2} - 5 = 8.4$
**8.** $\dfrac{x}{-3.5} + 4.1 = 7.2$

**Write and solve an equation for the problem situation.**

**9.** Hilda bought a flashlight. The flashlight cost \$9.50, and each battery cost \$1.75. She spent a total of \$13 on the flashlight and the batteries. How many batteries did she buy?

**10.** **Discuss and Write** Identify the properties of equality you used in solving the equation for exercise 4 above. Explain each use of a property of equality.

Go to PRACTICE BOOK Lesson 4-14 for exercise sets.

# 4-15 Rename Metric Units of Measure

**Objective** To rename metric units of length, capacity, and mass

PROBLEM-SOLVING STRATEGY 7

The strenuous Wild Cave Tour hike in Kentucky's Mammoth Cave National Park covers a distance of about 9170 meters. How many centimeters and how many kilometers are equivalent to 9170 meters?

To find the number of centimeters and kilometers, rename meters as centimeters and as kilometers.

► To convert from one metric unit to another, multiply or divide by a power of 10.

| Metric Units of Length | | | | | | |
|---|---|---|---|---|---|---|
| **thousands** | **hundreds** | **tens** | **ones** | **tenths** | **hundredths** | **thousandths** |
| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |
| kilometer (km) | hectometer (hm) | dekameter (dam) | **meter (m)** | decimeter (dm) | centimeter (cm) | millimeter (mm) |

*Multiply* by a power of 10 to rename larger units as smaller units. There will be *more* smaller units.

9170 m = ? cm

1 m = 100 cm

Multiply m by 100 to change m to cm.

9170(100) = 917 000, so 9170 m = 917 000 cm

*Divide* by a power of 10 to rename smaller units as larger units. There will be *fewer* larger units.

9170 m = ? km

1000 m = 1 km

Divide m by 1000 to change m to km.

9170 ÷ 1000 = 9.17, so 9170 m = 9.17 km

So 917 000 cm and 9.17 km are equivalent to 9170 m.

The meter is the base unit of measure for length.
The liter is the base unit of measure for capacity.
The gram is the base unit of measure for mass.

► Rename units of capacity and mass the same way you rename metric units of length.

| Metric Units of Capacity and Mass | | | | | | |
|---|---|---|---|---|---|---|
| **thousands** | **hundreds** | **tens** | **ones** | **tenths** | **hundredths** | **thousandths** |
| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |
| kiloliter (kL) | hectoliter (hL) | dekaliter (daL) | **liter (L)** | deciliter (dL) | centiliter (cL) | milliliter (mL) |
| kilogram (kg) | hectogram (hg) | dekagram (dag) | **gram (g)** | decigram (dg) | centigram (cg) | milligram (mg) |

## Examples

**1** Rename larger units as smaller units.

- 240 L ? cL
  1 L = 100 cL and (100)240 = 24,000
  So 240 L = 24 000 cL.
- 38 g = ? cg
  1 g = 100 cg and (100)38 = 3800
  So 38 g = 3800 cg.

**2** Rename smaller units as larger units.

- 240 L = ? kL
  1000 L = 1 kL and 240 ÷ 1000 = 0.240
  So 240 L = 0.24 kL.
- 38 g = ? kg
  1000 g = 1 kg and 38 ÷ 1000 = 0.038
  So 38 g = 0.038 kg.

► You can also use a shortcut to rename metric units. Move the decimal point the same number of places and in the same direction as the places between the given unit and the new unit on the metric units chart.

**Remember:**
To multiply by a power of 10, move the decimal point to the right.
To divide by a power of 10, move the decimal point to the left.

## Examples

**1** 5.8 hm = _?_ dm

**Think**
hectometers to decimeters →
Move 3 places to the right.

5.8 hm → 5.800. → 5800 dm

**2** 73.6 mm = _?_ dam

**Think**
millimeters to dekameters →
Move 4 places to the left.

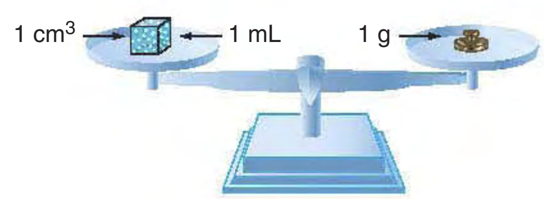
73.6 mm → 0.0073.6 → 0.00736 dam

► In science, some metric units are related to one another when measuring water.

- A cube that measures 1 centimeter along each edge has a volume of 1 cubic centimeter (1 $cm^3$).

  $1\ cm^3 = 1\ cm \cdot 1\ cm \cdot 1\ cm$

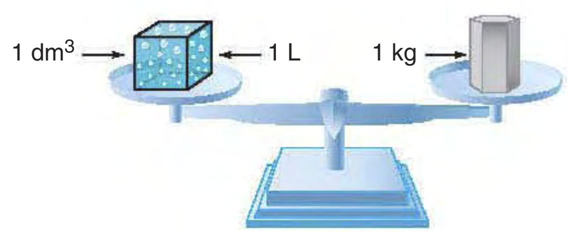
  The cube will hold 1 milliliter of water, which has a mass of 1 gram.




- A cube that measures 1 decimeter along each edge has a volume of 1 cubic decimeter (1 $dm^3$).

  $1000\ cm^3 = 1\ dm^3$
  $1\ dm^3 = 1\ dm \cdot 1\ dm \cdot 1\ dm$

  The cube will hold 1 liter of water, which has a mass of 1 kilogram.




## Try These

**Rename each unit of measure.**

**1.** 3 m = _?_ km
**2.** 99.5 m = _?_ cm
**3.** 0.41 g = _?_ kg
**4.** 6.1 kL = _?_ L
**5.** 2483 mL = _?_ L
**6.** 18.4 dag = _?_ dg

**Write the capacity and the mass of each cube of water.**

**7.** Volume of cube: 8 $cm^3$
**8.** Volume of cube: 15 $dm^3$

**9.** **Discuss and Write** You can divide by a power of 10 to rename smaller metric units as larger metric units. Explain why, when renaming 3.21 millimeters as meters, dividing by 1000 results in the same answer as multiplying by 0.001.

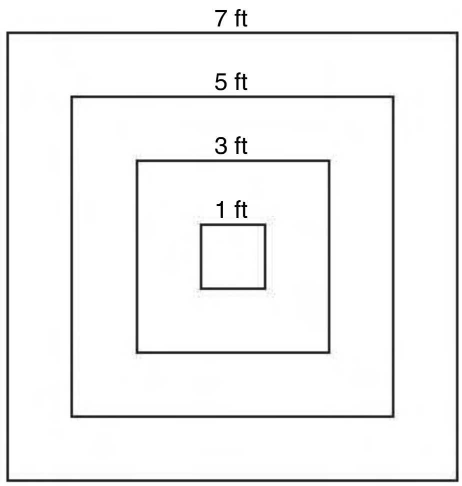
Go to PRACTICE BOOK Lesson 4-15 for exercise sets.

# 4-16 Problem Solving: Review of Strategies

Read › Plan › Solve › Check

**Objective** To solve problems using a variety of strategies

**Problem:** At the right is a part of a design made up of concentric squares (that is, squares with the same center). The entire design is made up of 50 squares. What is the sum of the perimeters of all 50 squares?




**Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** A design is made up of concentric squares with side lengths of 1 foot, 3 feet, 5 feet, 7 feet, 9 feet, and so on.

There is a total of 50 squares in the design.

**Question:** What is the sum of the perimeters of all the squares in the design?

**Select a strategy.**

There are many ways to approach this problem. Here are two possibilities:

- You can use the strategy *Find a Pattern*.
- You can use the strategy *Organize the Data* to analyze the situation.

**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Apply the strategy.**

### ► Method 1: Find a Pattern

Starting from the smallest square and moving outward, you find that the side lengths are the sequence of the first 50 odd numbers: 1, 3, 5, 7, . . . , 99. Because each square has four sides, if you add the numbers representing each side length in this sequence and then multiply the sum by 4, the result will be the total of the perimeters.

So you need to compute the sum $1 + 3 + 5 + 7 + \ldots + 99$. Find the sum of the first term, the sum of the first two terms, the sum of the first three terms, and so on. See if you can discover a pattern.

$1 = 1$
$1 + 3 = 4$
$1 + 3 + 5 = 9$
$1 + 3 + 5 + 7 = 16$
$1 + 3 + 5 + 7 + 9 = 25$

Notice that the sums are perfect squares. In fact, each sum is the square of the number of terms being added.

$1 = 1^2$
$1 + 3 = 2^2$
$1 + 3 + 5 = 3^2$
$1 + 3 + 5 + 7 = 4^2$
$1 + 3 + 5 + 7 + 9 = 5^2$

Extending this pattern, you find that the sum of all 50 terms will be $50^2$, or 2500. The total of the perimeters, therefore, is 4(2500 feet) = 10,000 feet.

### ▶ Method 2: Organize Data

As you did when using Method 1, first find the sum of the first 50 odd numbers.

$1 + 3 + 5 + 7 + 9 + 11 + \ldots + 89 + 91 + 93 + 95 + 97 + 99$

Adding these numbers in order could take a long time. However, if you look carefully, you can find a way to *organize* the terms to make computing the sum very simple. Notice that the sum of the first term and the last term, 1 + 99, is 100. The sum of the second term and the second to the last term, 3 + 97, is also 100. You can pair up all the terms in this way to form sums of 100.

$1 + 3 + 5 + 7 + \ldots + 93 + 95 + 97 + 99$

$7 + 93 = 100$

$5 + 95 = 100$

$3 + 97 = 100$

$1 + 99 = 100$

There are 50 odd numbers from 1 to 99, so there are 25 pairs, each with a sum of 100. So the total is 25(100), or 2500. The sum of one set of side lengths (that is, the length of one side of each of the 50 squares) is 2500 feet.

As you did when using Method 1, you can find the total perimeter by multiplying this sum by 4: 4(2500 feet) = 10,000 feet. The total perimeter is 10,000 feet.

**Check to make sure your answer makes sense.**

Because you found the same solution using two different strategies, you can be reasonably certain the solution is correct. If you wish, you can check by computing $1 + 3 + 5 + \ldots + 99$ on a calculator and multiplying the result by 4.

# Enrichment: Binary Numbers

**Objective** **To explore place value in the binary system • To convert between decimal and binary numbers • To relate binary numbers to "on-off" applications**

Computers operate at high speed because of the design of their electronic circuits. You can use binary numbers to describe "on-off" situations within the computer circuitry.

Place value in the binary system works just as it does in the decimal system. In the base-10 system, each place is a power of 10 (that is, $10^0$, $10^1$, $10^2$, and so on.) In the binary system, each place is a power of 2 (that is, $2^0$, $2^1$, $2^2$, and so on).

- The chart above at the right shows the place values of the digits in the base-10 number 46.
- In the binary system, the same number is written as $101110_{two}$. The chart at the right shows the place value of each digit in this number.

| thousands | hundreds | tens | ones |
|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | $10^0$ |
| | | 4 | 6 |

| thirty-twos | sixteens | eights | fours | twos | ones |
|---|---|---|---|---|---|
| $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 1 | 0 | 1 | 1 | 1 | 0 |

## Convert a Binary Number to a Decimal Number

Convert the binary number $101110_{two}$ to decimal form.

- Express the number in expanded binary notation and evaluate it.

$$\begin{aligned} 101110_{two} &= (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &= 32 + 0 + 8 + 4 + 2 + 0 \\ &= 46 \end{aligned}$$

## Convert a Decimal Number to a Binary Number

Convert 46 to binary form.

1. Find the greatest power of 2 that is less than or equal to the number. Subtract that power from the number.
2. Find the greatest power of 2 that is less than or equal to the difference and subtract that from the difference. Continue until the difference is 0.

$2^5$, or 32 ← $32 \le 46$
$46 - 32 = 14$
$2^3$, or 8 ← $8 \le 14$
$14 - 8 = 6$
$2^2$, or 4 ← $4 \le 6$
$6 - 4 = 2$
$2^1$, or 2 ← $2 \le 2$
$2 - 2 = 0$

3. Make a chart. Write 1s in the places for all the powers of 2 you subtracted. Write 0s in the other places.

| 1 | 0 | 1 | 1 | 1 | 0 |
|---|---|---|---|---|---|
| $2^5$ (32) | $2^4$ (16) | $2^3$ (8) | $2^2$ (4) | $2^1$ (2) | $2^0$ (1) |

So 46 in the base-10 system is $101110_{two}$ in the binary system.

## Try These

**Write each as a binary number.**

**1.** 93 **2.** 81 **3.** 64

**Write each as a decimal number.**

**4.** $100100_{two}$ **5.** $101101_{two}$ **6.** $111111_{two}$

**7. Discuss and Write** Compare the binary and decimal place-value systems.

Go to PRACTICE BOOK page 115 for exercise sets.

# Test Prep: Extended-Response Questions

## Strategy: Organize Information

Extended response questions often have two or more parts and usually require you to show or explain your work. When you solve an extended-response item, *make notes* to help organize your thinking about the steps you will follow.

### Sample Test Item

Students at a middle school need to raise \$2126.25 to pay for yearbooks for each student. There are 97 sixth graders, 112 seventh graders, and 106 eighth graders at the school.

**Part A**
What is the price of one yearbook?

**Part B**
A DVD adds \$0.95 to each yearbook's cost. What is the cost of a yearbook with a DVD?

**Part C**
Another 18 yearbooks will be ordered for faculty. What is the total cost of yearbooks with DVDs for students and faculty?
***Show all your work.***

**Read the whole test item carefully.**

- Reread the item to identify needed information.
- Make notes on the steps needed to solve the problem.
  1. Find how many students in all and the price of 1 yearbook.
  2. Find the cost of 1 yearbook with a DVD.
  3. Find the total cost of all yearbooks with DVDs.

### Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply an appropriate strategy for solving the problem.

To solve **Part A**, find the total number of students. Divide the total cost by the number of students to find the price of one yearbook.
97 + 112 + 106 = 315
\$2126.25 ÷ 315 = \$6.75
**Answer:** One yearbook costs \$6.75.

To solve **Part B**, add the cost of a DVD to the cost of each yearbook.
\$6.75 + \$0.95 = \$7.70
**Answer:** One yearbook with DVD costs \$7.70.

To solve **Part C**, add the number of faculty to the total number of students. Multiply the sum by the cost of one yearbook with DVD.
18 + 315 = 333
333 • \$7.70 = \$2564.10
**Answer:** The total cost of yearbooks is \$2564.10.

### Item Analysis

**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Estimate to be sure that your answer makes sense.

**Part A** There are about 300 students. One yearbook is about \$7. \$2100 ÷ 300 = \$7

**Part B** A yearbook with a DVD costs about \$1 more. \$7 + \$1 = \$8

**Part C** About 20 more yearbooks are needed. \$8(300 + 20) = \$2400 + \$160 = \$2560

The estimates are close to the actual answers, so the answers are reasonable.

**Try These** Item 1 is partially worked out for you.

**Solve. Make notes to help organize your thinking.**

**1.** The radius at the equator of the Sun is about $4.3 \times 10^5$ miles.

**Part A**
Write the radius of the Sun in standard notation.

**Part B**
The radius at the equator of Jupiter is about $\frac{1}{10}$ the radius of the Sun. Write the radius of Jupiter in standard notation and in scientific notation.
***Show all your work.***

**Read the test item for a general idea of the problem.**

- Reread the item.
- Make notes about the steps you must complete.
  1. Write the radius of the Sun in standard notation.
  2. Find the radius of Jupiter.
  3. Write the radius of Jupiter in standard notation and in scientific notation.

**Solve the problem.**

- Apply appropriate rules, definitions, and properties.

To solve **Part A**, apply the rules for rewriting numbers in standard notation.

To solve **Part B**, apply the rules for multiplying fractions.

## Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the items.**

- Analyze your answers. Do they make sense?
  Work backwards from the radius of Jupiter to find the radius of the Sun.

**2.** The table shows the change in four stock prices during a single day.

| Stock | Price Change from Opening to Closing ($) |
|---|---|
| A | −1.25 |
| B | 0.40 |
| C | 1.00 |
| D | −3.00 |

**Part A**
Which stock had the greatest change from opening to closing price? Explain.

**Part B**
Stock B started the day at $20 a share. An investor bought 50 shares of Stock B at its price at the end of the day. How much did the investor pay in all for the shares?
***Show all your work.***

**3.** The speed of light is $1.86 \times 10^5$ miles per second. If Earth is 92,900,000 miles from the Sun, about how long does it take light energy to travel from the Sun to Earth? Write your answer in standard notation and in scientific notation.

Go to PRACTICE BOOK pages 117–118 for exercise sets.

# Rational Numbers: Fractions

CHAPTER 5

## In This Chapter You Will:

- Find the prime factorization of a number
- Find the greatest common factor (GCF) and the least common multiple (LCM)
- Compare and order rational numbers
- Add, subtract, multiply, and divide positive and negative fractions and mixed numbers
- Use properties of rational numbers
- Solve equations and inequalities containing rational numbers
- Rename customary units of measure
- Apply the strategy: *Make a Drawing*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- A rational number is any number that can be written in the fractional form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$.
- A repeating decimal is a decimal in which a digit or sequence of digits repeats without end.
- Scientific notation shows a number as the product of two factors: $a$, any number greater than or equal to 1 but less than 10; and a power of 10 in exponent form, $10^b$.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 123–166

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

## Critical Thinking

Earth's moon has a mass of $7.3477 \times 10^{22}$ kg. Earth itself has a mass of $5.9742 \times 10^{24}$ kg. Triton, which is one of the moons of Neptune, has a mass of $2.147 \times 10^{22}$ kg, while Neptune has a mass of $1.0243 \times 10^{26}$ kg. Which moon's mass represents a greater part of the mass of its planet?

# 5-1 Prime Factorization

**Objective** To find the prime factorization of a number

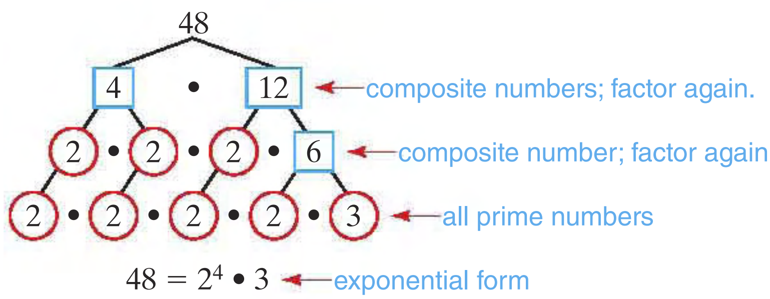
A prime number is a whole number greater than 1 that has *exactly two* factors, itself and 1. A composite number is a whole number greater than 1 that has *more than two* factors. The number 1, which has only one factor, is neither prime nor composite.

**Remember:**
Factors are numbers that are multiplied to find a product. You can *factor* a number or an expression by writing it as a product of its factors.

Prime factorization is a way of showing a composite number as the product of prime numbers. Except for the order of the factors, every composite number has a unique prime factorization.

► To find the prime factorization of a number, you can use a factor tree. Find the prime factorization of 48.

- Write 48 at the top of the tree.
- Choose any two whole number factors of 48.
- If any factor is *not* prime, rewrite it as a product of two factors.
- Write the prime factors that repeat in exponential form. List bases in least to greatest order.



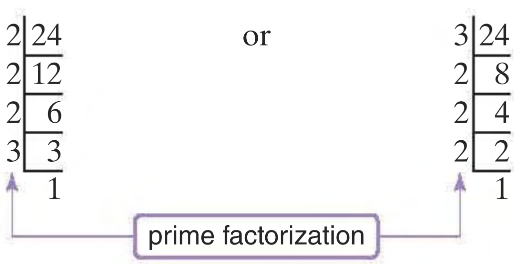

The tree is complete when the factors are all prime numbers.

The prime factorization of 48 is $2 \bullet 2 \bullet 2 \bullet 2 \bullet 3$, or $2^4 \bullet 3$.

If you had chosen 3 and 16 as the first pair of factors for 48, the prime factorization would remain $2^4 \bullet 3$.

► Another way to find the prime factorization is to divide by prime numbers until the quotient is 1.

Find the prime factorization of 24.




The prime factorization of 24 is $2 \bullet 2 \bullet 2 \bullet 3$ or $2^3 \bullet 3$.

► A number is *divisible* by another number if there is no remainder when you divide. You can use the divisibility rules to help you find the prime factorization of greater whole numbers.

| Divisibility Rules | |
|---|---|
| **A number is divisible by:** | |
| **2** → if it is an even number (ends in 0, 2, 4, 6, or 8). | **6** → if it is divisible by both 2 and 3. |
| **3** → if the sum of the digits is divisible by 3. | **8** → if the last three digits form a number divisible by 8. |
| **4** → if the last two digits form a number divisible by 4. | **9** → if the sum of the digits is divisible by 9. |
| **5** → if the ones-place digit is 5 or 0. | **10** → if the ones-place digit is 0. |

Find the prime factorization of 495.

**Method 1** Make a Factor Tree

495 ← ends in 5; divisible by 5
5 • 99 ← 9 + 9 = 18; divisible by 9
5 • 9 • 11 ← 9 is divisible by 3.
5 • 3 • 3 • 11 ← all prime numbers

**Method 2** Use Division

5 | 495 ← ends in 5; divisible by 5
3 | 99 ← 9 + 9 = 18; divisible by 3
3 | 33 ← 3 + 3 = 6; divisible by 3
11 | 11 ← 11 divisible by 11
1

So the prime factorization of 495 is $3^2 \cdot 5 \cdot 11$.

► When the divisibility rules for 2, 3, 5, or 9 do not work, try dividing by other prime numbers. To find the prime factorization of 9009, start by trying 7, 11, 13, 17, and 19.

**Method 1** Make a Factor Tree

9009 ← 9 + 9 = 18; divisible by 9
9 • 1001 ← divisible by 3
3 • 3 • 1001 ← divisible by 7
3 • 3 • 7 • 143 ← divisible by 11
3 • 3 • 7 • 11 • 13 ← all prime numbers

**Method 2** Use Division

3 | 9009 ← 9 + 9 = 18; divisible by 3
3 | 3003 ← 3 + 3 = 6; divisible by 3
7 | 1001 ← divisible by 7
11 | 143 ← divisible by 11
13 | 13 ← divisible by 13
1

So the prime factorization of 9009 is $3^2 \cdot 7 \cdot 11 \cdot 13$.

## Try These

**Tell whether each number is *prime* or *composite*.**

**1.** 41 **2.** 300 **3.** 264 **4.** 51 **5.** 67

**Write the prime factorization of each number in exponential form.**

**6.** 30 **7.** 80 **8.** 63 **9.** 52 **10.** 160

**11. Discuss and Write** Can the product of two prime numbers be a prime number? Explain. Give examples to support your explanation.

Go to PRACTICE BOOK Lesson 5-1 for exercise sets.

5-2

# Greatest Common Factor

**Objective** To find the greatest common factor (GCF) of two or more numbers • To simplify fractions by using the GCF and factoring • To form equivalent fractions

Students are planting 18 forsythia and 24 lilac bushes around the school in groups of equal number. Each group must have the same type of bush. What is the greatest number of bushes the students can plant in each group?

► To find the greatest number of equal groupings, find the greatest common factor of 18 and 24.

The greatest common factor (GCF) of two or more numbers is the greatest number that is a factor of each of those numbers.

Here are two ways to find the GCF of two numbers:

**Method 1** List the Factors

- List all the factors of each number.

  Factors of 18: 1, 2, 3, 6, 9, 18

  Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

- List all the common factors—factors that are the same for both numbers.

  1, 2, 3, 6

- Choose the greatest common factor.

The GCF of 18 and 24 is 6.

**Method 2** Use Prime Factorization

- Write the prime factorization of each number.

18

2 • 9

2 • 3 • 3

24

6 • 4

3 • 2 • 2 • 2

- Multiply the common prime factors.

  $2 \cdot 3 = 6$

The GCF of 18 and 24 is 6.

So the students can plant 6 bushes in each group.

## Example

**1** Find the GCF of 27, 36, and 63.

**Method 1** List the Factors

- List all the factors of each number.

  Factors of 27: 1, 3, 9, 27

  Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

  Factors of 63: 1, 3, 7, 9, 21, 63

- Find the common factors of 27, 36, and 63.

  1, 3, 9

- Choose the greatest common factor.

The GCF of 27, 36, and 63 is 9.

**Method 2** Use Prime Factorization

- Find the prime factors of each number.

  $27 = 3 \cdot 3 \cdot 3$

  $36 = 2 \cdot 2 \cdot 3 \cdot 3$

  $63 = 3 \cdot 3 \cdot 7$

- Find the common prime factors.

  $3 \cdot 3$ or $3^2$

- Multiply the common prime factors.

  $3 \cdot 3 = 9$

The GCF of 27, 36, and 63 is 9.

► Two numbers are relatively prime if their only common factor is 1. The numbers 3 and 4 are relatively prime.

► You can use the GCF to express fractions in simplest form. Write $\frac{45}{60}$ in simplest form.

**Method 1** Divide by the GCF

- Find the GCF of the numerator and the denominator.

  Factors of 45: 1, 3, 5, 9, 15, 45
  Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

  The GCF of 45 and 60 is 15.

- Divide the numerator and denominator by the GCF. $\frac{45}{60} = \frac{45 \div 15}{60 \div 15} = \frac{3}{4}$
- Check by multiplying the numerator and denominator of $\frac{3}{4}$ by the GCF you used in simplifying the original expression. $\frac{3}{4} = \frac{3 \bullet 15}{4 \bullet 15} = \frac{45}{60}$

**Remember:** A fraction is in *simplest form*, or *lowest terms*, when its numerator and denominator have no common factor other than 1.

**Method 2** Use Prime Factorization

$$\frac{45}{60} = \frac{3 \bullet 3 \bullet 5}{2 \bullet 2 \bullet 3 \bullet 5} \begin{array}{l} \leftarrow \text{prime factorization of 45} \\ \leftarrow \text{prime factorization of 60} \end{array}$$

$$= \frac{\cancel{3}^{1} \bullet 3 \bullet \cancel{5}^{1}}{2 \bullet 2 \bullet \cancel{3}_{1} \bullet \cancel{5}_{1}} \leftarrow \text{Simplify.}$$

$$= \frac{3}{4}$$

So $\frac{45}{60} = \frac{3}{4}$ in simplest form.

**Method 3** Use Division

| 5 | 45 | 60 |
|---|---|---|
| 3 | 9 | 12 |
| | 3 | 4 |

prime factors for both 45 and 60

► Two fractions that have the same value are called equivalent fractions. To find equivalent fractions, you can multiply or divide the numerator and denominator of a fraction by the same nonzero number.

PROBLEM-SOLVING STRATEGY 2

Find two fractions equivalent to $\frac{15}{27}$.

**Multiply:** $\frac{15 \bullet 2}{27 \bullet 2} = \frac{30}{54}$ **Divide:** $\frac{15 \div 3}{27 \div 3} = \frac{5}{9}$

So $\frac{30}{54}$ and $\frac{5}{9}$ are equivalent to $\frac{15}{27}$.

Multiplying or dividing both terms of a fraction by the same number is the same as multiplying or dividing the fraction by 1.

## Try These

**Find the greatest common factor. List the factors or use prime factorization.**

**1.** 20 and 45 **2.** 10 and 24 **3.** 6, 14, 28 **4.** 8, 16, 36

**Write each fraction in simplest form. Use the GCF or prime factorization.**

**5.** $\frac{49}{56}$ **6.** $\frac{12}{30}$ **7.** $\frac{22}{55}$ **8.** $\frac{16}{44}$

**9.** Find two fractions equivalent to $\frac{25}{85}$.

**10.** **Discuss and Write** Can the greatest common factor of 24 and 36 be greater than 24? Explain.

Go to PRACTICE BOOK Lesson 5-2 for exercise sets.

# 5-3 Least Common Multiple

**Objective** To find the least common multiple (LCM) of two or more numbers
• To find the least common denominator (LCD) of two or more fractions

The high school's soccer, field hockey, and baseball teams all had games today. The soccer team plays every 6 days, the field hockey team plays every 5 days, and the baseball team plays every 3 days. How many days from now will all three teams have games on the same day again?

To find the number of days from now that the teams will play again on the same day, find the *least common multiple* of 6, 5, and 3.

► A multiple is the product of a number and any whole number. Multiples shared by two or more whole numbers are common multiples. The least nonzero common multiple of two or more numbers is their least common multiple (LCM).

Find the LCM of 3, 5, and 6.

**Method 1** List the Multiples

• List the multiples of each number.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, . . .
Multiples of 5: 5, 10, 15, 20, 25, 30, . . .
Multiples of 6: 6, 12, 18, 24, 30, . . .

Extend each list until you find a multiple common to all the numbers.

• Identify the least multiple that is common to the three numbers: 30

So the LCM of 3, 5, and 6 is 30.

**Method 2** Use Prime Factorization

• Write the prime factorization of each composite number.

3 is prime.
5 is prime.
$6 = 2 \bullet 3$

• Write each prime factor the *greatest number of times* it appears in each of the prime factorizations. Then multiply the factors to find the LCM.

$2 \bullet 3 \bullet 5 = 30$

So the LCM of 3, 5, and 6 is 30.

The three teams will play again on the same day in 30 days.

► The LCM of relatively prime numbers is their product.

What is the least common multiple of 3 and 5?
The LCM of 3 and 5 is $3 \bullet 5$, or 15.

## Example

**1** Find the LCM of 6, 8, and 9.

**Method 1** List the Multiples

- List the multiples of each number.
  Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, . . .
  Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, . . .
  Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, . . .
- Identify the least multiple that is common to the three numbers: 72

So the LCM of 6, 8, and 9 is 72.

**Method 2** Use Prime Factorization

- Find the prime factorization of each number.
  $6 = 2 \bullet 3$
  $8 = 2 \bullet 2 \bullet 2$ or $2^3$
  $9 = 3 \bullet 3$ or $3^2$
- Write each base once, using the greatest exponent.
  $2^3 \bullet 3^2 = 72$

So the LCM of 6, 8, and 9 is 72.

► You can use the same methods for finding the LCM to find the *least common denominator (LCD)* of two or more fractions.

The least common denominator (LCD) is the least common multiple of the denominators of two or more fractions.

To find the LCD of $\frac{3}{8}$, $\frac{4}{5}$, and $\frac{7}{20}$:

**Method 1** List the Multiples

- List the multiples of each denominator until you find a multiple that is common to all three denominators.
  Multiples of 8: 8, 16, 24, 32, 40
  Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40
  Multiples of 20: 20, 40
- Identify the least multiple that is common to the three numbers: 40

**Method 2** Use Prime Factorization

- Write the prime factorization of each denominator. Then find the *greatest number of times* each factor appears in any prime factorization.
  $8 = 2 \bullet 2 \bullet 2$
  $5 = 5$
  $20 = 2 \bullet 2 \bullet 5$
- Multiply the factors: $2 \bullet 2 \bullet 2 \bullet 5 = 40$

Since the LCM of 8, 5, and 20 is 40, the LCD of $\frac{3}{8}$, $\frac{4}{5}$, and $\frac{7}{20}$ is 40.

## Try These

**Find the LCM of each set of numbers. List the multiples, or use prime factorization.**

**1.** 5, 7 **2.** 4, 5, and 10 **3.** 6, 9, and 12 **4.** 2, 3, 4, and 8

**Find the LCD of each set of fractions. List the multiples, or use prime factorization.**

**5.** $\frac{3}{8}$ and $\frac{3}{5}$ **6.** $\frac{9}{14}$, $\frac{1}{8}$, and $\frac{4}{7}$ **7.** $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{2}{3}$ **8.** $\frac{3}{10}$ and $\frac{1}{6}$

**9. Discuss and Write** If one number is a factor of the other, what can you say about the least common multiple of the two numbers?

Go to PRACTICE BOOK Lesson 5-3 for exercise sets.

# 5-4 Fraction Sense: Close to $-1, -\frac{1}{2}, 0, \frac{1}{2}$, or 1

**Objective** To determine whether a fraction is close to $-1, -\frac{1}{2}, 0, \frac{1}{2}$, or 1

Sonja's driving directions to her job interview told her to turn onto Allendale Road. The directions said she should stay on Allendale Road for $\frac{4}{10}$ mile before turning again. Is $\frac{4}{10}$ mile closest to $0, \frac{1}{2}$, or 1 mile?

To determine if the fraction of a mile is closest to $0, \frac{1}{2}$, or 1 mile, compare the absolute values of the numerator and the denominator of the fraction.

► To estimate if a fraction is close to $-1$, close to $-\frac{1}{2}$, close to 0, close to $\frac{1}{2}$, or close to 1, compare the absolute value of its numerator to the absolute value of its denominator.

The numbers $-1, -\frac{1}{2}, 0, \frac{1}{2}$, and 1 are *benchmarks*, or numbers that are convenient to use as standards of reference when making estimates and comparisons.

Since 5 is half of 10, the numerator 4 is almost half of 10. So $\frac{4}{10}$ mile is closest to $\frac{1}{2}$ mile.

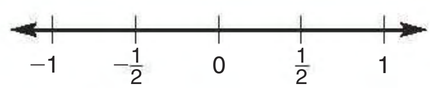
► A fraction is close to 0 when the absolute value of its numerator is *much less than* the absolute value of its denominator.

**Key Concept**

**Fraction Sense**

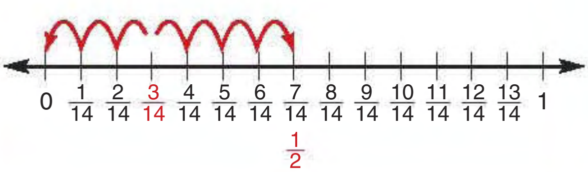
A fraction is:

- close to $-1$ or 1 when the absolute value of its numerator is about equal to the absolute value of its denominator.
- close to $-\frac{1}{2}$ or $\frac{1}{2}$ when double the absolute value of its numerator is about equal to the absolute value of its denominator.
- close to 0 when the absolute value of its numerator is much less than the absolute value of its denominator.

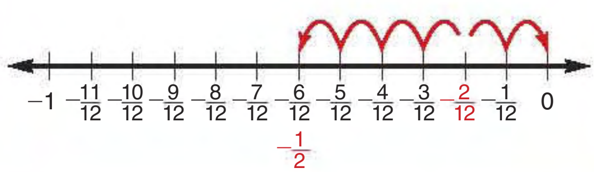



## Examples

**1** The fraction $\frac{3}{14}$ is close to 0 since the absolute value of the numerator, 3, is much less than the absolute value of the denominator, 14.



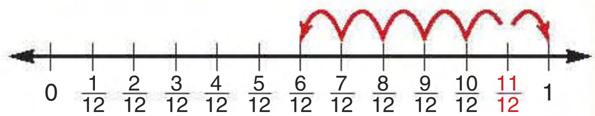

**2** The fraction $\frac{-2}{12}$ is close to 0 since the absolute value of the numerator, 2, is much less than the absolute value of the denominator, 12.



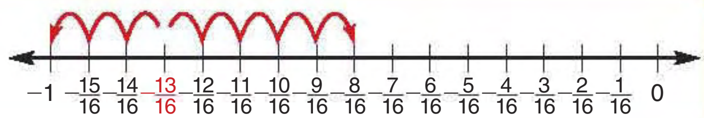
► A fraction is close to 1 or $-1$ when the absolute value of its denominator is *about equal* to the absolute value of its numerator.

## Examples

**1** $\frac{11}{12}$ is close to 1 since the absolute value of the numerator, 11, is about equal to the absolute value of the denominator, 12.




**2** $\frac{13}{-16}$ is close to $-1$ since the absolute value of the numerator, 13, is about equal to the absolute value of the denominator, 16.




► Another way to estimate the value of a fraction to the nearest $\frac{1}{2}$ is to use compatible numbers and mentally visualize a number line marked in intervals of $\frac{1}{2}$.

## Examples

**1** Estimate $\frac{9}{48}$ to the nearest $\frac{1}{2}$.

$\frac{9}{48}$ is close to $\frac{10}{50}$. ← Find compatible numbers.

$\frac{10 \div 10}{50 \div 10} = \frac{1}{5}$ ← Simplify.

$\frac{1}{5}$ is closer to 0 than to $\frac{1}{2}$.

So $\frac{9}{48}$ is close to 0 on a number line.

**2** Estimate $\frac{19}{24}$ to the nearest $\frac{1}{2}$.

$\frac{19}{24}$ is close to $\frac{18}{24}$. ← Find compatible numbers.

$\frac{18 \div 6}{24 \div 6} = \frac{3}{4}$ ← Simplify.

$\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1.

$\frac{19}{24} > \frac{3}{4}$ ← $\frac{19}{24} > \frac{18}{24}$ and $\frac{18}{24} = \frac{3}{4}$.

So $\frac{19}{24}$ is close to 1 on a number line.

## Try These

**Draw a number line to find the answer.**

1. Is $\frac{11}{16}$ closer to $\frac{1}{2}$ or to 1?
2. Is $-\frac{3}{8}$ closer to $-\frac{1}{2}$ or to 0?

**Tell whether the fraction is closest to $-1$, $-\frac{1}{2}$, 0, $\frac{1}{2}$, or 1.**

3. $\frac{15}{18}$
4. $\frac{-8}{14}$
5. $\frac{3}{28}$
6. $\frac{2}{14}$
7. $\frac{-33}{40}$

8. **Discuss and Write** How can you use compatible numbers to determine whether a fraction is closest to $-1$, $-\frac{1}{2}$, 0, $\frac{1}{2}$, or 1? Justify your answer with examples.

Go to PRACTICE BOOK Lesson 5-4 for exercise sets.

Update your skills. See page 413 XV.

# 5-5 Compare and Order Rational Numbers

**Objective** To compare and order rational numbers • To use the LCD to rename fractions • To rename mixed numbers as fractions greater than 1 • To rename fractions greater than 1 as mixed numbers • To name a rational number between any two rational numbers • To use cross products to compare fractions

In a gymnastics competition, Gina scored $7\frac{1}{4}$ points, and Mei scored $7\frac{3}{10}$ points. Who scored fewer points?

To find who scored fewer points, compare $7\frac{1}{4}$ and $7\frac{3}{10}$.

► To compare mixed numbers, first compare the integer parts. If needed, rename the fraction parts as equivalent fractions using the LCD, and then compare the fractions.

- Compare the integer parts of the mixed numbers.

  $7 = 7$

- The integers are equal. Compare the fractions. Use the LCD to rename the fractions with like denominators.

Find the LCD of $\frac{1}{4}$ and $\frac{3}{10}$.

Multiples of 4: 4, 8, 12, 16, 20, 24, . . .
Multiples of 10: 10, 20, . . .
The LCD of 4 and 10 is 20.

Rename $7\frac{1}{4}$ as a mixed number with a denominator of 20.

$7 + \frac{1 \cdot 5}{4 \cdot 5} = 7\frac{5}{20}$

So $7\frac{1}{4} = 7\frac{5}{20}$.

Rename $7\frac{3}{10}$ as a mixed number with a denominator of 20.

$7 + \frac{3 \cdot 2}{10 \cdot 2} = 7\frac{6}{20}$

So $7\frac{3}{10} = 7\frac{6}{20}$.

Compare the numerators of the fractions: $5 < 6$

Since $7\frac{1}{4}$ is less than $7\frac{3}{10}$, Gina scored fewer points than Mei.

► You can find a rational number between any two rational numbers.

**Density Property**
An infinite number of rational numbers can be found between any two rational numbers.

Name a number between $7\frac{5}{20}$ and $7\frac{6}{20}$.

$7\frac{5}{20}$, ___?___, $7\frac{6}{20}$

- Rename the fractions as equivalent fractions in greater terms that have the same denominator.

  $7\frac{5}{20} = 7\frac{5 \cdot 2}{20 \cdot 2} = 7\frac{10}{40}$

- Look at the numerators, and write the integers between them.

  $7\frac{6}{20} = 7\frac{6 \cdot 2}{20 \cdot 2} = 7\frac{12}{40}$

11 is between 10 and 12, so $7\frac{11}{40}$ is between $7\frac{1}{4}$ and $7\frac{3}{10}$.

► To compare and order fractions and decimals, first rename the fractions as equivalent decimals. Then compare and order.

Order $-\frac{5}{16}$, $-\frac{40}{100}$, and $-0.625$ from *least to greatest*.

- First rename the fractions as equivalent decimals. Divide the numerator by the denominator.

$-\frac{5}{16} \rightarrow -5 \div 16 = -0.3125 \qquad -\frac{40}{100} = -0.40$

- Then compare the decimals. Start with the greatest place, and move right as you compare the digits in each decimal place.

$-0.3125$ ← greatest value; closest to zero
$-0.4000$
$-0.6250$ ← least value; farthest from zero

**Check:** $-0.6250 < -0.4000$ and $-0.4000 < -0.3125$

The order from least to greatest is $-0.625, -\frac{40}{100}, -\frac{5}{16}$.

**Remember:** Write zeros so that decimals have the same number of decimal places.

► You can also compare fractions mentally by multiplying as shown below.

## Examples

**1** Compare: $\frac{7}{12}$ ? $\frac{3}{5}$

$\overset{35}{}\frac{7}{12} \times \frac{3}{5}\overset{36}{}$

$35 < 36$ so $\frac{7}{12} < \frac{3}{5}$.

**2** Compare: $\frac{-6}{11}$ ? $\frac{-5}{9}$

$\overset{-54}{}\frac{-6}{11} \times \frac{-5}{9}\overset{-55}{}$

$-54 > -55$ so $\frac{-6}{11} > \frac{-5}{9}$.

► Sometimes when you compare rational numbers, it is necessary to rename mixed numbers or fractions greater than or equal to 1.

Compare: $4\frac{1}{5}$ and $\frac{33}{5}$

**Method 1** Rename the mixed number as a fraction greater than 1.

Rename $4\frac{1}{5}$ as a fraction greater than 1.

$4\frac{1}{5} = \frac{(4 \bullet 5) + 1}{5} = \frac{21}{5}$ ← fraction greater than 1

$\frac{21}{5} < \frac{33}{5}$

So $4\frac{1}{5} < \frac{33}{5}$.

**Method 2** Rename the fraction greater than 1 as a mixed number.

Rename $\frac{33}{5}$ as a mixed number.

$\frac{33}{5} = 5\overline{)33}$ 6 R3 $= 6\frac{3}{5}$ ← mixed number

$4\frac{1}{5} < 6\frac{3}{5}$

So $4\frac{1}{5} < \frac{33}{5}$.

## Try These

1. Name a fraction between $-5\frac{3}{4}$ and $-5\frac{5}{8}$.

**Compare.**

2. $\frac{-2}{5}$ and $\frac{-2}{7}$
3. $8\frac{3}{20}$ and $8\frac{2}{15}$
4. $-0.44$ and $\frac{-2}{7}$
5. $-2\frac{4}{9}$ and $\frac{-17}{7}$

**Order from least to greatest.**

6. $\frac{3}{20}, \frac{2}{15}, \frac{1}{5}$
7. $\frac{-2}{3}, \frac{-3}{8}, \frac{-4}{5}$
8. $4\frac{1}{5}, 4\frac{3}{10}, 4.15$
9. $-3\frac{2}{3}, -4\frac{3}{5}, \frac{-11}{6}$

10. **Discuss and Write** Why is it helpful to first rename fractions in like terms to compare them?

Go to PRACTICE BOOK Lesson 5-5 for exercise sets.

Algebra

# 5-6

Update your skills. See page 414 XVI.

# Add and Subtract Fractions

**Objective** To add and subtract positive and negative fractions

PROBLEM-SOLVING STRATEGY 4

In this year's car race, the second-place car reached the finish line $\frac{1}{2}$ second behind the winner. The third-place car reached the finish line $\frac{5}{8}$ second after the second-place car. How many seconds behind the winner was the third-place car?

► To find the number of seconds, add $\frac{1}{2} + \frac{5}{8}$.

- The fractions have unlike denominators, so find the LCD of $\frac{1}{2}$ and $\frac{5}{8}$.

  Multiples of 2: 2, 4, 6, 8. . .

  Multiples of 8: 8, 16, 24. . .

  The LCD of 2 and 8 is 8.

- Rename one or both fractions using the LCD.

  $\frac{1}{2} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$

- Add. Express the sum in lowest terms.

  $\frac{4}{8} + \frac{5}{8} \rightarrow \frac{4+5}{8} \rightarrow \frac{9}{8} = 1\frac{1}{8}$ ← Write in simplest form.

A fraction greater than 1 is in simplest form, or lowest terms, when it is a mixed number and 1 is the only common factor of the numerator and denominator of its fraction part.

So the third-place car finished $1\frac{1}{8}$ seconds behind the winner.

► To add fractions with like or unlike signs, apply the same sign rules you use for adding integers. If the fractions have unlike denominators, use the LCD to find a common denominator.

**Remember:** $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

### Add with Unlike Denominators, Like Signs

Add: $-\frac{3}{7} + \left(\frac{-9}{14}\right)$

- Find the LCD of the fractions.

  Multiples of 7: 7, 14, 21. . .

  Multiples of 14: 14, 28. . .

  The LCD is 14.

- Use the LCD to rename one or both fractions.

  $-\frac{3}{7} = -\frac{3 \cdot 2}{7 \cdot 2} = -\frac{6}{14}$

- Add. Express the sum in lowest terms.

  $-\frac{6}{14} + \left(\frac{-9}{14}\right) = \frac{-6 + (-9)}{14} = -\frac{15}{14} = -1\frac{1}{14}$

  The addends are both negative, so the sum is negative.

So $-\frac{3}{7} + \left(\frac{-9}{14}\right) = -1\frac{1}{14}$.

### Add with Unlike Denominators, Unlike Signs

Add: $\frac{5}{6} + \left(\frac{-2}{9}\right)$

- Find the LCD of the fractions.

  Multiples of 6: 6, 12, 18, 24. . .

  Multiples of 9: 9, 18, 27. . .

  The LCD is 18.

- Use the LCD to rename one or both fractions.

  $\frac{5}{6} = \frac{5 \cdot 3}{6 \cdot 3} = \frac{15}{18}$

  $\frac{-2}{9} = -\frac{2 \cdot 2}{9 \cdot 2} = -\frac{4}{18}$

- Add. Express the sum in lowest terms.

  $\frac{15}{18} + \left(-\frac{4}{18}\right) = \frac{15 + (-4)}{18} = \frac{11}{18}$

  The positive addend has the greater absolute value, so the sum is positive.

So $\frac{5}{6} + \left(\frac{-2}{9}\right) = \frac{11}{18}$.

▶ To subtract fractions with like or unlike signs, apply the same rules you use for subtracting integers. If the fractions have unlike denominators, use the LCD to find a common denominator.

**Remember:** The Subtraction Principle:
$a - b = a + (-b)$

### Subtract with Like Denominators, Like Signs

Subtract: $\frac{-5}{11} - \left(\frac{-3}{11}\right)$

- Subtract by adding the opposite of the number being subtracted.

$$\frac{-5}{11} - \left(\frac{-3}{11}\right) = \frac{-5}{11} + \left(\frac{3}{11}\right) = \frac{-5+3}{11} = \frac{-2}{11}$$

The negative addend has the greater absolute value, so the answer is negative.

So $\frac{-5}{11} - \left(\frac{-3}{11}\right) = \frac{-2}{11}$.

### Subtract with Like Denominators, Unlike Signs

Subtract: $\frac{1}{5} - \left(\frac{-2}{5}\right)$

- Subtract by adding the opposite of the number being subtracted.

$$\frac{1}{5} - \left(\frac{-2}{5}\right) = \frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$$

Both addends are positive, so the answer is positive.

So $\frac{1}{5} - \left(\frac{-2}{5}\right) = \frac{3}{5}$.

### Subtract with Unlike Denominators, Like Signs

Subtract: $\frac{-7}{12} - \left(\frac{-1}{8}\right)$

- Subtract by adding the opposite of the number being subtracted.

$$\frac{-7}{12} - \left(\frac{-1}{8}\right) = \frac{-7}{12} + \frac{1}{8}$$

- Find the LCD.

Multiples of 12: 12, 24, 36. . .
Multiples of 8: 8, 16, 24, 32. . .

The LCD is 24.

- Use the LCD to rename any fractions.

$$\frac{-7}{12} = \frac{-7 \bullet 2}{12 \bullet 2} = \frac{-14}{24}$$

$$\frac{1}{8} = \frac{1 \bullet 3}{8 \bullet 3} = \frac{3}{24}$$

- Express the answer in lowest terms.

$$\frac{-14}{24} + \frac{3}{24} = \frac{-14+3}{24} = \frac{-11}{24}$$

The negative addend has the greater absolute value, so the answer is negative.

So $\frac{-7}{12} - \left(\frac{-1}{8}\right) = \frac{-11}{24}$.

### Subtract with Unlike Denominators, Unlike Signs

Subtract: $\frac{9}{16} - \left(\frac{-3}{4}\right)$

- Subtract by adding the opposite of the number being subtracted.

$$\frac{9}{16} - \left(\frac{-3}{4}\right) = \frac{9}{16} + \frac{3}{4}$$

- Find the LCD.

Multiples of 16: 16, 32. . .
Multiples of 4: 4, 8, 12, 16, 20. . .

The LCD is 16.

- Use the LCD to rename any fractions.

$$\frac{3}{4} = \frac{3 \bullet 4}{4 \bullet 4} = \frac{12}{16}$$

- Express the answer in lowest terms.

$$\frac{9}{16} + \frac{12}{16} = \frac{9+12}{16} = \frac{21}{16} = 1\frac{5}{16}$$

The addends are both positive, so the answer is positive.

So $\frac{9}{16} - \left(\frac{-3}{4}\right) = 1\frac{5}{16}$.

## Try These

**Add or subtract. Express your answer in lowest terms.**

**1.** $\frac{-5}{16} + \left(\frac{-5}{8}\right)$ **2.** $\frac{3}{5} + \left(\frac{-4}{15}\right)$ **3.** $\frac{7}{10} - \frac{4}{5}$ **4.** $\frac{-2}{10} - \frac{11}{20}$

**5. Discuss and Write** Explain how you would subtract $\frac{-4}{5}$ from $\frac{9}{10}$.

Go to PRACTICE BOOK Lesson 5-6 for exercise sets.

Algebra

# 5-7 Add and Subtract Mixed Numbers

**Objective** To add and subtract positive and negative mixed numbers

A company had a profit of $\$4\frac{1}{4}$ million in its first year. The company had a loss of $\$3\frac{1}{8}$ million in its second year. How much did the company gain or lose in all in its first 2 years?

To find the total profit or loss, add $4\frac{1}{4} + \left(-3\frac{1}{8}\right)$.

First estimate the sum by rounding to the nearest integer.

$4\frac{1}{4} \approx 4 \qquad -3\frac{1}{8} \approx -3 \qquad 4 + (-3) = 1$

► To add mixed numbers, rename each with a common denominator if necessary.

- Find the LCD of the fraction parts.
  Multiples of 4: 4, 8, 12. . .
  Multiples of 8: 8, 16, 24. . .
  The LCD of $\frac{1}{4}$ and $\frac{1}{8}$ is 8.
- Rename the fraction part of $4\frac{1}{4}$ using the LCD. $\frac{1 \cdot 2}{4 \cdot 2} = \frac{2}{8}$, so $4\frac{1}{4} = 4\frac{2}{8}$
- Add the mix numbers.
  First add the fraction parts.
  Then add the integer parts.
  Use the same rules as for adding integers.

$$\begin{array}{r} 4\frac{1}{4} \\ +\left(-3\frac{1}{8}\right) \\ \hline \end{array} \longrightarrow \begin{array}{r} 4\frac{2}{8} \\ +\left(-3\frac{1}{8}\right) \\ \hline 1\frac{1}{8} \end{array}$$

← simplest form

- Express the sum in simplest form.
- Check by comparing the answer to your estimate.

So the company gained $\$1\frac{1}{8}$ million.

**Think**
$1\frac{1}{8}$ is close to the estimate of 1.
The answer is reasonable.

► To write a sum in simplest form, sometimes you need to rename the sum when the fraction part is greater than or equal to 1.

Simplify the sum: $-5\frac{7}{12} + \left(-3\frac{11}{12}\right) = -8\frac{18}{12}$

- Divide the fraction part by the GCF. $-8\frac{18}{12} = -8\frac{18 \div 6}{12 \div 6} = -8\frac{3}{2}$ ← Divide by 6, the GCF of 18 and 12.
- Then rename the fraction part as a mixed number by dividing the numerator by the denominator. Write the quotient as the integer part and the remainder over the divisor as the fraction part.
  $\frac{-3}{2} = 2\overline{)-3}$ with quotient $-1$ R1
  $\frac{-3}{2} = -1\frac{1}{2}$
- Add the renamed fraction part to the original integer part, and write the sum. $-8 + \left(-1\frac{1}{2}\right) = -9\frac{1}{2}$ ← Add the integer and the renamed fraction part.

So $-8\frac{18}{12}$ in simplest form is $-9\frac{1}{2}$.

► To subtract a fraction from an integer or a mixed number, rename the integer or mixed number and subtract.

## Examples

**1** Subtract: $6 - \frac{4}{7}$

- Estimate: $6 - 1 = 5$
- Rename the integer as a mixed number.

$6 = 5 + 1 \rightarrow 6 = 5 + \frac{7}{7} = 5\frac{7}{7}$

- Subtract.

$5\frac{7}{7} - \frac{4}{7} = 5\frac{3}{7}$

**Check:** $5\frac{3}{7}$ is close to the estimate of 5.

**2** Subtract: $4 - 2\frac{5}{13}$

- Estimate: $4 - 2 = 2$
- Rename the integer as a mixed number.

$4 = 3 + 1 \rightarrow 4 = 3 + \frac{13}{13} = 3\frac{13}{13}$

- Subtract.

$3\frac{13}{13} - 2\frac{5}{13} = 1\frac{8}{13}$

**Check:** $1\frac{8}{13}$ is close to the estimate of 2.

**3** Subtract: $8\frac{1}{5} - \frac{4}{5}$

- Estimate: $8 - 1 = 7$
- Regroup $8\frac{1}{5}$ to subtract.

Think: $\frac{1}{5} < \frac{4}{5}$

$8\frac{1}{5} = 7 + 1 + \frac{1}{5}$

$= 7 + \frac{5}{5} + \frac{1}{5}$ ← Rename 1 as $\frac{5}{5}$.

$= 7 + \frac{6}{5} = 7\frac{6}{5}$

- Subtract: $7\frac{6}{5} - \frac{4}{5} = 7\frac{2}{5}$

**Check:** $7\frac{2}{5}$ is close to the estimate of 7.

**4** Subtract: $6\frac{1}{4} - 4\frac{3}{4}$

- Estimate: $6 - 5 = 1$
- Regroup $6\frac{1}{4}$ to subtract.

Think: $\frac{1}{4} < \frac{3}{4}$

$6\frac{1}{4} = 5 + 1 + \frac{1}{4}$

$= 5 + \frac{4}{4} + \frac{1}{4}$ ← Rename 1 as $\frac{4}{4}$.

$= 5 + \frac{5}{4}$

- Subtract: $5\frac{5}{4} - 4\frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

**Check:** $1\frac{1}{2}$ is close to the estimate of 1.

► To compute with two or more mixed numbers, apply the Subtraction Principle, and use the sign rules that you use in adding integers.

Evaluate $a + b - c - d$, when $a = 1\frac{7}{12}$, $b = -3\frac{7}{12}$, $c = -1\frac{3}{12}$, $d = 3\frac{7}{12}$.

$1\frac{7}{12} + \left(-3\frac{7}{12}\right) - \left(-1\frac{3}{12}\right) - 3\frac{7}{12}$ ← Substitute the known values.

$1\frac{7}{12} + \left(-3\frac{7}{12}\right) + 1\frac{3}{12} + \left(-3\frac{7}{12}\right)$ ← Apply the Subtraction Principle.

$\left(1\frac{7}{12} + 1\frac{3}{12}\right) + \left[\left(-3\frac{7}{12}\right) + \left(-3\frac{7}{12}\right)\right]$ ← Use the Commutative and Associative properties to group like signs.

$2\frac{10}{12} + \left(-6\frac{14}{12}\right)$ ← Simplify.

$-4\frac{4}{12} = -4\frac{1}{3}$

## Try These

**Add or subtract. Express each answer in simplest form.**

**1.** $1\frac{3}{10} - \left(\frac{-2}{5}\right)$ **2.** $-2\frac{7}{12} + \left(-3\frac{5}{8}\right)$ **3.** $1\frac{3}{5} + \left(-6\frac{7}{10}\right)$ **4.** $-3\frac{5}{6} - \left(-1\frac{3}{4}\right)$

**5.** $a + b$, when $a = 3\frac{2}{3}$ and $b = -5\frac{5}{8}$ **6.** $a + b - c$ when $a = -7\frac{3}{10}$, $b = -4\frac{2}{5}$, and $c = 3\frac{3}{10}$

**7. Discuss and Write** Is renaming the integer necessary to solve $-7 - 3\frac{5}{10}$? Explain.

Go to PRACTICE BOOK Lesson 5-7 for exercise sets.

Update your skills. See page 414 XVII.

# Multiply Fractions

**Objective** To multiply positive and negative fractions • To multiply positive and negative fractions and integers • To evaluate algebraic expressions involving multiplication of fractions

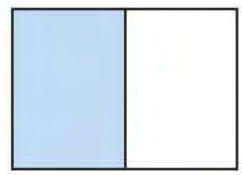
Star Middle School has designed a new flag to represent its sports teams. It is $\frac{1}{2}$ blue and $\frac{1}{2}$ white. It has stars on $\frac{2}{5}$ of the blue section. What fraction of the whole flag has stars?

To find the part with stars, $\frac{2}{5}$ of $\frac{1}{2}$, multiply $\frac{2}{5} \bullet \frac{1}{2}$.

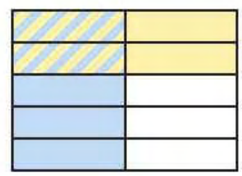
► You can draw an area model to show multiplication of fractions.

Find $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{2}{5} \bullet \frac{1}{2}$.

1. Divide a rectangle into halves. Shade $\frac{1}{2}$ blue.



2. Divide the rectangle into fifths. Shade $\frac{2}{5}$ yellow.



3. Count the parts that are shaded both yellow and blue. — 2 parts shaded both yellow and blue
4. Count the total number of parts. — 10 parts in all

So $\frac{2}{5}$ of $\frac{1}{2}$ is $\frac{2}{10}$, or $\frac{1}{5}$.

So $\frac{1}{5}$ of the flag has stars.

► To multiply positive and negative fractions, multiply numerators and denominators. Use the same sign rules as for multiplying integers.

Multiply: $\frac{2}{5}\left(\frac{1}{2}\right)$

$\frac{2}{5} \bullet \frac{1}{2} = \frac{2 \bullet 1}{5 \bullet 2}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{2}{10} = \frac{2 \div 2}{10 \div 2}$ ← Simplify. Divide by the GCF.

$= \frac{1}{5}$ ← Simplest form

**Key Concept**

**Multiply Fractions**

$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$, where $b \neq 0$ and $d \neq 0$

## Examples

**1** Multiply: $\left(\frac{-5}{8}\right)\left(\frac{-1}{4}\right)$

$\left(\frac{-5}{8}\right)\left(\frac{-1}{4}\right) = \frac{-5 \bullet (-1)}{8 \bullet 4}$ ← Multiply.

$= \frac{5}{32}$ ← Both signs are negative, so the product is positive.

**2** Multiply: $\left(\frac{-4}{5}\right)\left(\frac{2}{3}\right)$

$\left(\frac{-4}{5}\right)\left(\frac{2}{3}\right) = \frac{(-4) \bullet 2}{5 \bullet 3}$ ← Multiply.

$= \frac{-8}{15}$ ← The signs are unlike, so the product is negative.

► Sometimes you can divide the numerator of one fraction and the denominator of another by their GCF before multiplying.

Evaluate: $abc$, when $a = \frac{5}{8}$, $b = \frac{-2}{3}$, and $c = \frac{7}{10}$.

$$\left(\frac{5}{8}\right)\left(\frac{-2}{3}\right)\left(\frac{7}{10}\right) = \frac{(\cancel{5}^{\,1})(\cancel{-2}^{\,-1})(7)}{(\cancel{8}_{\,4})(3)(\cancel{10}_{\,2})} \leftarrow \text{Divide 5 and 10 by their GCF, 5. Divide } -2 \text{ and 8 by their GCF, 2.}$$

$$= \frac{(1)(-1)(7)}{(4)(3)(2)} = \frac{-7}{24} \leftarrow \text{simplest form}$$

► To multiply an integer and a positive or negative fraction, first rename the integer as a fraction. Then multiply.

**Remember:** You can write any integer $a$ as a fraction with a denominator of 1: $a = \frac{a}{1}$.

## Examples

**1** Multiply: $(7)\left(\frac{3}{10}\right)$

$$(7)\left(\frac{3}{10}\right) = \left(\frac{7}{1}\right)\left(\frac{3}{10}\right) \leftarrow 7 = \frac{7}{1}$$
$$= \frac{(7)(3)}{(1)(10)}$$
$$= \frac{21}{10}$$
$$= 2\frac{1}{10}$$

**2** Multiply: $(-8)\left(\frac{-5}{16}\right)$

$$(-8)\left(\frac{-5}{16}\right) = \left(\frac{-8}{1}\right)\left(\frac{-5}{16}\right) \leftarrow -8 = \frac{-8}{1}$$
$$= \frac{(\cancel{-8}^{\,-1})(-5)}{(1)(\cancel{16}_{\,2})}$$
$$= \frac{(-1)(-5)}{(1)(2)}$$
$$= \frac{5}{2} = 2\frac{1}{2}$$

**3** Find $\frac{3}{5}$ of 70.

**Remember:** *Of* means "multiply."

$$\left(\frac{3}{5}\right)\left(\frac{70}{1}\right) = \frac{(3)(\cancel{70}^{\,14})}{(\cancel{5}_{\,1})(1)}$$
$$= \frac{(3)(14)}{(1)(1)}$$
$$= 42$$

**4** Find $\frac{1}{4}$ of \$5.00.

$$\left(\frac{1}{4}\right)\left(\frac{5}{1}\right) = \frac{5}{4} \leftarrow \$5.00 = \frac{5}{1}$$
$$= 1\frac{1}{4}$$
$$= 1.25 \leftarrow \frac{1}{4} = 0.25$$

So $\frac{1}{4}$ of \$5.00 is \$1.25.

## Try These

**Simplify each expression. Express the product in simplest form.**

**1.** $\frac{7}{8} \bullet \frac{1}{2}$ **2.** $\frac{-3}{8}\left(\frac{-2}{8}\right)$ **3.** $\left(-\frac{7}{8}\right)\left(\frac{2}{21}\right)$ **4.** $\frac{-3}{8} \bullet \frac{-5}{6}$ **5.** $\frac{3}{8} \bullet \frac{-4}{5}$

**6.** $24\left(\frac{3}{8}\right)$ **7.** $-30 \bullet \frac{-2}{3}$ **8.** $\frac{5}{8} \bullet 13$ **9.** $\frac{4}{5}$ of 50 **10.** $\frac{5}{6}$ of $-30$

**11.** Evaluate $xyz$, when $x = \frac{-5}{6}$, $y = \frac{4}{5}$, and $z = \frac{-6}{7}$.

**Solve.**

**12.** At the mineral show, $\frac{2}{7}$ of the 35 minerals are unpolished gemstones. How many minerals are unpolished gemstones?

**13. Discuss and Write** Why is it a good idea to simplify first before multiplying fractions?

Go to PRACTICE BOOK Lesson 5-8 for exercise sets.

# Multiply Mixed Numbers

**Objective** To estimate the product of mixed numbers • To multiply positive and negative mixed numbers • To evaluate algebraic expressions involving multiplication of fractions and mixed numbers

A mile is approximately $1\frac{3}{5}$ kilometers. Eli jogs $2\frac{1}{4}$ miles every afternoon. About how many kilometers does he jog?

To find the number of kilometers, multiply: $2\frac{1}{4} \bullet 1\frac{3}{5}$

► First estimate by rounding to the nearest integer.

$2\frac{1}{4} \approx 2$ $\quad 1\frac{3}{5} \approx 2$ $\quad 2 \bullet 2 = 4$ ← estimated product

► To multiply mixed numbers, rename each mixed number as a fraction greater than 1.

$2\frac{1}{4} \bullet 1\frac{3}{5} = \frac{9}{4} \bullet \frac{8}{5}$ ← Rename each as a fraction greater than 1.

$= \frac{9 \bullet \cancel{8}^{2}}{\cancel{4}_{1} \bullet 5}$ ← Simplify. Divide by the GCF.

$= \frac{9 \bullet 2}{1 \bullet 5}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{18}{5}$ ← Divide to rename as a mixed-number equivalent.

$= 3\frac{3}{5}$

**Remember:** To rename a mixed number as a fraction: $a\frac{b}{c} = \frac{(a \bullet c) + b}{c}, c \neq 0$

**Check:** $3\frac{3}{5}$ is close to the estimate of 4. The answer is reasonable.

Eli jogs $3\frac{3}{5}$ kilometers.

► To multiply positive and negative mixed numbers, apply the sign rules for multiplying integers. To multiply a mixed number and an integer or a fraction, rename the mixed number.

## Examples

**1** Multiply: $-2\frac{2}{5} \bullet \left(-3\frac{1}{2}\right)$

Estimate first: $-2\frac{2}{5} \approx -2$ $\quad -3\frac{1}{2} \approx -4$ $\quad -2 \bullet (-4) = 8$

Then multiply: $-2\frac{2}{5} \bullet \left(-3\frac{1}{2}\right) = \frac{-12}{5} \bullet \left(\frac{-7}{2}\right)$ ← Rename each as a fraction greater than 1.

$= \frac{\cancel{-12}^{-6}}{5} \bullet \left(\frac{-7}{\cancel{2}_{1}}\right)$ ← Simplify. Divide by the GCF.

$= \frac{-6 \bullet (-7)}{5 \bullet 1}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{42}{5}$ ← Divide to rename as a mixed number.

$= 8\frac{2}{5}$

**Check:** $8\frac{2}{5}$ is close to the estimate of 8. The answer is reasonable.

**2** Multiply: $-3\frac{3}{5} \bullet 2\frac{1}{4}$

Estimate first: $-3\frac{3}{5} \approx -4$ $\quad 2\frac{1}{4} \approx 2$ $\quad -4 \bullet 2 = -8$

Then multiply:

$-3\frac{3}{5} \bullet 2\frac{1}{4} = \frac{-18}{5} \bullet \frac{9}{4}$ ← Rename each mixed number.

$= \frac{-18 \bullet 9}{5 \bullet 4}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{-162}{20}$ ← Divide to rename as a mixed number.

$= -8\frac{2}{20} \rightarrow -8\frac{1}{10}$ ← Express the fraction part in simplest form.

**Think** $-8\frac{1}{10}$ is close to $-8$.

**3** Multiply: $4\frac{9}{10} \bullet 9$

Estimate first. Then multiply.

$4\frac{9}{10} \approx 5$ $\quad 5 \bullet 9 = 45$

$4\frac{9}{10} \bullet 9 = \frac{49}{10} \bullet \frac{9}{1}$ ← Rename the factors, then multiply.

$= \frac{441}{10}$ ← Divide to rename.

$= 44\frac{1}{10}$

**Think** $44\frac{1}{10}$ is close to 45.

**4** Multiply: $2\frac{1}{3} \bullet \frac{7}{8}$

Estimate first. Then multiply.

$2\frac{1}{3} \approx 2$ $\quad \frac{7}{8} \approx 1$ $\quad 2 \bullet 1 = 2$

$2\frac{1}{3} \bullet \frac{7}{8} = \frac{7}{3} \bullet \frac{7}{8}$ ← Rename $2\frac{1}{3}$, then multiply.

$= \frac{49}{24}$ ← Divide to rename.

$= 2\frac{1}{24}$

**Think** $2\frac{1}{24}$ is close to 2.

► You can evaluate algebraic expressions involving multiplication of fractions.

Evaluate $a \bullet b$, when $a = \frac{-6}{7}$ and $b = \frac{-2}{3}$.

$\frac{-6}{7} \bullet \left(\frac{-2}{3}\right) = \frac{\overset{-2}{\cancel{-6}}(-2)}{7 \bullet \underset{1}{\cancel{3}}}$ ← Substitute the known values.

$= \frac{-2(-2)}{7 \bullet 1}$ ← Multiply.

$= \frac{4}{7}$

Evaluate $a \bullet b$, when $a = 3\frac{2}{5}$ and $b = \frac{-2}{3}$.

$3\frac{2}{5} \bullet \left(\frac{-2}{3}\right) = \frac{17}{5} \bullet \left(\frac{-2}{3}\right)$ ← Substitute the known values.

$= \frac{17(-2)}{5 \bullet 3}$ ← Multiply.

$= \frac{-34}{15}$ ← Divide to rename.

$= -2\frac{4}{15}$

## Try These

**Estimate. Then multiply. Express the product in simplest form.**

**1.** $4\frac{3}{5} \bullet 4\frac{1}{2}$ **2.** $3\frac{1}{2}\left(-1\frac{1}{2}\right)$ **3.** $-1\frac{3}{4}\left(-1\frac{2}{3}\right)$ **4.** $-2\frac{1}{6} \bullet 1\frac{7}{8}$ **5.** $3\frac{1}{2}(-7)$

**Evaluate.**

**6.** $a \bullet b$, when $a = \frac{-2}{3}, b = \frac{7}{9}$ **7.** $a \bullet b$, when $a = -4\frac{3}{4}, b = \frac{-3}{8}$

**8. Discuss and Write** Explain why estimation is or is not helpful when multiplying various kinds of fractional numbers. Support your answer with examples.

Go to PRACTICE BOOK Lesson 5-9 for exercise sets.

Algebra

# 5-10 Divide Fractions

Update your skills. See page 415 XVIII.

**Objective** To divide positive and negative fractions • To divide positive and negative fractions in complex fraction form

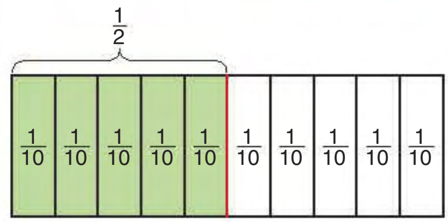
The Swanson Running Track is 1 mile long. It is marked in tenths of a mile from the starting line. How many $\frac{1}{10}$-mile sections are there in the first half of the track?

To find the number of $\frac{1}{10}$-mile sections, divide $\frac{1}{2}$ by $\frac{1}{10}$.

► You can draw an area model to represent division of fractions.

Divide: $\frac{1}{2} \div \frac{1}{10}$

**Think** How many tenths are in $\frac{1}{2}$?




1. Draw a rectangle, and divide it into 2 equal parts to represent halves.
2. Shade $\frac{1}{2}$ of the rectangle.
3. Divide the whole rectangle into 10 equal parts to represent tenths.
4. Count the number of $\frac{1}{10}$ parts in the shaded $\frac{1}{2}$.

   There are 5, so $\frac{1}{2} \div \frac{1}{10} = 5$.

There are 5 $\frac{1}{10}$-mile sections in the first half of the track.

► To divide fractions, multiply the *reciprocal* of the divisor by the dividend. The reciprocal of a number is its multiplicative inverse. Zero has no reciprocal.

**Key Concept**

**Divide Fractions**

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$, when $b, c, d \neq 0$

$\frac{a}{b} \bullet \frac{b}{a} = 1$, when $a$ and $b \neq 0$

## Example

**1** Divide: $\frac{1}{2} \div \frac{1}{10}$

reciprocals

$\frac{1}{2} \div \frac{1}{10} = \frac{1}{2} \bullet \frac{10}{1}$ ← Rewrite division as multiplication with the reciprocal of the divisor.

$= \frac{1}{\cancel{2}_{1}} \bullet \frac{\cancel{10}^{5}}{1}$ ← Simplify using the GCF.

$= \frac{1 \bullet 5}{1 \bullet 1}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{5}{1} = 5$ ← Divide.

**Think** $\frac{1}{\cancel{10}_{1}} \bullet \frac{\cancel{10}^{1}}{1} = \frac{1}{1} = 1$

**Remember:** $\frac{5}{1}$ means $5 \div 1$.

► To divide signed fractions, use the same sign rules as you use for dividing integers. You can also divide an integer by a fraction or a fraction by an integer.

## Examples

**1** Divide: $\frac{-2}{9} \div \left(\frac{-1}{8}\right)$

$\frac{-2}{9} \div \left(\frac{-1}{8}\right) = \frac{-2}{9} \bullet \left(\frac{8}{-1}\right)$ ← Multiply by the reciprocal.

$= \frac{-2 \bullet 8}{9 \bullet (-1)}$

$= \frac{-16}{-9}$ ← The signs are the same, so the quotient is positive.

$= 1\frac{7}{9}$ ← Express as a mixed number.

**2** Divide: $\frac{-7}{20} \div \frac{1}{5}$

$\frac{-7}{20} \div \frac{1}{5} = \frac{-7}{20} \bullet \frac{5}{1}$ ← Multiply by the reciprocal.

$= \frac{-7}{\cancel{20}_{4}} \bullet \frac{\cancel{5}^{1}}{1}$

$= \frac{-7 \bullet 1}{4 \bullet 1}$

$= \frac{-7}{4}$ ← The signs are different, so the quotient is negative.

$= -1\frac{3}{4}$ ← Express as mixed number.

**3** Divide: $-10 \div \frac{1}{2}$

$-10 \div \frac{1}{2} = \frac{-10}{1} \div \frac{1}{2}$

$= \frac{-10}{1} \bullet \frac{2}{1}$ ← Multiply the reciprocal of the divisor by the dividend.

$= \frac{-10 \bullet 2}{1 \bullet 1}$

$= \frac{-20}{1} = -20$

**4** Divide: $\frac{2}{3} \div 20$

$\frac{2}{3} \div 20 = \frac{2}{3} \div \frac{20}{1}$

$= \frac{2}{3} \bullet \frac{1}{20}$ ← Multiply the reciprocal of the divisor by the dividend.

$= \frac{\cancel{2}^{1}}{3} \bullet \frac{1}{\cancel{20}_{10}}$

$= \frac{1 \bullet 1}{3 \bullet 10} = \frac{1}{30}$

► A complex fraction has a fraction or mixed number in the numerator, the denominator, or both. To simplify, divide the numerator by the denominator.

## Examples

**1** $\frac{\frac{3}{4}}{12} = \frac{3}{4} \div 12$

$= \frac{3}{4} \bullet \frac{1}{12}$

$= \frac{\cancel{3}^{1}}{4} \bullet \frac{1}{\cancel{12}_{4}} = \frac{1}{16}$

**2** $\frac{\frac{9}{1}}{\frac{3}{5}} = \frac{9}{1} \div \frac{3}{5}$

$= \frac{9}{1} \bullet \frac{5}{3}$

$= \frac{\cancel{9}^{3}}{1} \bullet \frac{5}{\cancel{3}_{1}} = \frac{15}{1} = 15$

**3** $\frac{\frac{8}{9}}{\frac{5}{6}} = \frac{8}{9} \div \frac{5}{6}$

$= \frac{8}{9} \bullet \frac{6}{5}$

$= \frac{8}{\cancel{9}_{3}} \bullet \frac{\cancel{6}^{2}}{5} = \frac{16}{15} = 1\frac{1}{15}$

## Try These

**Divide. Express your answer in simplest form.**

1. $\frac{7}{8} \div \frac{1}{2}$
2. $\frac{-5}{6} \div \frac{7}{10}$
3. $\frac{5}{6} \div (-30)$
4. $\frac{-4}{5} \div \frac{1}{2}$
5. $\frac{\frac{-3}{8}}{\frac{-3}{4}}$
6. **Discuss and Write** Explain the difference in how you simplify $\frac{1}{2} \div \frac{3}{8}$ and $\frac{1}{2} \bullet \frac{3}{8}$.

Go to PRACTICE BOOK Lesson 5-10 for exercise sets.

Algebra **5-11**

*Update your skills. See page 413 XV.*

# Divide Mixed Numbers

**Objective** To divide positive and negative mixed numbers • To evaluate algebraic expressions involving division of fractions and mixed numbers • To simplify complex fractions containing mixed numbers

Jerome measured his frog's leap at $6\frac{1}{8}$ inches. This was $1\frac{3}{4}$ times as far as his leap yesterday. How far did the frog leap yesterday?

To find the distance of yesterday's leap, divide $6\frac{1}{8}$ by $1\frac{3}{4}$.

First estimate by rounding to the nearest integer.

$6\frac{1}{8} \approx 6 \qquad 1\frac{3}{4} \approx 2 \qquad 6 \div 2 = 3$ ← estimated quotient

► To divide mixed numbers, first rename each mixed number as a fraction.

$6\frac{1}{8} \div 1\frac{3}{4} = \frac{49}{8} \div \frac{7}{4}$ ← Rename as fractions.

$\frac{49}{8} \div \frac{7}{4} = \frac{49}{8} \bullet \frac{4}{7}$ ← Multiply the reciprocal of the divisor by the dividend.

$= \frac{\overset{7}{\cancel{49}}}{\underset{2}{\cancel{8}}} \bullet \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{7}}}$ ← Simplify using the GCF, then multiply.

$= \frac{7}{2} = 3\frac{1}{2}$ ← Divide to rename as a mixed number.

So yesterday's leap was $3\frac{1}{2}$ inches long.

**Remember:** A reciprocal is the multiplicative inverse of a number.
$\frac{a}{b} \bullet \frac{b}{a} = 1$, when $a$ and $b \neq 0$

**Think**
$3\frac{1}{2}$ is close to the estimate of 3.
The answer is reasonable.

► To divide positive and negative mixed numbers, use the same sign rules you use for dividing integers.

• Divide: $-3\frac{1}{4} \div 2\frac{1}{2}$

$-3\frac{1}{4} \div 2\frac{1}{2} = \frac{-13}{4} \div \frac{5}{2}$ ← Rename as fractions.

$= \frac{-13}{4} \bullet \frac{2}{5}$ ← reciprocal of divisor

$= \frac{-13}{\underset{2}{\cancel{4}}} \bullet \frac{\overset{1}{\cancel{2}}}{5}$

$= \frac{-13}{10} = -1\frac{3}{10}$ ← unlike signs, negative quotient

• Divide: $-2\frac{1}{8} \div \left(-1\frac{1}{4}\right)$

$-2\frac{1}{8} \div \left(-1\frac{1}{4}\right) = \frac{-17}{8} \div \left(\frac{-5}{4}\right)$ ← Rename as fractions.

$= \frac{-17}{8} \bullet \left(\frac{4}{-5}\right)$ ← reciprocal of divisor

$= \frac{-17}{\underset{2}{\cancel{8}}} \bullet \left(\frac{\overset{1}{\cancel{4}}}{-5}\right)$

$= \frac{-17}{-10} = 1\frac{7}{10}$ ← like signs, positive quotient

► You can evaluate algebraic expressions involving division and fractions.

Evaluate $a \div b$, when $a = \frac{6}{7}$ and $b = \frac{-3}{8}$.

$a \div b = \frac{6}{7} \div \left(\frac{-3}{8}\right)$

$= \frac{6}{7} \bullet \left(\frac{8}{-3}\right)$ ← reciprocal of divisor

$= \frac{\cancel{6}^{2}}{7} \bullet \left(\frac{8}{\cancel{-3}_{-1}}\right)$

$= \frac{16}{-7} = -2\frac{2}{7}$

Evaluate $a \div b$, when $a = 1\frac{2}{3}$ and $b = \frac{5}{6}$.

$a \div b = 1\frac{2}{3} \div \frac{5}{6}$

$= \frac{5}{3} \bullet \frac{6}{5}$ ← reciprocal of divisor

$= \frac{\cancel{5}^{1}}{\cancel{3}_{1}} \bullet \frac{\cancel{6}^{2}}{\cancel{5}_{1}}$

$= \frac{2}{1} = 2$

► To divide a mixed number and an integer, rename the mixed number *and* the integer as fractions greater than or equal to 1 before computing.

Divide: $5\frac{1}{3} \div 8$

$5\frac{1}{3} \div 8 = \frac{16}{3} \div \frac{8}{1}$ ← Rename as fractions.

$= \frac{\cancel{16}^{2}}{3} \bullet \frac{1}{\cancel{8}_{1}} = \frac{2 \bullet 1}{3 \bullet 1}$ ← Multiply by the reciprocal of the divisor. Simplify.

$= \frac{2}{3}$ ← Express in simplest form.

► To simplify a complex fraction, rename the divisor and dividend so that both are in fraction form. Then divide the numerator by the denominator.

• Simplify: $\dfrac{1\frac{3}{4}}{\frac{3}{8}}$

$\dfrac{1\frac{3}{4}}{\frac{3}{8}} = \dfrac{\frac{7}{4}}{\frac{3}{8}} = \frac{7}{4} \div \frac{3}{8}$

$= \frac{7}{\cancel{4}_{1}} \bullet \frac{\cancel{8}^{2}}{3}$

$= \frac{14}{3} = 4\frac{2}{3}$

• Simplify: $\dfrac{2\frac{2}{5}}{3\frac{3}{5}}$

$\dfrac{2\frac{2}{5}}{3\frac{3}{5}} = \dfrac{\frac{12}{5}}{\frac{18}{5}} = \frac{12}{5} \div \frac{18}{5}$

$= \frac{\cancel{12}^{2}}{\cancel{5}_{1}} \bullet \frac{\cancel{5}^{1}}{\cancel{18}_{3}}$

$= \frac{2}{3}$

## Try These

**Divide. Express your answer in simplest form.**

**1.** $2\frac{7}{8} \div \frac{1}{2}$ **2.** $7\frac{2}{3} \div 3$ **3.** $-21 \div 1\frac{1}{6}$ **4.** $-6\frac{5}{8} \div \left(-2\frac{1}{4}\right)$ **5.** $\dfrac{1\frac{5}{7}}{6\frac{3}{4}}$

**Evaluate.**

**6.** $a \div b$, when $a = \frac{2}{3}, b = \frac{-1}{6}$

**7.** $m \div n$, when $m = -1\frac{3}{8}, n = 2\frac{1}{2}$

**8. Discuss and Write** Is the quotient of $3\frac{1}{4} \div \frac{1}{8}$ greater than the dividend or less than the dividend? Explain your answer.

Go to PRACTICE BOOK Lesson 5-11 for exercise sets.

Algebra

# 5-12 Properties of Rational Numbers

**Objective** To identify properties of addition and multiplication of rational numbers
• To use the properties to compute mentally with rational numbers

► Properties can help you compute. Let $a$, $b$, and $c$ represent any rational numbers.

**Commutative Property of Addition**
Changing the *order* of the addends does *not* change the sum.
$a + b = b + a$
*Example*: $\frac{2}{3} + \frac{1}{4} = \frac{1}{4} + \frac{2}{3}$

**Commutative Property of Multiplication**
Changing the *order* of the factors does *not* change the product.
$ab = ba$
*Example*: $\frac{-5}{7}\left(\frac{-1}{8}\right) = \left(\frac{-1}{8}\right)\frac{-5}{7}$

Think
"order"

**Associative Property of Addition**
Changing the *grouping* of the addends does *not* change the sum.
$(a + b) + c = a + (b + c)$
*Example*: $\left(\frac{-2}{3} + \frac{1}{3}\right) + \frac{1}{5} = \frac{-2}{3} + \left(\frac{1}{3} + \frac{1}{5}\right)$

**Associative Property of Multiplication**
Changing the *grouping* of the factors does *not* change the product.
$(ab)c = a(bc)$
*Example*: $\left(\frac{3}{4} \bullet \frac{5}{9}\right)\frac{3}{5} = \frac{3}{4}\left(\frac{5}{9} \bullet \frac{3}{5}\right)$

Think
"grouping"

**Identity Property of Addition**
The *additive identity element* is 0. Adding 0 to any number does not change the value of the number.
$a + 0 = a$ and $0 + a = a$
*Examples*: $\frac{3}{8} + 0 = \frac{3}{8}$ and $0 + \frac{3}{8} = \frac{3}{8}$

**Identity Property of Multiplication**
The *multiplicative identity element* is 1. Multiplying 1 and any number does not change the value of the number.
$a \bullet 1 = a$ and $1 \bullet a = a$
*Examples*: $\frac{1}{10} \bullet 1 = \frac{1}{10}$ and $1 \bullet \frac{1}{10} = \frac{1}{10}$

Think
"same number"

**Inverse Property of Addition**
The *additive inverse*, or *opposite*, of $a$ is $-a$.
$a + (-a) = 0$ and $-a + a = 0$
*Examples*: $\frac{1}{2} + \left(-\frac{1}{2}\right) = 0$ and $-\frac{1}{2} + \frac{1}{2} = 0$

**Inverse Property of Multiplication**
The *multiplicative inverse*, or *reciprocal*, of $a$ is $\frac{1}{a}$, $a \neq 0$.
$a\left(\frac{1}{a}\right) = 1$ and $\left(\frac{1}{a}\right)a = 1$
*Examples*: $9\left(\frac{1}{9}\right) = 1$ and $\left(\frac{1}{9}\right)9 = 1$

**Zero Property of Multiplication**
The product of 0 and any number is 0.
$a \bullet 0 = 0$ and $0 \bullet a = 0$
$\frac{0}{b}\left(\frac{c}{d}\right) = 0$, when $b$ and $d \neq 0$
*Examples*: $0\left(\frac{5}{12}\right) = 0$ and $\left(\frac{5}{12}\right)0 = 0$

**Multiplicative Property of −1**
The product of −1 and any number is the additive inverse of the number.
$a \bullet (-1) = -(a)$
*Example*: $\frac{4}{9}(-1) = -\frac{4}{9}$ ← $\frac{4}{9}$ and $-\frac{4}{9}$ are opposites.

## Examples

Identify the properties used.

**1** $\left(\frac{-1}{8} + \frac{1}{5}\right) + 2\frac{1}{4} = \frac{-1}{8} + \left(\frac{1}{5} + 2\frac{1}{4}\right)$ ← grouping changed

Associative Property of Addition

**2** $\frac{-4}{9} \bullet \frac{3}{5} = \frac{3}{5} \bullet \frac{-4}{9}$ ← order changed

Commutative Property of Multiplication

► You can use the Distributive Property to compute with rational numbers.

**Distributive Property of Multiplication Over Addition**

$a(b + c) = (a \bullet b) + (a \bullet c)$

$2\frac{1}{2}\left(1\frac{3}{4} + 2\frac{3}{4}\right) = \left(2\frac{1}{2} \bullet 1\frac{3}{4}\right) + \left(2\frac{1}{2} \bullet 2\frac{3}{4}\right)$

$= \left(\frac{5}{2} \bullet \frac{7}{4}\right) + \left(\frac{5}{2} \bullet \frac{11}{4}\right)$

$= \frac{35}{8} + \frac{55}{8}$

$= \frac{90}{8} = 11\frac{2}{8} = 11\frac{1}{4}$

**Think**
Distribute the number for $a$ across the sum or difference.

**Distributive Property of Multiplication Over Subtraction**

$a(b - c) = (a \bullet b) - (a \bullet c)$

$\frac{2}{3}\left(\frac{1}{2} - \frac{1}{6}\right) = \left(\frac{2}{3} \bullet \frac{1}{2}\right) - \left(\frac{2}{3} \bullet \frac{1}{6}\right)$

$= \frac{2}{6} - \frac{2}{18}$

$= \left(\frac{2 \bullet 3}{6 \bullet 3}\right) - \frac{2}{18} = \frac{6}{18} - \frac{2}{18}$

$= \frac{4}{18} = \frac{2}{9}$

► Sometimes the Distributive Property can help you compute mentally when one of the factors is a mixed number.

## Examples

**1** Multiply: $2\frac{1}{2} \bullet (-14)$

**Think**
$2\frac{1}{2} = 2 + \frac{1}{2}$

$2\frac{1}{2}(-14) = 2(-14) + \frac{1}{2}(-14)$

$= -28 + (-7)$

$= -35$

**2** Multiply: $3\frac{1}{2} \bullet \frac{2}{3}$

**Think**
$3\frac{1}{2} = 3 + \frac{1}{2}$

$3\frac{1}{2}\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + \frac{1}{2}\left(\frac{2}{3}\right)$

$= \frac{6}{3} + \frac{2}{6}$

$= 2 + \frac{1}{3} = 2\frac{1}{3}$

## Try These

**Name the property used.**

**1.** $\frac{-2}{3}\left(\frac{3}{-2}\right) = 1$ **2.** $\frac{-5}{7} \bullet 1 = \frac{-5}{7}$ **3.** $1 = \frac{1}{8} \bullet 8$ **4.** $\frac{1}{6} \bullet \left(\frac{-2}{3}\right) \bullet \frac{0}{1} \bullet \frac{3}{4} = 0$

**What value for the variable makes each sentence true? Name the property used.**

**5.** $\frac{-4}{5} \bullet \frac{1}{4} = \frac{1}{4} \bullet g$ **6.** $\frac{2}{9} + 0 = h$ **7.** $\frac{3}{5}\left(5 - \frac{3}{10}\right) = \left(\frac{3}{5} \bullet 5\right) - \left(\frac{3}{5} \bullet k\right)$

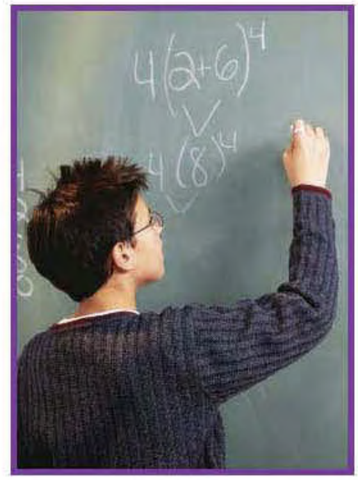
**8. Discuss and Write** Explain how you can use the Distributive Property to simplify the expression $\left(5\frac{1}{2} \bullet 1\frac{1}{6}\right) + \left(5\frac{1}{2} \bullet 2\frac{5}{6}\right)$. Simplify the expression to support your answer.

Go to PRACTICE BOOK Lesson 5-12 for exercise sets.

Algebra **5-13**

# Order of Operations with Rational Numbers

**Objective** To use the order of operations to simplify numerical expressions containing rational numbers • To use a calculator to check solutions



The order of operations is used to simplify expressions that contain any form of rational numbers.

Simplify: $(-0.4)^3 \bullet \left(\frac{-5}{2}\right)^4$ ← Simplify each power before multiplying the factors.

$(-0.4)(-0.4)(-0.4) \bullet \left(\frac{-5}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{-5}{2}\right)$ ← Multiply like terms.

$-0.064 \bullet \frac{625}{16}$ ← Multiply.

$\frac{-0.064 \bullet 625}{16}$

$\frac{-40}{16} = -2\frac{1}{2}$ ← Simplify.

► To simplify expressions that contain more than one grouping symbol, begin computing with the innermost set.

Simplify: $-6\left(\frac{2}{3} - \frac{5}{9}\right) \div [(2.4 \bullet 5)(-1)^9]$

$-6\left(\frac{2}{3} - \frac{5}{9}\right) \div [(2.4 \bullet 5)(-1)^9]$ ← Compute within parentheses.

$-6\left(\frac{1}{9}\right) \div [12(-1)^9]$ ← Simplify the power.

$-6\left(\frac{1}{9}\right) \div [12(-1)]$ ← Multiply.

$\frac{-6}{9} \div (-12)$

$\frac{\cancel{-6}^{\,-2}}{\cancel{9}_{\,3}} \div (-12)$ ← Simplify.

$\frac{-2}{3} \bullet \frac{-1}{12}$ ← Divide, multiply by the reciprocal.

$\frac{\cancel{-2}^{\,-1}}{3} \bullet \frac{-1}{\cancel{12}_{\,6}} = \frac{1}{18}$ ← Simplify.

► Some calculators have a fraction key, [Ab/c], and an exponent key, [^], which you can use to check your solution to an expression involving fractions and powers.

**Check:** $-6\left(\frac{2}{3} - \frac{5}{9}\right) \div (2.4 \bullet 5)(-1)^9$

Press [(−)] 6 [×] [(] 2 [Ab/c] 3 [−] 5 [Ab/c] 9 [)] [÷] [(] 2.4 [×] 5 [)] [×] [(] [(−)] 1 [)] [^] 9 [ENTER]

1/18

So $-6\left(\frac{2}{3} - \frac{5}{9}\right) \div (2.4 \bullet 5)(-1)^9 = \frac{1}{18}$.

**Remember:**
**Order of Operations**
- First, compute operations within grouping symbols.
- Next, simplify exponents.
- Then multiply or divide from left to right.
- Last, add or subtract from left to right.

**To Key Fractions with [Ab/c]**

$\frac{2}{3}$ Press 2 [Ab/c] 3

$7\frac{5}{8}$ Press 7 [Ab/c] 5 [Ab/c] 8

► Remember that the division bar is a grouping symbol. To simplify an expression with a division bar, first simplify above the bar, next simplify below the bar, and finally divide.

## Examples

**1** Simplify: $\dfrac{5^2 - 7\frac{2}{10}}{(2-15)+2^3}$

$\dfrac{5^2 - 7\frac{2}{10}}{(2-15)+2^3}$ ← Simplify the power.

$\dfrac{25 - 7\frac{2}{10}}{(2-15)+2^3}$ ← Subtract and rewrite the difference in simplest form.

$\dfrac{17\frac{4}{5}}{(2-15)+2^3}$ ← Compute within parentheses.

$\dfrac{17\frac{4}{5}}{-13+2^3}$ ← Simplify the power.

$\dfrac{17\frac{4}{5}}{-13+8}$ ← Add.

$\dfrac{17\frac{4}{5}}{-5} = 17\frac{4}{5} \div (-5)$ ← Rewrite in horizontal form.

$\frac{89}{5} \div \frac{-5}{1}$ ← Rename as fractions.

$\frac{89}{5} \bullet \frac{1}{-5}$ ← Multiply by the reciprocal to divide.

$\frac{89}{-25} = -3\frac{14}{25}$

**2** Simplify: $\dfrac{-12.5+0.5}{\frac{3}{4} \bullet 0.5}$

$\dfrac{-12.5+0.5}{\frac{3}{4} \bullet 0.5}$ ← Add.

$\dfrac{-12}{\frac{3}{4} \bullet 0.5}$ ← Rename 0.5 as $\frac{1}{2}$.

$\dfrac{-12}{\frac{3}{4} \bullet \frac{1}{2}}$ ← Multiply.

$\dfrac{-12}{\frac{3}{8}}$ ← Simplify.

$\frac{-12}{1} \div \frac{3}{8}$ ← Rewrite in horizontal form.

$\frac{-12}{1} \bullet \frac{8}{3}$ ← Multiply by the reciprocal to divide.

$\frac{\overset{-4}{\cancel{-12}}}{1} \bullet \frac{8}{\underset{1}{\cancel{3}}}$ ← Simplify.

$\frac{-32}{1} = -32$

## Try These

**Simplify.**

**1.** $(-0.2)^4 \bullet (-0.3)^2$

**2.** $\left(-\frac{1}{2}\right)^3 \div \left(\frac{3}{4}\right)^2$

**3.** $(-0.5)^3 \bullet \left(-\frac{2}{3}\right)^3$

**4.** $-3(4.6 + 0.3) \div (-3)^2$

**5.** $\frac{2}{25} + 21 \div 7 + 4\frac{1}{5}$

**6.** $[-24 + (-16)] \div 2^2 + 0.5$

**7.** $\dfrac{-2.4 + (-4.6)}{(-7)(10)}$

**8.** $\dfrac{3\frac{1}{2} + \left(8\frac{3}{4} + 3\frac{1}{2}\right)}{2}$

**9.** $\dfrac{3\frac{3}{4} + \left(8\frac{3}{4} + 3\frac{1}{2}\right)}{6 + (-2)^2}$

**10. Discuss and Write** For the expression $\dfrac{-12.5+0.5}{\frac{3}{4} \bullet \frac{1}{2}}$, is it easier to solve it the way it is, to rename the decimals as fractions, or to rename the fractions as decimals? Show examples to justify your answer.

Go to PRACTICE BOOK Lesson 5-13 for exercise sets.

Algebra 5-14

Update your skills. See page 414 XVI.

# Addition and Subtraction Equations with Fractions

**Objective** To apply the Subtraction Property of Equality to solve addition equations with fractions • To apply the Addition Property of Equality to solve subtraction equations with fractions

Herman packages nails for a hardware company. He weighs them instead of counting the individual nails. If the scale already shows $\frac{3}{8}$ pound, what fraction of a pound of nails does he need to make a $\frac{9}{16}$-pound package?

► To find the amount, write an addition equation, and apply the Subtraction Property of Equality.

Let $x$ = the fraction of a pound of nails needed.

**Solve:** $x + \frac{3}{8} = \frac{9}{16}$

$x + \frac{3}{8} - \frac{3}{8} = \frac{9}{16} - \frac{3}{8}$ ← Subtract $\frac{3}{8}$ from both sides to isolate $x$.

$x = \frac{9}{16} - \frac{6}{16}$ ← Rename as fractions with a common denominator, then subtract.

$x = \frac{3}{16}$ ← Express in simplest form.

**Remember:**
**Subtraction Property of Equality**
If $a = b$, then $a - c = b - c$.

**Check:** $x + \frac{3}{8} = \frac{9}{16}$

$\frac{3}{16} + \frac{3}{8} \stackrel{?}{=} \frac{9}{16}$ ← Substitute $\frac{3}{16}$ for $x$.

$\frac{3}{16} + \frac{6}{16} \stackrel{?}{=} \frac{9}{16}$ ← Rename as fractions with a common denominator, then add.

$\frac{9}{16} = \frac{9}{16}$ True

So Herman needs $\frac{3}{16}$ pound of nails to make a $\frac{9}{16}$-pound package.

► To solve an addition equation with signed fractions, follow the same rules as you use to add and subtract integers.

**Solve:** $n + \left(\frac{-6}{15}\right) = \frac{3}{15}$

$n + \left(\frac{-6}{15}\right) - \left(\frac{-6}{15}\right) = \frac{3}{15} - \left(\frac{-6}{15}\right)$ ← Subtract $\frac{-6}{15}$ from both sides to isolate $n$.

$n = \frac{3}{15} + \frac{6}{15}$ ← Use the Subtraction Principle. Add.

$n = \frac{9}{15}$

$n = \frac{9 \div 3}{15 \div 3}$ ← Simplify. Divide the numerator and denominator by their GCF, 3.

$n = \frac{3}{5}$

**Check:** $n + \left(\frac{-6}{15}\right) = \frac{3}{15}$

$\frac{3}{5} + \left(\frac{-6}{15}\right) \stackrel{?}{=} \frac{3}{15}$

$\frac{9}{15} + \left(\frac{-6}{15}\right) \stackrel{?}{=} \frac{3}{15}$

$\frac{3}{15} = \frac{3}{15}$ True

► To solve a subtraction equation with positive and negative fractions, apply the Addition Property of Equality.

**Remember:**
**Addition Property of Equality**
If $a = b$, then $a + c = b + c$.

**Solve:** $\frac{1}{2} = n - \frac{3}{8}$

$\frac{1}{2} + \frac{3}{8} = n - \frac{3}{8} + \frac{3}{8}$ ← Add $\frac{3}{8}$ to both sides to isolate $n$.

$\frac{4}{8} + \frac{3}{8} = n$ ← Rename as fractions with a common denominator, then add.

$\frac{7}{8} = n$

**Check:** $\frac{1}{2} = n - \frac{3}{8}$

$\frac{1}{2} \stackrel{?}{=} \frac{7}{8} - \frac{3}{8}$ ← Substitute $\frac{7}{8}$ for $n$, then subtract.

$\frac{1}{2} \stackrel{?}{=} \frac{4}{8}$

**Think** $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$ True

► To solve an addition or subtraction equation with mixed numbers, rename so the fraction parts have like denominators.

**Solve:** $n - 4\frac{3}{10} = 17\frac{2}{5}$

$n - 4\frac{3}{10} + 4\frac{3}{10} = 17\frac{2}{5} + 4\frac{3}{10}$ ← Add $4\frac{3}{10}$ to both sides to isolate $n$.

$n = 17\frac{4}{10} + 4\frac{3}{10}$ ← Rename, using a common denominator, then add.

$n = 21\frac{7}{10}$

**Check:** $n - 4\frac{3}{10} = 17\frac{2}{5}$

$21\frac{7}{10} - 4\frac{3}{10} \stackrel{?}{=} 17\frac{2}{5}$ ← Substitute $21\frac{7}{10}$ for $n$.

$17\frac{4}{10} \stackrel{?}{=} 17\frac{2}{5}$ ← Subtract.

**Think** $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

$17\frac{2}{5} = 17\frac{2}{5}$ True

## Try These

**Solve for $x$. Then use substitution to check.**

**1.** $x + \frac{1}{2} = \frac{3}{4}$

**2.** $y + \left(\frac{-5}{8}\right) = \frac{1}{4}$

**3.** $\frac{1}{2} = m - \frac{3}{5}$

**4.** $\frac{3}{4} = x - \frac{1}{12}$

**5.** $x - \left(\frac{-3}{10}\right) = \frac{3}{5}$

**6.** $x + 7\frac{1}{4} = 13\frac{3}{8}$

**7. Discuss and Write** What steps would you take to solve for $k$?

$k - \frac{3}{10} - \frac{4}{5} + \frac{1}{2} = \frac{7}{10}$

Go to PRACTICE BOOK Lesson 5-14 for exercise sets.

Algebra

# 5-15 Multiplication and Division Equations with Fractions

Update your skills. See page 414 XVII.

**Objective** To apply the Division Property of Equality to solve multiplication equations with fractions • To apply the Multiplication Property of Equality to solve division equations with fractions

On May 6, Kiera volunteered for 12 hours at the natural history museum. This was $\frac{3}{4}$ as much time as she had volunteered the day before. How many hours did she spend volunteering at the natural history museum on May 5?

To find the number of hours, write and solve a multiplication equation.

Let $x$ = the number of hours volunteered on May 5.

$\frac{3}{4}x = 12$

**Think**
$\frac{3}{4}$ is the coefficient of $x$.

► To solve a multiplication equation with fractions, use either of these methods:

**Method 1** Multiply by the Reciprocal

**Solve:** $\frac{3}{4}x = 12$

**Think**
$\frac{3}{4}x = \frac{3}{4} \bullet x$

$\frac{4}{3} \bullet \frac{3}{4}x = \frac{4}{3} \bullet 12$ ← Multiply both sides by the reciprocal of $\frac{3}{4}$.

$x = \frac{4}{\cancel{3}_1} \bullet \frac{\cancel{12}^4}{1}$ ← Simplify.

$x = \frac{4 \bullet 4}{1 \bullet 1}$ ← Multiply.

$x = 16$

**Method 2** Use the Division Property of Equality

**Remember:**
**Division Property of Equality**
If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

**Solve:** $\frac{3}{4}x = 12$

$\frac{\frac{3}{4}x}{\frac{3}{4}} = \frac{12}{\frac{3}{4}}$ ← Divide both sides by $\frac{3}{4}$ to isolate $x$.

$\frac{3}{4}x \bullet \frac{4}{3} = 12 \bullet \frac{4}{3}$ ← Divide, multiply by the reciprocal.

$x = \frac{\cancel{12}^4}{1} \bullet \frac{4}{\cancel{3}_1}$ ← Simplify.

$x = \frac{4 \bullet 4}{1 \bullet 1}$ ← Multiply.

$x = 16$

Both methods result in the same solution.

**Check:** $\frac{3}{4}x = 12$

$\frac{3}{4} \bullet 16 \stackrel{?}{=} 12$ ← Substitute 16 for $x$.

$\frac{3}{\cancel{4}_1} \bullet \frac{\cancel{16}^4}{1} \stackrel{?}{=} 12$ ← Simplify.

$\frac{3 \bullet 4}{1 \bullet 1} \stackrel{?}{=} 12$ ← Multiply.

$12 = 12$ True

So Kiera spent 16 hours volunteering at the natural history museum on May 5.

► To solve a division equation with fractions, apply the Multiplication Property of Equality.

**Remember:**
**Multiplication Property of Equality**
If $a = b$, then $a \bullet c = b \bullet c$

**Solve:** $\dfrac{n}{\frac{1}{4}} = 8$

Think: $\dfrac{n}{\frac{1}{4}} = 8 \rightarrow n \div \frac{1}{4} = 8$

$\frac{1}{4} \bullet \dfrac{n}{\frac{1}{4}} = \frac{1}{4} \bullet 8$ ← Multiply both sides by $\frac{1}{4}$ to isolate $n$.

$n = \frac{8}{4}$ ← Simplify.

$n = 2$

**Check:** $\dfrac{n}{\frac{1}{4}} = 8$

$n \div \frac{1}{4} = 8$ ← Rewrite, using the ÷ symbol.

$2 \div \frac{1}{4} \stackrel{?}{=} 8$ ← Substitute 2 for $n$.

$2 \bullet \frac{4}{1} \stackrel{?}{=} 8$ ← Divide, multiply by the reciprocal of the divisor.

$8 = 8$ True

► To solve multiplication or division equations with positive and negative fractions, use the sign rules for multiplying and dividing integers.

## Examples

**1** **Solve:** $\frac{3}{7}c = -21$

$\frac{3}{7}c \bullet \frac{7}{3} = -21 \bullet \frac{7}{3}$

$c = \dfrac{\cancel{-21}^{\,-7}}{1} \bullet \dfrac{7}{\cancel{3}_{\,1}}$

$c = \frac{-49}{1}$

$c = -49$ ← The dividend and the divisor have different signs, so the product is negative.

**Check:** $\frac{3}{7}c = -21$

$\left(\frac{3}{7}\right)(-49) \stackrel{?}{=} -21$ ← Substitute $-49$ for $c$.

$\dfrac{3}{\cancel{7}_{\,1}} \bullet \dfrac{\cancel{-49}^{\,-7}}{1} \stackrel{?}{=} -21$

$-21 = -21$ True

**2** **Solve:** $-5 = -g \div \frac{1}{2}$

$-5 \bullet \frac{1}{2} = -g \div \frac{1}{2} \bullet \frac{1}{2}$

$\frac{-5}{2} = -g$

$-2\frac{1}{2} = -g$

$(-1)\left(-2\frac{1}{2}\right) = (-1)(-g)$ ← Use the Multiplicative Property of $-1$ to isolate $g$.

$2\frac{1}{2} = g$

**Check:** $-5 = -g \div \frac{1}{2}$

$-5 \stackrel{?}{=} -2\frac{1}{2} \div \frac{1}{2}$ ← Substitute $2\frac{1}{2}$ for $g$.

$-5 \stackrel{?}{=} \frac{-5}{2} \div \frac{1}{2}$

$-5 \stackrel{?}{=} \frac{-5}{2} \bullet \frac{2}{1}$

$-5 \stackrel{?}{=} \frac{-10}{2}$

$-5 = -5$ True

## Try These

**Solve and check.**

**1** $\frac{2}{3}t = 16$ **2.** $z \div \frac{1}{3} = 8$ **3.** $-\frac{1}{3}k = 2$ **4.** $30 = c \div \left(-\frac{1}{4}\right)$

**5. Discuss and Write** Is it possible to estimate a value for the unknown in a multiplication or division equation that contains fractions? Justify your answer with examples.

Go to PRACTICE BOOK Lesson 5-15 for exercise sets.

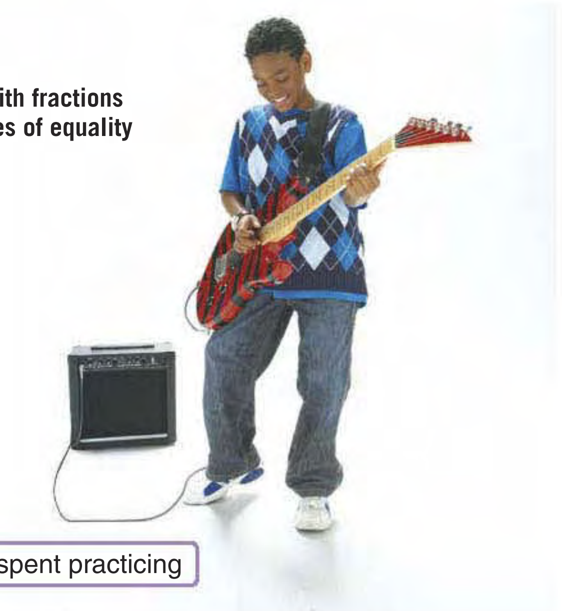
Algebra

5-16

Update your skills. See page 415 XVIII.

# Solve Two-Step Equations with Fractions

**Objective** **To solve two-step algebraic equations with fractions and mixed numbers by applying the appropriate properties of equality and the Inverse Properties of Addition and Multiplication**



Sean spends $\frac{1}{2}$ hour playing songs he knows. Then he spends $\frac{3}{4}$ hour practicing each new song he is learning. If Sean spends $2\frac{3}{4}$ hours practicing today, how many new songs is he learning?

► To find the number of new songs Sean is learning, write and solve a two-step equation.

Let $s$ = the number of new songs Sean is learning.

$$\frac{1}{2} + \frac{3}{4}s = 2\frac{3}{4}$$

total number of hours spent practicing

hours spent on old songs | hours spent on each new song | number of new songs

**Solve:** $\frac{1}{2} + \frac{3}{4}s = 2\frac{3}{4}$

$\frac{1}{2} - \frac{1}{2} + \frac{3}{4}s = 2\frac{3}{4} - \frac{1}{2}$ ← Use the Subtraction Property of Equality to "undo" addition.

$\frac{3}{4}s = 2\frac{1}{4}$ ← Simplify.

$\left(\frac{4}{3}\right)\frac{3}{4}s = \left(\frac{4}{3}\right)2\frac{1}{4}$ ← Multiply both sides by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$, to isolate $s$.

$s = \frac{4}{3} \bullet \frac{9}{4}$ ← Rename the mixed number.

$s = \frac{\cancel{4}^{1}}{\cancel{3}_{1}} \bullet \frac{\cancel{9}^{3}}{\cancel{4}_{1}}$ ← Simplify, then multiply.

$s = 3$

**Check:** $\frac{1}{2} + \frac{3}{4}s = 2\frac{3}{4}$

$\frac{1}{2} + \frac{3}{4}(3) \stackrel{?}{=} 2\frac{3}{4}$ ← Substitute 3 for $s$.

$\frac{1}{2} + \frac{9}{4} \stackrel{?}{=} 2\frac{3}{4}$ ← Multiply.

$\frac{2}{4} + \frac{9}{4} \stackrel{?}{=} 2\frac{3}{4}$

$2\frac{3}{4} = 2\frac{3}{4}$ True

So Sean is learning 3 new songs.

## Examples

**1** **Solve:** $21 = \frac{1}{4}c + 5$

$21 - 5 = \frac{1}{4}c + 5 - 5$ ← Subtract 5 from both sides.

$16 = \frac{1}{4}c$ ← Simplify.

$\left(\frac{4}{1}\right)16 = \left(\frac{4}{1}\right)\left(\frac{1}{4}\right)c$ ← Multiply by the reciprocal of $\frac{1}{4}$.

$64 = c$

**Check:** $21 = \frac{1}{4}c + 5$

$21 \stackrel{?}{=} \frac{1}{4}(64) + 5$ ← Substitute 64 for $c$.

$21 \stackrel{?}{=} 16 + 5$

$21 = 21$ True

**2** **Solve:** $\frac{1}{3}n - 7 = 8$

$\frac{1}{3}n - 7 + 7 = 8 + 7$ ← Add 7 to both sides.

$\frac{1}{3}n = 15$ ← Simplify.

$\left(\frac{3}{1}\right)\left(\frac{1}{3}\right)n = \left(\frac{3}{1}\right)15$ ← Multiply by the reciprocal.

$n = 45$

**Check:** $\frac{1}{3}n - 7 = 8$

$\frac{1}{3}(45) - 7 \stackrel{?}{=} 8$ ← Substitute 45 for $n$.

$15 - 7 \stackrel{?}{=} 8$

$8 = 8$ True

**3** **Solve:** $\frac{a}{3} + \frac{7}{8} = -16\frac{1}{2}$

$\frac{a}{3} + \frac{7}{8} - \frac{7}{8} = -16\frac{1}{2} - \frac{7}{8}$ ← Subtract $\frac{7}{8}$ from both sides.

$\frac{a}{3} = -17\frac{3}{8}$ ← Simplify.

$(3)\frac{a}{3} = (3)\left(-17\frac{3}{8}\right)$ ← Multiply both sides by 3.

$a = -51\frac{9}{8} = -52\frac{1}{8}$

**Check:** $\frac{a}{3} + \frac{7}{8} = -16\frac{1}{2}$

$\frac{-52\frac{1}{8}}{3} + \frac{7}{8} \stackrel{?}{=} -16\frac{1}{2}$ ← Substitute $-52\frac{1}{8}$ for $a$.

$-17\frac{3}{8} + \frac{7}{8} \stackrel{?}{=} -16\frac{1}{2}$

$-16\frac{1}{2} = -16\frac{1}{2}$ True

**4** Webster made 5 pounds of fruit salad for a party. He used $1\frac{1}{4}$ pounds of strawberries. He added $\frac{5}{8}$ pound of each of the other ingredients. How many other ingredients were there?

Let $n$ = the number of other ingredients.

**Solve:** $5 = \frac{5}{8}n + 1\frac{1}{4}$

$5 - 1\frac{1}{4} = \frac{5}{8}n + 1\frac{1}{4} - 1\frac{1}{4}$

$3\frac{3}{4} = \frac{5}{8}n$

$3\frac{3}{4} \div \frac{5}{8} = \frac{5}{8}n \div \frac{5}{8}$

$\left(3\frac{3}{4}\right)\frac{8}{5} = n$

$\frac{\cancel{15}^{3}}{\cancel{4}_{1}} \bullet \frac{\cancel{8}^{2}}{\cancel{5}_{1}} = n$

$6 = n$

**Check:** $5 = \frac{5}{8}n + 1\frac{1}{4}$

$5 \stackrel{?}{=} \frac{5}{8}(6) + 1\frac{1}{4}$

$5 \stackrel{?}{=} \frac{30}{8} + 1\frac{1}{4}$

$5 \stackrel{?}{=} 3\frac{6}{8} + 1\frac{1}{4}$

$5 \stackrel{?}{=} 3\frac{3}{4} + 1\frac{1}{4}$

$5 = 5$ True

## Try These

**Solve and check.**

**1.** $\frac{1}{3}n - 6 = 9$ **2.** $30 = \frac{1}{5}c + 10$ **3.** $\frac{x}{4} + \frac{3}{5} = 8\frac{1}{4}$ **4.** $\frac{a}{5} - \frac{1}{2} = -6\frac{3}{8}$

**5. Discuss and Write** Justify the steps you used in solving exercise 2 above.

Go to PRACTICE BOOK Lesson 5-16 for exercise sets.

## 5-17

*Update your skills. See page 416 XXI.*

# Rename Customary Units of Measure

**Objective** To rename customary units of length, capacity, and weight
• To compute with units of measure

A basketball court is $16\frac{2}{3}$ yards wide.
How many feet wide is a basketball court?

To find how many feet, rename $16\frac{2}{3}$ yards as feet.

► You can rename units of length in the customary measurement system by multiplying or dividing. Use the table of equivalent units on page 439 to help you.

To rename larger units as smaller units, *multiply*.
To rename smaller units as larger units, *divide*.

$16\frac{2}{3}$ yd = $\underline{\ ?\ }$ ft  **Think** 1 yard $>$ 1 foot

$16\frac{2}{3} \bullet 3$ ← 1 yd = 3 ft, so multiply by 3.

$\frac{50}{\cancel{3}_{1}} \bullet \frac{\cancel{3}^{1}}{1}$ ← Rename the mixed number as a fraction.

50 ← Multiply. Write in simplest form.

So a basketball court is 50 feet wide.

**Customary Units of Length**
1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 ft or 36 in.
1 mile (mi) = 5280 ft or 1760 yd

## Examples

**1** How many feet are in $\frac{3}{4}$ mile?

$\frac{3}{4}$ mi = $\underline{\ ?\ }$ ft  **Think** 1 mile $>$ 1 foot

$\frac{3}{4} \bullet 5280$ ← 1 mi = 5280 ft, so multiply by 5280.

$\frac{3}{\cancel{4}_{1}} \bullet \frac{\cancel{5280}^{1320}}{1}$ ← Simplify.

3960

So there are 3960 feet in $\frac{3}{4}$ mile.

**2** How many yards are in 90 inches?

90 in. = $\underline{\ ?\ }$ yd  **Think** 1 inch $<$ 1 yard

$\frac{90}{36}$ ← 36 in. = 1 yd, so divide by 36.

$\frac{\cancel{90}^{5}}{\cancel{36}_{2}}$ ← Simplify.

$\frac{5}{2} = 2\frac{1}{2}$

So there are $2\frac{1}{2}$ yards in 90 inches.

► Rename units of weight using the same process as you did for units of length.

How many pounds are in 60 ounces?

60 oz = $\underline{\ ?\ }$ lb  **Think** 1 oz $<$ 1 lb

$\frac{60}{16}$ ← 1 pound = 16 ounces, so divide by 16.

$\frac{\cancel{60}^{15}}{\cancel{16}_{4}} = \frac{15}{4} = 3\frac{3}{4}$

So there are $3\frac{3}{4}$ pounds in 60 ounces.

**Customary Units of Weight**
1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2000 lb

▶ Rename units of capacity by using the same process as you did for units of length or weight.

**Customary Units of Capacity**
1 cup (c) = 8 fluid ounces (fl oz)
1 pint (pt) = 2 c
1 quart (qt) = 2 pt
1 gallon (gal) = 4 qt

## Examples

**1** How many cups are in 30 fluid ounces?

30 fl oz = ? c

Think: 1 fl oz < 1 c

$\frac{30}{8}$ ← 8 fl oz = 1 c, so divide by 8.

$\frac{\cancel{30}^{15}}{\cancel{8}_{4}} = \frac{15}{4} = 3\frac{3}{4}$

So there are $3\frac{3}{4}$ cups in 30 fluid ounces.

**2** How many quarts are in $6\frac{1}{2}$ gallons?

$6\frac{1}{2}$ gal = ? qt

Think: 1 gal > 1 qt

$6\frac{1}{2} \bullet 4$ ← 1 gal = 4 qt, so multiply by 4.

$\frac{13}{\cancel{2}_{1}} \bullet \frac{\cancel{4}^{2}}{1} = 26$ ← Rename the numbers as fractions.

So there are 26 quarts in $6\frac{1}{2}$ gallons.

▶ You can compute with measurements that have two or more different units if the units all measure the same property: length, weight, or capacity.

### Add or Subtract

- Add or subtract like units.
- Regroup to rename units.

$$\begin{array}{rr} 3\text{ ft} & 7\text{ in.} \\ +4\text{ ft} & 9\text{ in.} \\ \hline 7\text{ ft} & 16\text{ in.} \end{array}$$

7 ft 16 in. = 8 ft 4 in.

$$\begin{array}{rrr} & 1 & 18 \\ 3\text{ yd} & \cancel{2}\text{ ft} & \cancel{6}\text{ in.} \\ -1\text{ yd} & 1\text{ ft} & 9\text{ in.} \\ \hline 2\text{ yd} & 0\text{ ft} & 9\text{ in.} \end{array}$$

### Multiply

- Multiply each unit.
- Regroup to rename units.

OR

- Rename all units as like units.
- Multiply and then regroup.

$$\begin{array}{rr} 2\text{ ft} & 8\text{ in.} \\ \times & 4 \\ \hline 8\text{ ft} & 32\text{ in.} \end{array}$$

8 ft 32 in. = 10 ft 8 in.

$$\begin{array}{rr} 2\text{ ft}\ \ 8\text{ in.} & \rightarrow 32\text{ in.} \\ \times \qquad 4 & \rightarrow \times\ 4 \\ \hline & 128\text{ in.} \end{array}$$

128 in. = 10 ft 8 in.

### Divide

- Rename all units as like units, and combine.
- Divide and then regroup.

1 yd 1 ft 9 in. ÷ 3 ← Rename as like units.
(36 in. + 12 in. + 9 in.) ÷ 3 ← Combine like units.
57 in. ÷ 3 ← Divide.
19 in. = 1 ft 7 in. ← Rename as feet and inches.

## Try These

**Rename each unit of measure.**

**1.** $2\frac{1}{2}$ lb = ? oz **2.** 30 in. = ? ft **3.** $6\frac{1}{2}$ c = ? fl oz

**4.** $\frac{3}{4}$ T = ? lb **5.** $4\frac{1}{2}$ gal = ? pt **6.** 12,600 lb = ? T

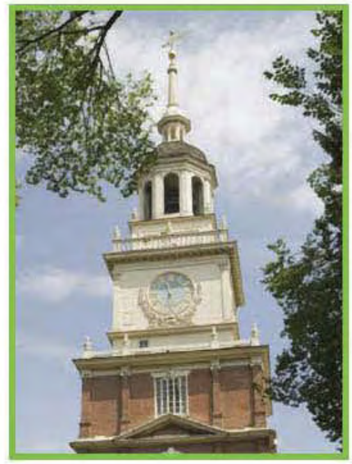
**7.** Compute. **a.** (8 mi 3 yd) • 7 **b.** (1 yd 1 ft 8 in.) ÷ 8 **c.** (1 gal 2 qt 3 c) − (3 qt 1 pt 2 c)

**8. Discuss and Write** Why do you multiply to rename larger units as smaller units and divide to rename smaller units as larger units?

Go to PRACTICE BOOK Lesson 5-17 for exercise sets.

# 5-18 Problem-Solving Strategy: Make a Drawing

**Objective** To solve problems using the strategy *Make a Drawing*



**Problem 1:** The clock tower in Liberty Square, known for its accuracy, chimes its bell every hour on the hour at equal intervals. If the clock strikes 6 chimes at 6 o'clock in 6 seconds, how long would it take for the clock to strike 12 chimes at 12 o'clock? (To complete the problem, assume that the chime itself takes no time.) *Hint:* The answer is not 12 seconds.

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Each chime occurs within an equal interval.
6 chimes strike in 6 seconds at 6 o'clock.
The answer is not 12 seconds.

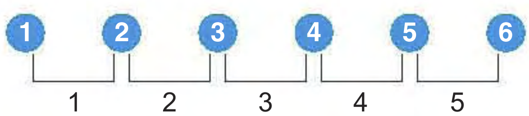
**Question:** How long would it take for the clock to strike 12 chimes at 12 o'clock?

**Problem-Solving Strategies**
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. **Make a Drawing**
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**
Using the strategy *Make a Drawing* will help you understand the situation.

**Solve** **Apply the strategy.**

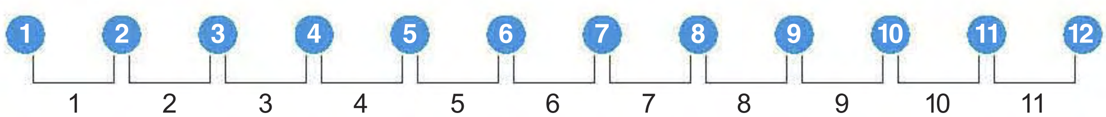
- First make a drawing that relates to the facts. Use dots to represent the chimes that occur at 6 o'clock.




The 6 chimes occur in 6 seconds. There are 5 intervals throughout the ringing of 6 chimes, therefore each interval must take $\frac{6}{5}$ seconds.

**Think** $\frac{6}{5} \cdot 5 = 6$

- Make a drawing to represent the chimes that occur at 12 o'clock.




You can see that there are 11 intervals when there are 12 chimes. If each interval takes $\frac{6}{5}$ seconds, then multiply $\frac{6}{5}$ by 11 to find how long it takes for 12 chimes.

$11 \cdot \frac{6}{5} = 13\frac{1}{5}$

The clock takes $13\frac{1}{5}$ seconds to strike 12 chimes.

**Check** **Check to make sure your answer makes sense.**
There are twice as many chimes, so there ought to be about twice as much time used. It appears to be so.

**Problem 2:** There are 240 seventh graders at Kingston Middle School. Of these students, $\frac{1}{6}$ walk to school. Of those who do not walk, $\frac{3}{4}$ take the bus to school. Of those who do not walk or take the bus, half ride their bikes. How many seventh graders ride their bikes to school?


**Read** **Read to understand what is being asked.**
List the facts and restate the question.

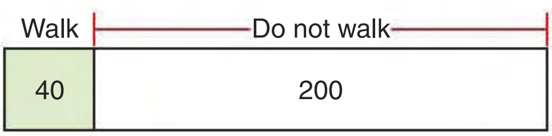
**Facts:** There are 240 seventh graders in all.
$\frac{1}{6}$ walk to school.
$\frac{3}{4}$ of those who do not walk take the bus.
$\frac{1}{2}$ of those who do not walk or take the bus ride their bikes.

**Question:** How many seventh graders ride their bikes to school?

**Plan** **Select a strategy.**
This problem has a lot of information. To make this information easier to understand, you can use the strategy *Make a Drawing*.

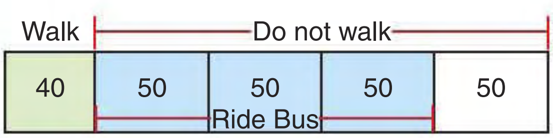
**Solve** **Apply the strategy.**

- Draw a rectangle to represent the entire seventh grade. Divide the rectangle to show those who walk and those who do not.




**Think** $\frac{1}{6}$ of 240 is 40.

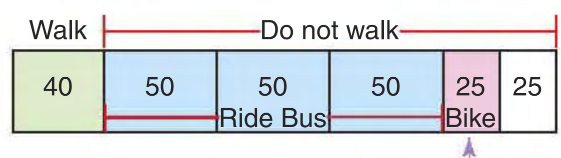
- Divide the section representing those who do not walk into fourths. Shade $\frac{3}{4}$ to represent those who take the bus. The unshaded part represents those who do not walk or take the bus.




**Think** $\frac{1}{4}$ of 200 is 50.

- Divide the unshaded section to represent those who ride their bikes.

So 25 students ride their bikes to school.




$\frac{1}{2}$ of 50 is 25.

**Check** **Check to make sure your answer makes sense.**
Look back at the final drawing, and make sure the numbers that represent each section satisfy the conditions in the problem.

The total is $40 + 50 + 50 + 50 + 25 + 25 = 240$ students. ✓

40 students walk. This is $\frac{1}{6}$ of the 240 students. ✓

150 students ride the bus. This is $\frac{3}{4}$ of the 200 who do not walk. ✓

25 students ride their bikes. This is $\frac{1}{2}$ of the 50 who do not walk or ride the bus. ✓

**Go to** PRACTICE BOOK **Lesson 5-18 for exercise sets.**

ONLINE www.progressinmathematics.com
More Enrichment Topics

# Enrichment:
## Different Ways to Find the GCF

**Objective** To explore different algorithms for finding the greatest common factor (GCF) of two numbers

Finding the GCF of two numbers by listing factors can be time-consuming. Since the time of ancient Greece, people have used other methods to find the GCF.

► Look at these two ways to find the GCF of 10 and 18.

**Method 1** Use Division

- Divide the greater number by the lesser number.
- If the remainder is 0, the lesser number is the GCF. If not, divide the divisor by the remainder.
- Continue this process until the remainder is 0. The last divisor is the GCF.

18 ÷ 10 = 1R8
10 ÷ 8 = 1R2
8 ÷ 2 = 4R0

The GCF is 2.

**Method 2** Use Subtraction

- Subtract the lesser number from the greater number.
- Then compare the three numbers (the two numbers and the difference) and subtract the least number from the next least number.
- Continue until two of the three numbers are the same. That number is the GCF.

18 − 10 = 8
10 − 8 = 2
8 − 2 = 6
6 − 2 = 4
4 − 2 = 2

The GCF is 2.

As you use these methods, you divide or subtract until you find the result you want. These methods are *iterative*. An iterative process repeats over and over.

► Now use the two methods above to find the GCF of 1,989 and 2,691.

**Method 1** Use Division

2,691 ÷ 1,989 = 1R702

1,989 ÷ 702 = 2R585

702 ÷ 585 = 1R117

585 ÷ 117 = 5 R0

The GCF is 117.

**Method 2** Use Subtraction

2,691 − 1,989 = 702
1,989 − 702 = 1,287
1,287 − 702 = 585
702 − 585 = 117
585 − 117 = 468
468 − 117 = 351
351 − 117 = 234
234 − 117 = 117

The GCF is 117.

### Try These

**Use any method (or more than one) to find the GCF of these pairs of numbers.**

**1.** 21, 28 **2.** 202, 2,002 **3.** 17, 68 **4.** 54, 180 **5.** 45, 16

**6.** **Discuss and Write** For one of the problems, you found a GCF of 1. What does that mean?



Go to PRACTICE BOOK page 159 for exercise sets.

# Test Prep: Short-Answer Questions

## Strategy: Show All Your Work

Short-answer questions give you an opportunity to *explain your thinking*. Showing all your work demonstrates your understanding of how to solve the problem. You should include written explanations as you work through the solution.

**Sample Test Item**

Bryce is making smoothies for a party. He needs a total of 60 fluid ounces of orange juice for the smoothie recipe. He only has a $1\frac{1}{2}$-cup measuring cup. How many times must he fill the measuring cup with juice to make the smoothies? ***Show all your work.***

Look at the sample test item.

**Read the whole test item carefully.**

- Reread the test item. Try to relate the question to similar problems. Think about relationships between units of measure.
- Make a plan to explain your thinking.
  1. Use measurement conversions to find the number of fluid ounces the measuring cup holds.
  2. Divide the total amount needed by the amount in each full measuring cup.

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply appropriate rules, definitions, properties, or strategies. Use measurement conversions.

$$8 \bullet 1\frac{1}{2} = 8 \bullet \frac{3}{2}$$
$$= 12 \text{ fluid ounces}$$

Bryce needs 60 fluid ounces. Since $60 \div 12 = 5$, he needs to fill the cup 5 times.
**Answer:** Bryce needs to fill the measuring cup 5 times.

**Think**
There are 8 fluid ounces in 1 cup, so $1\frac{1}{2}$ cups equals how many fluid ounces?

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answer. Does it make sense? Try another method.

Convert the total amount Bryce needs from fluid ounces to cups, then divide by the capacity of the measuring cup.

$$60 \div 8 = 7\frac{1}{2} \text{ cups}$$

$$7\frac{1}{2} \div 1\frac{1}{2} = \frac{15}{2} \bullet \frac{2}{3}$$
$$= 5 \checkmark$$

## Try These

**Solve. Explain your thinking.**

**1.** Evaluate the expression.

$$\frac{16 - 8\left(1\frac{1}{2} + 1\frac{1}{4}\right)}{3}$$

***Show all your work.***

**Read the whole test item carefully.**

- Reread the test item. Try to relate the question to similar problems.
- Make a plan to explain your thinking.
  1. Follow the order of operations.
  2. Use a common denominator to add the mixed numbers.

**Solve the problem.**

- Apply appropriate rules, properties, and operations. Follow the order of operations to evaluate the expression.

### Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Think**
First evaluate within grouping symbols. The parentheses and the fraction bar are both grouping symbols.

## Item Analysis

**Check your work. Review your notes.**
**Make sure you have completed all parts of the items.**

- Analyze your answers. Do they make sense? Look back at the expression. Use estimation to check that your answer is reasonable.

**2.** Graph the solution set of the inequality shown below.
$-6x < 18$
***Show all your work.***

**3.** Solve for $y$ in the equation shown below.
$\frac{3}{4}y + 5 = -4$
***Show all your work.***

**4.** What is the greatest common factor of 32, 72, and 120? Explain how you found your answer.
***Show all your work.***

**5.** Order the numbers from least to greatest.
$-\frac{9}{2}, -4.3, -3\frac{1}{4}, -3.\overline{4}$
***Show all your work.***

**6.** Simplify the expression.
$(-9 + 2)^3 \div (-2 + 9)^3$
***Show all your work.***

Go to PRACTICE BOOK pages 161–162 for exercise sets.

# Ratio and Proportion

CHAPTER 6

## In This Chapter You Will:

- Identify and compare ratios
- Find and compare unit rates, unit costs
- Write proportions and find the missing term in a proportion
- Solve direct and inverse proportions
- Use dimensional analysis
- Solve problems involving similar figures
- Solve indirect measurement problems
- Apply the strategy: *Solve a Simpler Problem*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- Prime factorization is a way of showing a composite number as the product of prime numbers.
- The greatest common factor (GCF) of two or more numbers is the greatest number that is a factor of each of the numbers.
- The least common multiple (LCM) of two or more numbers is the least nonzero number that is a multiple of each of the numbers.
- The least common denominator (LCD) is the least common multiple of the denominators of two or more fractions.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 167–196

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

## Critical Thinking

Alison and Cynthia are each making a quilt of the same size for their parents' anniversary. Last week Alison completed $\frac{2}{5}$ of her quilt, and Cynthia completed $\frac{2}{3}$ of hers. This week Alison completed $\frac{3}{4}$ of the remaining part of her quilt, and Cynthia completed $\frac{5}{8}$ of the rest of hers. What fraction of each quilt remains to be completed? Who has the larger portion of work remaining?

Algebra

# 6-1

Update your skills. See pages 411 XI, 412 XII.

# Ratio

**Objective** To express a ratio in different forms • To identify equivalent ratios • To express ratios in simplest form

The distance from one base to the next on a softball diamond is 55 feet. The distance from one base to the next on a baseball diamond is 90 feet. What ratio compares the base-to-base distances on a softball diamond with the base-to-base distances on a baseball diamond?

► A **ratio** is a comparison of two quantities, $a$ and $b$, by division, where $b \neq 0$. These quantities are called the **terms** of the ratio.

**Writing Ratios** The order of terms in a ratio is the same order as stated in the comparison.

To compare the distances, use 55 and 90 to write the ratio. You can express the same ratio in different forms.

| **Word Form** $a$ to $b$ | **Ratio Form** $a : b$ | **Fraction Form** $\frac{a}{b}$, $b \neq 0$ |
|---|---|---|
| 55 to 90 | $55 : 90$ | $\frac{55}{90}$ |
| softball distance / baseball distance | | |

So to compare the base-to-base distances on a softball diamond to the base-to base distances on a baseball diamond, write the ratio as 55 to 90, $55 : 90$, or $\frac{55}{90}$.

► Ratios may compare *like* or *unlike* quantities.

## Examples

**1** Compare 7 pairs of yellow sneakers to 12 pairs of green sneakers.

7 to 12 $\quad$ $7 : 12$ $\quad$ $\frac{7}{12}$

**2** Compare 25 calories burned in 3 minutes.

25 to 3 $\quad$ $25 : 3$ $\quad$ $\frac{25}{3}$

► **Equivalent ratios** have the same value. To find equivalent ratios, multiply or divide both terms of the ratio by the same nonzero number.

## Examples

**1** Multiply to form equivalent ratios.

$$\frac{3}{7} = \frac{3 \bullet 2}{7 \bullet 2} = \frac{6}{14}$$

$$\frac{3}{7} = \frac{3 \bullet 3}{7 \bullet 3} = \frac{9}{21}$$

So $\frac{6}{14}$ and $\frac{9}{21}$ are equivalent to $\frac{3}{7}$.

**2** Divide to form equivalent ratios.

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

$$\frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}$$

So $\frac{9}{12}$ and $\frac{6}{8}$ are equivalent to $\frac{18}{24}$.

► A ratio can be simplified by using the greatest common factor (GCF).

**Remember:** The greatest common factor (GCF) of two numbers is the greatest number that is a factor of both numbers.

## Examples

**1** Simplify the ratio 55 : 90.

$55 : 90 = \frac{55}{90}$ ← Write the ratio in fraction form.

$= \frac{55 \div 5}{90 \div 5}$ ← Divide each term by the GCF.

$= \frac{11}{18}$ ← simplest form

So in simplest form, 55 : 90 is 11 : 18.

**2** Simplify the ratio $1\frac{7}{8} : 3\frac{3}{4}$.

$1\frac{7}{8} : 3\frac{3}{4} = \frac{15}{8} \div \frac{15}{4}$ ← Rename each term as a fraction greater than 1.

$= \frac{15}{8} \bullet \frac{4}{15}$ ← Multiply by the reciprocal of the divisor.

$= \frac{60 \div 60}{120 \div 60}$ ← Divide by the GCF.

$= \frac{1}{2}$ ← simplest form

So in simplest form, $1\frac{7}{8} : 3\frac{3}{4}$ is 1 : 2.

**3** Is 1.2 : 1.8 equivalent to 2.4 : 3.6?

$\frac{1.2(10)}{1.8(10)} \stackrel{?}{=} \frac{2.4(10)}{3.6(10)}$ ← Write the ratio in fraction form. Multiply each term by 10 to form integers.

$\frac{12}{18} \stackrel{?}{=} \frac{24}{36}$ ← Simplify both ratios.

$\frac{2}{3} = \frac{2}{3}$ ← True

**Think**
$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$ and $\frac{24 \div 12}{36 \div 12} = \frac{2}{3}$

So 1.2 : 1.8 is equivalent to 2.4 : 3.6.

## Try These

**Express each ratio in simplest form.**

**1.** 36 to 9 **2.** 14 : 16 **3.** 12 : 15 **4.** 625 : 125 **5.** 2.7 : 0.6 **6.** $8 : \frac{4}{5}$

**7.** Write the ratio to compare 11 striped shirts to 8 plaid shirts in three different ways.

**Write three equivalent ratios for each. Explain how you found each ratio.**

**8.** $\frac{4}{9}$ **9.** $\frac{5}{8}$ **10.** $\frac{32}{16}$ **11.** $\frac{0.3}{0.12}$ **12.** $\frac{0.5}{0.25}$ **13.** $\frac{4}{9} : 1\frac{1}{3}$

**14.** Compare the ratios $\frac{1.2}{14}$ and $\frac{1.4}{16}$. Write <, =, or >.

**15.** An animal shelter has 12 cats and 9 dogs. If 1 cat and 1 dog are adopted, would the ratio of cats to dogs remain the same? Explain.

**16.** **Discuss and Write** Explain why $1\frac{3}{7}$ does not represent a ratio and $1\frac{3}{7} : \frac{3}{7}$ does represent a ratio. Then tell how you express $1\frac{3}{7}$ as a ratio.

Go to PRACTICE BOOK Lesson 6-1 for exercise sets.

Algebra

6-2

Update your skills. See pages 412 XIII, 413 XIV.

# Unit Rate and Unit Cost

**Objective** To write rates • To find unit rates • To use unit cost to determine the better or best buy • To compare rates

The price list at the right shows the sale prices for DVDs in three different video stores. Which store has the best buy?

To find the best buy, use the given price at each store to find the *unit rate*, or the cost of one DVD. Then compare the unit costs.

**DVD PRICES**

| Stores | Sale Prices |
|---|---|
| Abe's DVD | 3 DVDs for $27.99 |
| DVD Exchange | 4 DVDs for $33 |
| DVD City | 5 DVDs for $47.50 |

**Key Concept**

Rate is a ratio that compares two *unlike* quantities.

Unit rate is a ratio that is simplified to have a denominator of 1 unit. It compares an amount, $x$, to *one unit:* $\frac{x}{1}$.

Unit cost is the price per unit of an item.

1 Write each rate. Then divide the total cost by the number of units to find the unit cost.

**Abe's DVD**

Rate: $\frac{\text{cost}}{\text{number of}} = \frac{\$27.99}{3}$

$\frac{\$27.99}{3} = \frac{\$9.33}{1}$

Unit rate: $9.33 per DVD

Unit cost: $9.33

**DVD Exchange**

Rate: $\frac{\text{cost}}{\text{number of}} = \frac{\$33}{4}$

$\frac{\$33}{4} = \frac{\$8.25}{1}$

Unit rate: $8.25 per DVD

Unit cost: $8.25

**DVD City**

Rate: $\frac{\text{cost}}{\text{number of}} = \frac{\$47.50}{5}$

$\frac{\$47.50}{5} = \frac{\$9.50}{1}$

Unit rate: $9.50 per DVD

Unit cost: $9.50

2 Compare: $9.33, $8.25, $9.50 ⟶ The least cost for one DVD is $8.25.

So DVD Exchange has the best buy.

## Example

**1** Which is the better buy?

- Write each rate. Divide to find the unit rate and unit cost.

$$\frac{\text{cost}}{\text{number of tickets}} \rightarrow \frac{\$42.67}{7} \approx \frac{\$6.096}{1} \approx \$6.10$$

$$\frac{\text{cost}}{\text{number of tickets}} \rightarrow \frac{\$63.18}{9} = \frac{\$7.02}{1} = \$7.02$$

- Compare unit costs: $6.10, $7.02
  $6.10 < $7.02


So buying a group of 7 tickets is the better buy.

► Rates can be written in two ways.

| | Rate Shown Two Ways | |
|---|---|---|
| Alec counts 225 heartbeats in 3 minutes. | $\frac{\text{beats}}{\text{minutes}}$ $\frac{225}{3}$ | $\frac{\text{minutes}}{\text{beats}}$ $\frac{3}{225}$ |
| Mrs. Jones burns 480 calories in 2 hours. | $\frac{\text{calories}}{\text{hours}}$ $\frac{480}{2}$ | $\frac{\text{hours}}{\text{calories}}$ $\frac{2}{480}$ |
| Gasoline costs $38 for 10 gallons. | $\frac{\text{dollars}}{\text{gallons}}$ $\frac{38}{10}$ | $\frac{\text{gallons}}{\text{dollars}}$ $\frac{10}{38}$ |
| Rhiannon does 25 sit-ups in 30 seconds. | $\frac{\text{sit-ups}}{\text{seconds}}$ $\frac{25}{30}$ | $\frac{\text{seconds}}{\text{sit-ups}}$ $\frac{30}{25}$ |

► To write the ratio of two measures as a fraction without unit names, both units must be the same. Change the larger unit of measure to the smaller unit of measure.

| Ratio with Different Units | Change to Smaller Unit of Measure | Ratio with Same Units |
|---|---|---|
| **4 nickels to 3 dimes**<br>$\frac{\text{4 nickels}}{\text{3 dimes}}$ | **Change dimes to nickels.**<br>Since 1 dime = 2 nickels, multiply the number of dimes by 2.<br>3 • 2 = 6 nickels<br>So 3 dimes = 6 nickels. | **4 nickels to 6 nickels**<br>$\frac{4}{6} = \frac{2}{3}$ ← simplest form |
| **2 gallons to 5 quarts**<br>$\frac{\text{2 gallons}}{\text{5 quarts}}$ | **Change gallons to quarts.**<br>Since 1 gallon = 4 quarts, multiply the number of gallons by 4.<br>2 • 4 = 8 quarts<br>So 2 gallons = 8 quarts. | **8 quarts to 5 quarts**<br>$\frac{8}{5}$ |
| **4 days to 2 weeks**<br>$\frac{\text{4 days}}{\text{2 weeks}}$ | **Change weeks to days.**<br>Since 1 week = 7 days, multiply the number of weeks by 7.<br>2 • 7 = 14 days<br>So 2 weeks = 14 days. | **4 days to 14 days**<br>$\frac{4}{14} = \frac{2}{7}$ ← simplest form |

## Try These

**Express each as a unit rate.**

**1.** 70 min to stock 14 shelves
**2.** 140 mi on 4 gal of gas
**3.** 720 words in 60 lines
**4.** 5 oranges for $1.20
**5.** $6.00 for 12 lb of spaghetti
**6.** 4 qt of soup for $3.80

**Find the better buy.**

**7.** 3 pt for $1.08 or 5 pt for $1.95
**8.** 3 cans for $4 or 4 cans for $5.50

**9.** A 30-ounce jar of salad dressing costs $2.39. A 24-ounce jar costs $2.14, and a 13-ounce jar costs $1.42. To the nearest cent, what is the cost per ounce of each size jar? Which is the best buy?

**10.** **Discuss and Write** Explain how to find the best buy when the price given is not the unit cost.

Go to PRACTICE BOOK Lesson 6-2 for exercise sets.

Algebra

# 6-3 Write and Solve Proportions

**Objective** To write proportions • To use the cross-products rule to determine whether ratios form a proportion • To find the missing term in a proportion

Elena uses 9 gallons of gas when she drives 225 miles on the highway. Latif uses 7 gallons of gas when he drives 175 miles on the highway. Is the average number of miles per gallon of gas the same for Elena's car as for Latif's?

► To find out if the two average rates are the same, write two ratios, and determine if they are equivalent—that is, if they form a *proportion*. A proportion is an *equation* stating that two ratios are equivalent.

Two ways to determine if a proportion is true are to use the Cross-Products Rule to determine if the cross products are equal or to compare simplified ratios.

## Key Concept

**Proportion**

means

$a : b = c : d$

extremes

The middle terms, $b$ and $c$, are called the *means*. The end terms, $a$ and $d$, are called the *extremes*.

**Cross-Products Rule**

Product of Means = Product of Extremes

extremes means

$\frac{a}{b} = \frac{c}{d}$ $(b, d, \neq 0) \rightarrow ad = bc$ $ad$ and $bc$ are cross products.

In a proportion, the order of the labels in both ratios needs to be the same.

**Think**

$\frac{\text{Elena's miles}}{\text{Elena's gallons}} = \frac{\text{Latif's miles}}{\text{Latif's gallons}}$ or $\frac{\text{Elena's miles}}{\text{Latif's miles}} = \frac{\text{Elena's gallons}}{\text{Latif's gallons}}$

**Method 1** Use the Cross-Products Rule

- Write the information as a proportion.

$$\frac{\text{Elena's miles}}{\text{Elena's gallons}} \stackrel{?}{=} \frac{\text{Latif's miles}}{\text{Latif's gallons}}$$

$$\frac{225}{9} \stackrel{?}{=} \frac{175}{7}$$

- Use the Cross-Products Rule.

$$\frac{225}{9} \stackrel{?}{=} \frac{175}{7} \rightarrow 225 \bullet 7 \stackrel{?}{=} 9 \bullet 175$$

$$1575 = 1575$$ True

**Method 2** Compare Simplified Ratios

- Write each ratio as a fraction in simplest form.

$$\frac{\text{Elena's miles}}{\text{Elena's gallons}} \qquad \frac{\cancel{225}^{25}}{\cancel{9}_{1}} = \frac{25}{1}$$

$$\frac{\text{Latif's miles}}{\text{Latif's gallons}} \qquad \frac{\cancel{175}^{25}}{\cancel{7}_{1}} = \frac{25}{1}$$

- Compare ratios.

$$\frac{25}{1} = \frac{25}{1}$$ True

Since the ratios 225 : 9 and 175 : 7 are equivalent, they form a proportion.

The average number of miles per gallon of gas for Elena's car is the same as for Latif's.

## Examples

Use the Cross-Products Rule or simplification to determine if the ratios form a proportion.

**1 Use the Cross-Products Rule:**

$56 : 8 \stackrel{?}{=} 35 : 5$

$\frac{56}{8} \stackrel{?}{=} \frac{35}{5}$ ← Cross multiply.

$56 \bullet 5 \stackrel{?}{=} 8 \bullet 35$

$280 = 280$ True

**2 Compare Simplified Ratios:**

$\frac{14}{28} \stackrel{?}{=} \frac{12}{36}$

$\frac{\cancel{14}^{1}}{\cancel{28}_{2}} = \frac{1}{2}$ $\qquad$ $\frac{\cancel{12}^{1}}{\cancel{36}_{3}} = \frac{1}{3}$

$\frac{1}{2} \neq \frac{1}{3}$ not a proportion

► Here are two ways to find the missing term in a proportion. The missing term in a proportion can be located in any of the four positions.

**Method 1** Use the Cross-Products Rule

**Solve:** $\frac{n}{16} = \frac{18}{32}$

$\frac{n}{16} = \frac{18}{32}$ ← Cross multiply.

$32n = 16 \bullet 18$

$\frac{32n}{32} = \frac{288}{32}$ ← Divide both sides by 32 to isolate $n$.

$n = 9$

**Check:** $\frac{9}{16} \stackrel{?}{=} \frac{18}{32}$ ← Substitute 9 for $n$.

$\frac{9}{16} \stackrel{?}{=} \frac{\cancel{18}^{9}}{\cancel{32}_{16}}$ ← Simplify.

$\frac{9}{16} = \frac{9}{16}$ True

**Method 2** Use Proportional Reasoning and Mental Math

**Solve:** $\frac{n}{16} = \frac{18}{32}$

$n = 9$ ← $9 \bullet 2 = 18$

**Think**
$16 \bullet 2 = 32$, so what number times 2 equals 18?

**Check:** $\frac{9}{16} \stackrel{?}{=} \frac{18}{32}$ ← Substitute 9 for $n$.

$\frac{9}{16} \stackrel{?}{=} \frac{18}{32}$ ← Cross multiply.

$9 \bullet 32 \stackrel{?}{=} 16 \bullet 18$

$288 = 288$ True

## Try These

**Does the pair of ratios form a proportion? Use cross products, simplification, or proportional reasoning.**

1. $\frac{6}{12} \stackrel{?}{=} \frac{10}{18}$
2. $\frac{6}{36} \stackrel{?}{=} \frac{6}{1}$
3. $4\frac{1}{2} : 9 \stackrel{?}{=} \frac{2}{5} : \frac{4}{5}$
4. $1.2 : 1.8 \stackrel{?}{=} 2.4 : 3.6$
5. $3.6 : 4.2 \stackrel{?}{=} 4.5 : 5.6$
6. $1.5 : 2.4 \stackrel{?}{=} 2.7 : 3.6$

**Find the missing term in each proportion. Then check your work to justify your answer.**

7. $\frac{5}{6} = \frac{n}{48}$
8. $\frac{9}{t} = \frac{36}{8}$
9. $\frac{0.9}{3.6} = \frac{1.2}{y}$

10. **Discuss and Write** Explain why you could set up a proportion for the opening problem as $\frac{9}{7} = \frac{225}{175}$, but you could not set it up as $\frac{9}{7} = \frac{175}{225}$.

Go to PRACTICE BOOK Lesson 6-3 for exercise sets.

Algebra

# 6-4 Direct Proportion

**Objective** To understand and apply the concept of direct proportion

Tracy works at home reading and correcting manuscript for a small local newsletter publisher. If she works for 5 hours, she earns \$70. At that rate, how many hours must she work to earn \$630?

The amount Tracy earns depends *directly* on the number of hours she works.

To find how many hours she must work in order to earn \$630, write and solve a *direct proportion*.

► Two quantities have a direct proportion relationship when an increase or decrease in one quantity causes *the same kind of change* in the other quantity.

Let $h$ = the number of hours Tracy must work to earn \$630.

**Method 1** Use *Unlike* Units for Each Ratio

$$\frac{\text{hours worked}}{\text{dollars earned}} = \frac{\text{hours worked}}{\text{dollars earned}}$$

$$\frac{5 \text{ hours}}{\$70} = \frac{h}{\$630} \leftarrow \text{direct proportion}$$

**Solve:** $\frac{5}{70} = \frac{h}{630}$

$\frac{5}{70} = \frac{h}{630}$ ← Use the Cross-Products Rule to solve for the missing term.

$5 \bullet 630 = 70h$ ← Multiply.

$3150 = 70h$

$\frac{\cancel{3150}^{45}}{\cancel{70}_{1}} = \frac{\cancel{70}^{1}h}{\cancel{70}_{1}}$ ← Divide both sides by 70 to isolate $h$.

$45 = h$

**Check:** $\frac{5}{70} \stackrel{?}{=} \frac{45}{630}$ ← Substitute 45 for $h$ in the original equation.

$5 \bullet 630 \stackrel{?}{=} 70 \bullet 45$ ← Cross multiply.

$3150 = 3150$ True

**Method 2** Use *Like* Units for Each Ratio

$$\frac{\text{hours worked}}{\text{hours worked}} = \frac{\text{dollars earned}}{\text{dollars earned}}$$

$$\frac{5 \text{ hours}}{h} = \frac{\$70}{\$630} \leftarrow \text{direct proportion}$$

**Solve:** $\frac{5}{h} = \frac{70}{630}$

$\frac{5}{h} = \frac{70}{630}$ ← Use the Cross-Products Rule to solve for the missing term.

$5 \bullet 630 = 70h$ ← Multiply.

$3150 = 70h$

$\frac{\cancel{3150}^{45}}{\cancel{70}_{1}} = \frac{\cancel{70}^{1}h}{\cancel{70}_{1}}$ ← Divide both sides by 70 to isolate $h$.

$45 = h$

**Check:** $\frac{5}{45} \stackrel{?}{=} \frac{70}{630}$ ← Substitute 45 for $h$ in the original equation.

$5 \bullet 630 \stackrel{?}{=} 45 \bullet 70$ ← Cross multiply.

$3150 = 3150$ True

So Tracy must work 45 hours to earn \$630.

## Example

**1** PROBLEM-SOLVING STRATEGY 7

A mass hanging on a spring causes the spring to stretch. The length of the spring when stretched is in direct proportion to the force applied by the mass. If a 3-kilogram mass stretches a spring so that it is 63 centimeters long, what will be the length of the spring if the force of a 5-kilogram mass is used?

Let $x$ = the length of the spring if a 5-kg mass is used.

**Method 1** Use Like Units

$\frac{\text{length}}{\text{length}} = \frac{\text{mass}}{\text{mass}} \rightarrow \frac{63 \text{ cm}}{x} = \frac{3 \text{ kg}}{5 \text{ kg}}$

**Solve:** $\frac{63}{x} = \frac{3}{5}$ ← Use the Cross-Products Rule.

$63 \bullet 5 = 3x$

$315 = 3x$

$\frac{315}{3} = \frac{3x}{3}$ ← Divide both sides by 3 to isolate $x$.

$x = 105$

**Check:** $\frac{63}{105} \stackrel{?}{=} \frac{3}{5}$ ← Substitute 105 for $x$.

$63 \bullet 5 \stackrel{?}{=} 105 \bullet 3$ ← Cross multiply.

$315 = 315$ True

**Method 2** Use Unlike Units

$\frac{\text{length}}{\text{mass}} = \frac{\text{length}}{\text{mass}} \rightarrow \frac{63 \text{ cm}}{3 \text{ kg}} = \frac{x}{5 \text{ kg}}$

**Solve:** $\frac{63}{3} = \frac{x}{5}$ ← Use the Cross-Products Rule.

$5(63) = 3x$

$315 = 3x$

$\frac{315}{3} = \frac{3x}{3}$ ← Divide both sides by 3 to isolate $x$.

$x = 105$

**Check:** $\frac{63}{3} \stackrel{?}{=} \frac{105}{5}$ ← Substitute 105 for $x$.

$63(5) \stackrel{?}{=} (105)3$ ← Cross multiply.

$315 = 315$ True

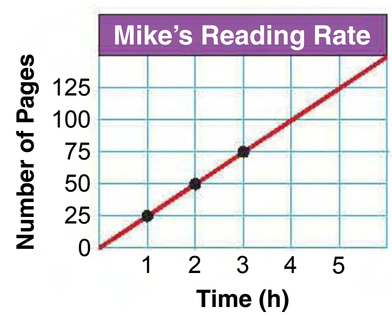
The spring will be 105 cm long if a 5-kg mass is used.

▶ You can represent a direct proportion as a graph.

Write the equivalent ratios. Then plot the coordinates.

| | | |
|---|---|---|
| 25 pages read in 1 hour | 25 : 1 | (1, 25) |
| 50 pages read in 2 hours | 50 : 2 | (2, 50) |
| 75 pages read in 3 hours | 75 : 3 | (3, 75) |

The ratios form a direct proportion: $\frac{25}{1} = \frac{50}{2} = \frac{75}{3}$.




The graph is a straight line slanting upward from left to right.
The number of pages Mike reads and the number of hours are in direct proportion.

## Try These

**Write and solve a proportion to solve the problem.**

1. A sample of paint contains 3 ounces of blue paint and 8 ounces of yellow paint. If you have a 24-ounce can of the blue paint, how much yellow paint should you mix with it in order to make the same color as the sample?

2. **Discuss and Write** Explain how a rate of 20 pages an hour would affect the graph in the example above.

Go to PRACTICE BOOK Lesson 6-4 for exercise sets.

Algebra

# 6-5 Proportion by Parts

**Objective** To model solutions of proportions
• To solve proportions using part-to-whole ratios

Bob and Leo were running for the position of team captain. For every 5 votes Leo received from his teammates, Bob got 3 votes. If each one of the 32 players voted once, how many votes did each boy receive?


To find the number of votes each boy received, use a 5 to 3 ratio.

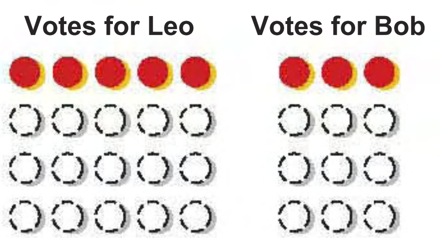
► You can use a model, a table, or proportions to solve the problem.

**Method 1** Use a Model

• Use 32 counters. Make two sets based on the ratio of the parts.

• Add the counters to both sets according to the ratio 5 : 3 until you reach a total of 32 counters.

Ratio: 5 votes for Leo to 3 votes for Bob




| Votes for Leo | + | Votes for Bob | = | Total |
|---|---|---|---|---|
| 20 | + | 12 | = | 32 |

So the model shows that Leo received 20 votes and Bob received 12 votes.

**Method 2** Make a Table

• Use the ratio 5 : 3.

• Multiply each term to find equivalent ratios until you reach a total of 32 votes.

| | Ratio | Equivalent Ratios | | |
|---|---|---|---|---|
| Votes for Leo / Votes for Bob | $\frac{5}{3}$ | $\frac{5 \cdot 2}{3 \cdot 2} = \frac{10}{6}$ | $\frac{5 \cdot 3}{3 \cdot 3} = \frac{15}{9}$ | $\frac{5 \cdot 4}{3 \cdot 4} = \frac{20}{12}$ |
| Total Votes | 8 | 16 | 24 | **32** |

20 votes for Leo + 12 votes for Bob = 32

So the table shows that Leo received 20 votes and Bob received 12 votes.

A proportion can compare parts to parts or parts to the whole quantity.

**Method 3** Use Proportions

- Find the sum of the parts to find the total: $5 + 3 = 8$
- Write a proportion with two part-to-whole ratios for both Leo and Bob.

**Votes for Leo**

Let $l$ = the total number of votes for Leo.

**Solve:** $\frac{5}{8} = \frac{l}{32}$ ← part ← whole

$5 \bullet 32 = 8l$ ← Cross multiply.

$\frac{\overset{20}{\cancel{160}}}{\underset{1}{\cancel{8}}} = \frac{\overset{1}{\cancel{8}}l}{\underset{1}{\cancel{8}}}$ ← Divide both sides by 8 to isolate $l$. Simplify.

$20 = l$

**Check:** $\frac{5}{8} \stackrel{?}{=} \frac{20}{32}$ ← Substitute 20 for $l$.

$5 \bullet 32 \stackrel{?}{=} 8 \bullet 20$

$160 = 160$ True

**Votes for Bob**

Let $b$ = the total number of votes for Bob.

**Solve:** $\frac{3}{8} = \frac{b}{32}$ ← part ← whole

$3 \bullet 32 = 8b$ ← Cross multiply.

$\frac{\overset{12}{\cancel{96}}}{\underset{1}{\cancel{8}}} = \frac{\overset{1}{\cancel{8}}b}{\underset{1}{\cancel{8}}}$ ← Divide both sides by 8 to isolate $b$. Simplify.

$12 = b$

**Check:** $\frac{3}{8} \stackrel{?}{=} \frac{12}{32}$ ← Substitute 12 for $b$.

$3 \bullet 32 \stackrel{?}{=} 8 \bullet 12$

$96 = 96$ True

So the proportions show that Leo received 20 votes and Bob received 12 votes.

## Example

**1** Pat mixes 5 quarts yellow, 2 quarts orange, and 1 quart white paint. If she wants to make 16 quarts of this mixture, how many quarts of each color does she need?

Let $y$ = quarts of yellow needed

$\frac{5}{8} = \frac{y}{16}$ ← part ← whole

$5 \bullet 16 = 8y$

$80 = 8y$

$10 = y$

Let $o$ = quarts of orange needed

$\frac{2}{8} = \frac{o}{16}$ ← part ← whole

$2 \bullet 16 = 8o$

$32 = 8o$

$4 = o$

Let $w$ = quarts of white needed

$\frac{1}{8} = \frac{w}{16}$ ← part ← whole

$1 \bullet 16 = 8w$

$16 = 8w$

$2 = w$

**Check:** Write and solve an equation.

Let $q$ = the number by which each part is multiplied to make the correct 16-quart mixture.

$5q + 2q + q = 16 \rightarrow 8q = 16 \rightarrow q = 2$

white paint = 2
yellow paint = 5(2) = 10
orange paint = 2(2) = 4

So Pat needs 10 quarts of yellow, 4 quarts of orange, and 2 quarts of white paint to make a 16-quart mixture.

## Try These

1. Jack and Li shared the driving during a trip across the country. For every 150 miles that Li drove, Jack drove 200 miles. If the total driving distance was 2800 miles, how far did each person drive?

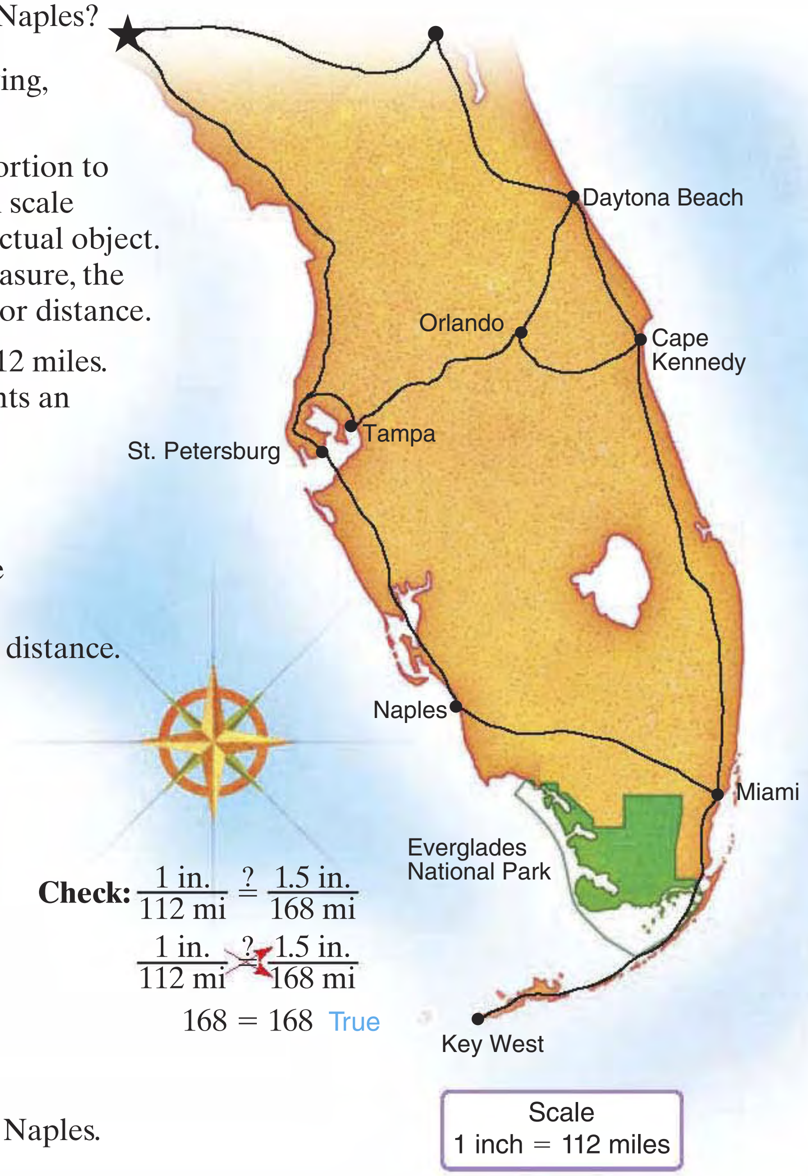
2. **Discuss and Write** Describe the meaning of a part-to-whole ratio.

Go to PRACTICE BOOK Lesson 6-5 for exercise sets.

# Scale Drawings and Models

**Objective** To use proportions to solve scale-drawing and scale-model problems • To use a map scale • To use a scale factor to make a scale-model

About how many miles is St. Petersburg from Naples?




► To find how many miles, use the scale drawing, the map scale, and proportional reasoning.

A two-dimensional drawing that is in proportion to an actual object is called a scale drawing. A scale drawing can be larger or smaller than the actual object. The scale gives the ratio of the *pictured* measure, the distance *on the map*, to the *actual* measure or distance.

The scale for the Florida map is 1 inch = 112 miles. This means that 1 inch on the map represents an actual distance of 112 miles.

To find the actual distance:

- Identify the two cities on the map.
- Use an inch ruler to measure the distance between the two points on the map.
- Choose a variable to represent the actual distance.
- Write and solve a proportion.

The scale distance measures 1.5 inches.

Let $d$ = the actual distance between St. Petersburg and Naples.

**Solve:** $\frac{1 \text{ in.}}{112 \text{ mi}} = \frac{1.5 \text{ in.}}{d \text{ mi}}$ ← scale measure ← actual measure

$\frac{1 \text{ in.}}{112 \text{ mi}} \times \frac{1.5 \text{ in.}}{d \text{ mi}}$ ← Cross multiply.

$d = 1.5 \cdot 112$ ← Simplify.

$d = 168$ mi

**Check:** $\frac{1 \text{ in.}}{112 \text{ mi}} \stackrel{?}{=} \frac{1.5 \text{ in.}}{168 \text{ mi}}$

$\frac{1 \text{ in.}}{112 \text{ mi}} \stackrel{?}{\times} \frac{1.5 \text{ in.}}{168 \text{ mi}}$

$168 = 168$ True

It is about 168 miles from St. Petersburg to Naples.

## Example

**1** What is the actual length of a hallway, $h$, that measures 2.5 cm on a scale drawing? Use the scale at the right.

Scale
1 cm = 3 m

**Solve:** $\frac{1 \text{ cm}}{3 \text{ m}} = \frac{2.5 \text{ cm}}{h \text{ m}}$

$h = 3 \cdot 2.5$

$h = 7.5$

**Check:** $\frac{1 \text{ cm}}{3 \text{ m}} \stackrel{?}{=} \frac{2.5 \text{ cm}}{7.5 \text{ cm}}$

$1 \cdot 7.5 \stackrel{?}{=} 3 \cdot 2.5$

$7.5 = 7.5$ True

The actual length of the hallway is 7.5 meters.

A scale model is a three-dimensional model that accurately represents a real object. A scale written as a ratio in simplest form is called the scale factor. It tells how much larger or smaller the scale model is than the actual object it represents. You can make a *net*, or a two-dimensional pattern to make a scale model of a three-dimensional figure.

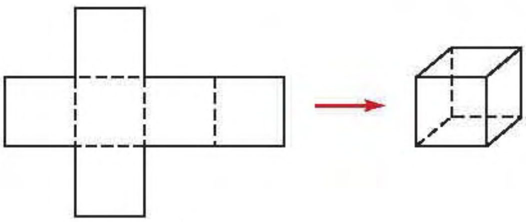
**Key Concept**

**Scale Factor**

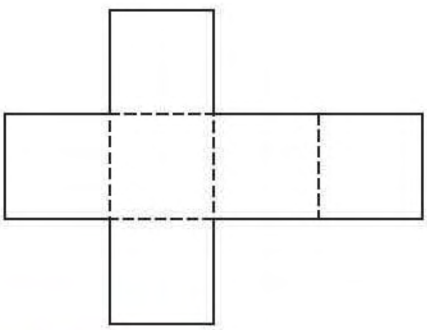
scale factor $= \dfrac{\text{scale model}}{\text{actual object}}$

The terms of a scale factor are always like units.

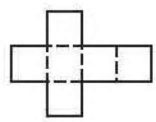
- This net can be folded to form a cube.



- The net below shows an enlargement of the net above using the scale factor $\frac{1.5}{1}$. You can fold it to form a larger scale model of the original cube.



- The net below shows a reduction of the net above using the scale factor $\frac{1}{2}$. You can fold it to form a smaller scale model of the original cube.



## Technology

**Varying the Scale**

This spreadsheet shows that for an actual distance of 30 ft and a scale factor of $\frac{1}{100}$, the scale drawing will be 3.6 inches in length. When you enter new scale factors and dimensions, the spreadsheet automatically calculates new values.

C2 = =(A2*12)/B2

| | A | B | C |
|---|---|---|---|
| 1 | Actual Measure (ft) | Scale Factor | Scale Drawing Measure (in.) |
| 2 | 5 | 60 | 1.0 |
| 3 | 30 | 100 | 3.6 |
| 4 | 100 | 250 | 4.8 |
| 5 | | | |
| 6 | | | |
| 7 | | | |

Sheet 4 Sheet 5 Sheet 6

Multiply the actual measure by 12 to convert feet to inches. Then divide by the scale factor.

## Try These

**1.** A scale drawing of a pentagonal building measures 6 cm on each side. The scale used is 1 cm = 5.5 m. How long is each side of the building?

**2. Discuss and Write** Draw a net on grid paper for a scale model of a cube that is 3 cm on each edge. Use a scale factor of $\frac{2}{1}$. Explain how you can enlarge the model.

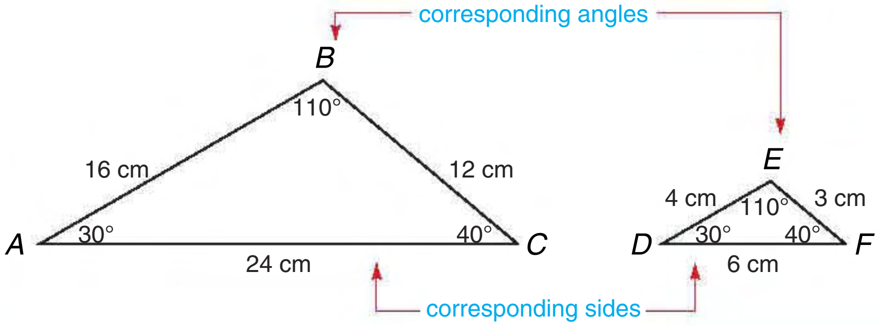
Go to PRACTICE BOOK Lesson 6-6 for exercise sets.

Algebra

# 6-7 Similarity

**Objective** To determine similarity • To name corresponding parts of similar figures • To use proportions to find missing dimensions

Is triangle $ABC$ similar to triangle $DEF$?




► Similar figures have the same shape but may be a different size. If two figures are similar, all the pairs of *corresponding angles* are congruent, and the ratios of the lengths of all the pairs of *corresponding sides* are equal.

Corresponding angles of similar figures are angles that are in the same relative position. Corresponding sides of similar figures are sides that are in the same relative position.

**Remember:** Congruent figures have the same shape and the same size.

► To determine whether the two triangles are similar:

- Compare the measures of corresponding angles.

| | | |
|---|---|---|
| $\angle A$ corresponds to $\angle D$. | $30° = 30°$ | $\angle A \cong \angle D$ |
| $\angle B$ corresponds to $\angle E$. | $110° = 110°$ | $\angle B \cong \angle E$ |
| $\angle C$ corresponds to $\angle F$. | $40° = 40°$ | $\angle C \cong \angle F$ |

**Symbols**

$\cong$ means "is congruent to."
$\overline{AB}$ means "side $AB$."
$AB$ means "the measure of side $AB$."
$\sim$ means "is similar to."

So the corresponding angles of the two triangles are congruent.

- Compare the ratios of the lengths of corresponding sides.

$\overline{AB}$ corresponds to $\overline{DE}$. $\quad \frac{AB}{DE} = \frac{16}{4} = \frac{4}{1}$

$\overline{BC}$ corresponds to $\overline{EF}$. $\quad \frac{BC}{EF} = \frac{12}{3} = \frac{4}{1}$

$\overline{AC}$ corresponds to $\overline{DF}$. $\quad \frac{AC}{DF} = \frac{24}{6} = \frac{4}{1}$

Each pair of corresponding sides has a ratio 4 : 1. So the ratios of the lengths of the corresponding sides are equal.

Triangle $ABC$ is similar to triangle $DEF$ since the corresponding angles are congruent and the ratios of the corresponding sides are equal.

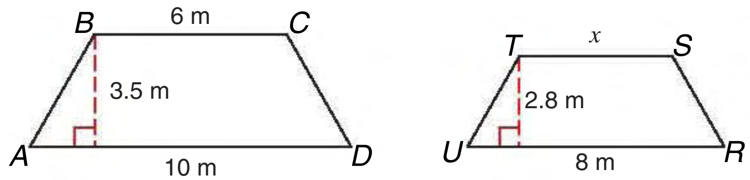
► You can use symbols to show that triangle $ABC$ is similar to triangle $DEF$.

$\triangle ABC \sim \triangle DEF$ ← List the letters in order of the corresponding parts.

**Read as:** triangle $ABC$ is similar to triangle $DEF$.

► In similar figures, the lengths of the corresponding sides are proportional. Therefore, you can use proportions to find missing dimensions of similar figures.

What is the length of $\overline{TS}$, if figure $ABCD$ is similar to figure $UTSR$?




To find the length of $\overline{TS}$:

- Identify pairs of corresponding sides.
  $\overline{AD}$ corresponds to $\overline{UR}$.
  $\overline{BC}$ corresponds to $\overline{TS}$ .
- Write and solve a proportion.
  $\frac{AD}{UR} = \frac{BC}{TS}$
  Let $x$ = the length of side $TS$.

**Solve:** $\frac{10 \text{ m}}{8 \text{ m}} = \frac{6 \text{ m}}{x}$

$10x = 48$ ← Cross multiply.

$10x \div 10 = 48 \div 10$ ← Divide both sides by 10.

$x = 4.8$

**Check:** $\frac{10 \text{ m}}{8 \text{ m}} \stackrel{?}{=} \frac{6 \text{ m}}{4.8 \text{ m}}$

$10(4.8) \stackrel{?}{=} 8(6)$

$4.8 = 4.8$ True

So the length of side $\overline{TS}$ is 4.8 meters.

## Example

**1** $\overline{AC}$ and $\overline{BC}$ of $\triangle BAC$ are equal in length. If $\triangle GFH$ and $\triangle BAC$ are similar, what is the length of $\overline{HF}$?

$\frac{GF}{BA} = \frac{HF}{CA}$

Let $y$ = the length of side $HF$.

**Solve:** $\frac{1.5}{2} = \frac{y}{4}$

$1.5 \bullet 4 = 2y$

$6 \div 2 = 2y \div 2$

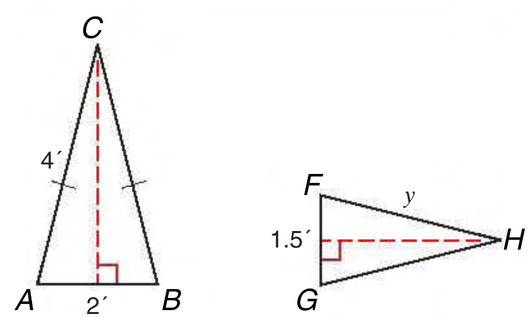
$3 = y$

**Check:** $\frac{1.5}{2} \stackrel{?}{=} \frac{3}{4}$

$1.5(4) \stackrel{?}{=} 2(3)$

$6 = 6$ True

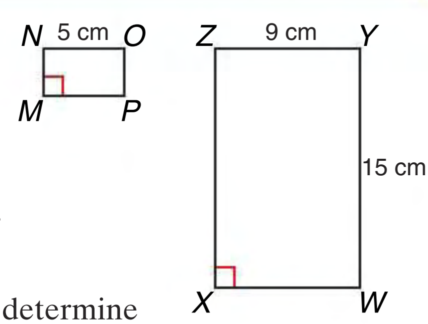
So the length of $\overline{HF}$ is 3 feet.




**Think**
Side $GF$ corresponds to side $BA$.
Side $HF$ corresponds to side $CA$.
Side $GH$ corresponds to side $BC$.

## Try These

1. Figure $MNOP$ is similar to figure $ZYWX$. Find the length of $\overline{NM}$.




2. $\triangle FGH \sim \triangle TUV$. Angle $F$ measures 90°, angle $G$ measures 30°, and angle $H$ measures 60°. What is the measure of angle $U$?

3. **Discuss and Write** How can you use the lengths of the sides to determine whether two figures are similar? Use an example to explain your answer.

Go to PRACTICE BOOK Lesson 6-7 for exercise sets.

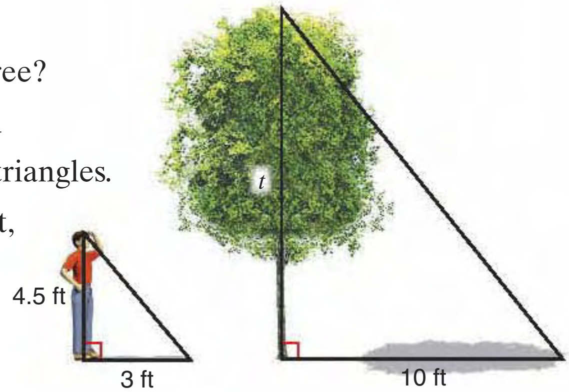
Algebra

# 6-8 Indirect Measurement

Update your skills. See page 410.

**Objective** To solve problems involving indirect measurement by using similar right triangles

A tree casts a 10-ft shadow. At the same time, a 4.5-ft-tall boy standing nearby casts a 3-ft shadow. How tall is the tree?




► To find the height of the tree, use *indirect measurement* to write and solve a proportion involving similar right triangles.

Indirect measurement is used when the distance, height, or length of an object is difficult to measure directly.

$\frac{\text{boy's shadow}}{\text{tree's shadow}} = \frac{\text{boy's height}}{\text{tree's height}}$ ← Set up a proportion.

Let $t$ = the tree's height.

**Solve:** $\frac{3}{10} = \frac{4.5}{t}$

$\frac{3}{10} = \frac{4.5}{t}$ ← Use the Cross-Products Rule.

$3t = 10 \bullet 4.5$ ← Multiply.

$\frac{3t}{3} = \frac{45}{3}$ ← Divide both sides by 3.

$t = 15$

So the tree is 15 ft tall.

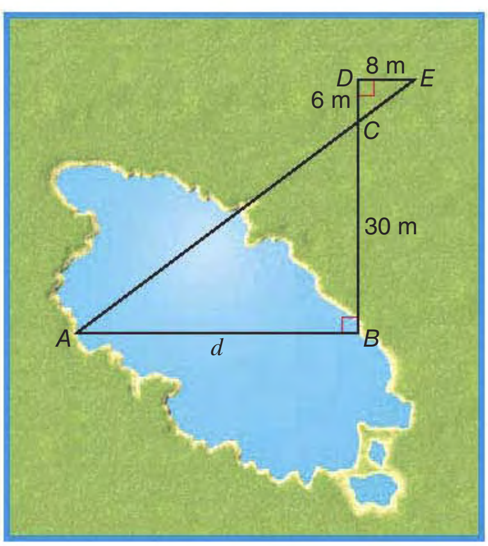
**Check:** $\frac{3}{10} \stackrel{?}{=} \frac{4.5}{15}$ ← Substitute 15 for $t$ in the original equation.

$3(15) \stackrel{?}{=} 10(4.5)$

$45 = 45$ True

## Examples

**1** Triangle $ABC$ and triangle $EDC$ are similar. Find the distance across the lake along $\overline{AB}$.




$\frac{AB}{ED} = \frac{BC}{DC}$ ← Set up a proportion.

Let $d$ = the length of $\overline{AB}$.

**Solve:** $\frac{d}{8} = \frac{30}{6}$ ← Substitute.

$\frac{d}{8} = \frac{30}{6}$ ← Use the Cross-Products Rule.

$6d = 8 \bullet 30$ ← Multiply.

$\frac{6d}{6} = \frac{240}{6}$ ← Divide both sides by 6 to isolate $d$.

$d = 40$

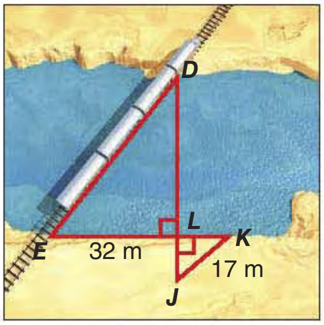
**Check:** $\frac{40}{8} \stackrel{?}{=} \frac{30}{6}$ ← Substitute 40 for $d$ in the original equation.

$40(6) \stackrel{?}{=} 8(30)$

$240 = 240$ True

So the distance across the lake along $\overline{AB}$ is 40 meters.

**2** Triangle $DEL$ and triangle $JKL$ are similar right triangles. What is the distance of the track span across the gorge if $\overline{KL}$ is 8 meters?




$\frac{DE}{JK} = \frac{EL}{KL}$ ← Set up a proportion.

Let $x$ = the length of $\overline{DE}$.

**Solve:** $\frac{x}{17} = \frac{32}{8}$

$8x = 17 \bullet 32$ ← Cross multiply.

$\frac{8x}{8} = \frac{544}{8}$ ← Divide both sides by 8 to isolate $x$.

$x = 68$

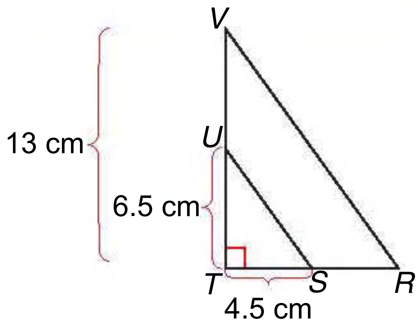
So the length of the track over the gorge is 68 meters.

**Check:** $\frac{68}{17} \stackrel{?}{=} \frac{32}{8}$

$68(8) \stackrel{?}{=} 17(32)$

$544 = 544$

**3** Triangle $RTV$ is similar to triangle $STU$. Side $ST$ has a length of 4.5 cm, side $TU$ has a length of 6.5 cm, and side $TV$ has a length of 13 cm. What is the length of side $RT$?




$\frac{ST}{RT} = \frac{TU}{TV}$ ← Set up a proportion.

Let $x$ = the length of $\overline{RT}$.

**Solve:** $\frac{4.5}{x} = \frac{6.5}{13}$

$4.5 \bullet 13 = 6.5x$ ← Cross multiply.

$\frac{58.5}{6.5} = \frac{6.5x}{6.5}$ ← Divide both sides by 6.5 to isolate $x$.

$9 = x$

So the length of side $RT$ is 9 cm.

**Check:** $\frac{4.5}{9} \stackrel{?}{=} \frac{6.5}{13}$

$4.5(13) \stackrel{?}{=} 9(6.5)$

$58.5 = 58.5$ True

## Try These

**1.** A tower that is 75 feet high casts a shadow. At the same time, a 250-ft-high building next to the tower casts a shadow that is 25 feet long. How long is the shadow cast by the tower?

**2.** A lamppost is 6.5 meters high. Next to it, a 1.2-meter-high mailbox casts a shadow 3.6 meters long. How long is the shadow of the lamppost?

**3.** Similar triangular sails are raised on two sailboats. If the smaller sail is 1.5 m wide and 4.5 m tall and the larger sail is 6 m wide, how tall is the larger sail?

**4.** How long is the shadow cast by a 50.5-meter-tall building if, at the same time, the shadow cast by a 1.5-meter-tall passerby is 3 meters long?

**5.** **Discuss and Write** Give some examples of when you would use indirect measurement.

Go to PRACTICE BOOK Lesson 6-8 for exercise sets.

Algebra

6-9

Update your skills. See pages 414 XVII, 415 XVIII.

# Inverse Proportion

**Objective** To solve inverse proportions

When an increase or decrease in one quantity causes the *opposite* kind of change in the other quantity, an inverse proportion can be formed.

► You can use a table to help visualize the concept of inverse proportions.

A farmer has enough food to feed 2 goats for 32 days. If the farmer doubles the number of goats, what happens to the number of days of available food?

- As the number of goats *increases*, the number of days of available food *decreases*.
- The number of goats changed by a factor of 2.
- The number of days changed by a factor of $\frac{1}{2}$.
- The two quantities, goats and days, changed by reciprocal factors.
- The two quantities are *inversely* proportional.

| Goats ↑ | Days ↓ |
|---|---|
| 2 | 32 |
| 4 | 16 |
| 8 | 8 |
| 16 | 4 |
| 32 | 2 |

If the number of goats doubles, the number of days of available food decreases by a factor of $\frac{1}{2}$. So if the farmer has enough food for 2 goats for 32 days, then doubling the goats to 4 means there is only enough food for 16 days.

## Example

**1** It takes a group of 4 teenagers a half hour to wash 10 cars. How long would it take a group of 8 teenagers to wash 10 cars?

Let $t$ = the time it would take 8 teenagers to wash 10 cars.

- Write the two comparisons in words.

4 teenagers work $\frac{1}{2}$ hour = 8 teenagers work $t$ hours ← Both groups wash 10 cars

- Write a proportion.

$$\frac{\text{more time}}{\text{less time}} = \frac{\text{more teenagers}}{\text{fewer teenagers}} \rightarrow \frac{\frac{1}{2}\text{ hour}}{t} = \frac{8\text{ teenagers}}{4\text{ teenagers}}$$

**Solve:** $\frac{\frac{1}{2}}{t} = \frac{8}{4}$

$\frac{1}{2} \bullet 4 = 8t$ ← Cross multiply.

$2 = 8t$

$\frac{2}{8} = \frac{8t}{8}$ ← Divide both sides by 8 to isolate $t$.

$\frac{1}{4} = t$

**Check:** $\frac{\frac{1}{2}}{\frac{1}{4}} \stackrel{?}{=} \frac{8}{4}$ ← Substitute $\frac{1}{4}$ for $t$.

$\frac{1}{2} \bullet 4 \stackrel{?}{=} \frac{1}{4} \bullet 8$ ← Cross multiply.

$2 = 2$ True

**Think**
It would take 8 teenagers *less time* to wash 10 cars than it would 4 teenagers.

It would take 8 teenagers $\frac{1}{4}$ hour to wash the same 10 cars.

As with direct proportion problems, you can use either like or unlike units for each ratio when solving inverse proportion problems.

Hilda walks 3 miles on her treadmill every morning. At the rate of 3 miles per hour, it takes her an hour to walk that distance on the treadmill. If she increases her rate of speed to 4 miles per hour, how long will it take her to walk the same 3 miles?

**Think.** As the rate of speed *increases*, the amount of time on the treadmill *decreases*.

Let $h$ = the number of hours Hilda will walk at a rate of 4 miles per hour.

**Method 1** Use *Like* Units for Each Ratio

$$\frac{\text{fewer miles per hour}}{\text{more miles per hour}} = \frac{\text{less time on treadmill}}{\text{more time on treadmill}} \rightarrow \frac{3 \text{ mi/h}}{4 \text{ mi/h}} = \frac{h}{1 \text{ hour}}$$

**Solve:** $\frac{3}{4} = \frac{h}{1}$

$\frac{3}{4} \times \frac{h}{1}$ ← Cross multiply.

$3 = 4h$

$3 \div 4 = 4h \div 4$ ← Divide both sides by 4 to isolate $h$.

$h = \frac{3}{4}$

**Check:** $\frac{3}{4} \stackrel{?}{=} \frac{\frac{3}{4}}{1}$ ← Substitute $\frac{3}{4}$ for $h$.

$3 \bullet 1 \stackrel{?}{=} 4 \bullet \frac{3}{4}$ ← Cross multiply.

$3 = 3$ True

**Method 2** Use *Unlike* Units for Each Ratio

$$\frac{\text{fewer miles per hour}}{\text{less time on treadmill}} = \frac{\text{more miles per hour}}{\text{more time on treadmill}} \rightarrow \frac{3 \text{ mi/h}}{h} = \frac{4 \text{ mi/h}}{1 \text{ hour}}$$

**Solve:** $\frac{3}{h} = \frac{4}{1}$

$\frac{3}{h} \times \frac{4}{1}$ ← Cross multiply.

$3 = 4h$

$3 \div 4 = 4h \div 4$ ← Divide both sides by 4 to isolate $h$.

$h = \frac{3}{4}$

**Check:** $\frac{3}{\frac{3}{4}} \stackrel{?}{=} \frac{4}{1}$ ← Substitute $\frac{3}{4}$ for $h$.

$3 \bullet 1 \stackrel{?}{=} \frac{3}{4} \bullet 4$ ← Cross multiply.

$3 = 3$ True

So it will take Hilda $\frac{3}{4}$ hour to walk 3 miles at a rate of 4 miles per hour.

## Try These

**Write and solve an inverse proportion to solve each problem.**

1. If Cora drives at the rate of 50 miles per hour, it takes her $1\frac{1}{2}$ hours to make a 75-mile trip. How long does the trip take if she drives at a rate of 40 miles per hour?

2. **Discuss and Write** Explain why the car-wash problem on page 164 is an inverse proportion. Make a table to support your answer.

Go to PRACTICE BOOK Lesson 6-9 for exercise sets.

# 6-10 Dimensional Analysis

**Objective** To apply dimensional analysis • To use unit ratios to convert currency, time, and Customary Units of length, capacity, and weight

Dan and Lin rode a high speed train that traveled at a rate of 180 miles per hour (mi/h). What was the train's speed in feet per second (ft/s)?

To find the speed in feet per second, convert miles per hour using *dimensional analysis*.

► **Dimensional analysis** is the conversion from one unit system to another.

You are given the units for the rate of speed as the ratio: $\frac{180\text{ mi}}{1\text{ h}}$.

In order to convert systems, you need to know:

- How many feet are in 1 mile?
- How many minutes are in 1 hour?
- How many seconds are in 1 minute?

Use these unit ratios: 1 mi : 5280 ft
1 h : 60 min
1 min : 60 s

These unit ratios are called **conversion factors**, the unit ratios needed to convert the given units.

**Unit Conversions**

| Customary Units of Length |
|---|
| 1 foot (ft) = 12 inches (in.) |
| 1 yard (yd) = 3 ft or 36 in. |
| 1 mile (mi) = 5280 ft or 1760 yd |
| **Customary Units of Capacity** |
| 1 cup (c) = 8 fluid ounces (fl oz) |
| 1 pint (pt) = 2 c |
| 1 quart (qt) = 2 pt |
| 1 gallon (gal) = 4 qt = 128 fl oz |
| **Customary Units of Weight** |
| 1 pound (lb) = 16 ounces (oz) |
| 1 ton (T) = 2000 lb |

**Method 1** Convert by multiplying the given rate by the conversion factors.

$$180\text{ mi/h} = \frac{\overset{3}{\cancel{180}}\ \cancel{\text{mi}}}{1\ \cancel{\text{h}}} \cdot \frac{\overset{88}{\cancel{5280}}\text{ ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{\underset{1}{\cancel{60}}\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{\underset{1}{\cancel{60}}\text{ s}}$$

Cross off units that are the same in the numerator and the denominator.

$$= \frac{264\text{ ft}}{1\text{ s}}$$

$$= 264\text{ ft/s}$$

**Method 2** Convert using separate calculations.

- Convert miles to feet.
  If 1 mi = 5280 ft, then 180 mi = (180)(5280) ft.
  180 mi = 950,400 ft
- Convert hours to seconds.
  If 1 h = 60 min and 1 min = 60 s, then 1 h = (60)(60) s.
  1 h = 3600 s

So $\frac{180\text{ mi}}{1\text{ h}} = \frac{950{,}400\text{ ft}}{3600\text{ s}} = \frac{264\text{ ft}}{1\text{ s}}$.

The high speed train traveled at 264 feet per second.

## Example

**1** How many pounds are in 60 oz?

**Method 1** Divide to Convert

- Identify the unit conversion.
  1 lb = 16 oz
- Divide to find how many 16 ounces are in 60 ounces.

$\frac{60}{16} = 3\frac{12}{16} = 3\frac{3}{4}$

So 60 oz = 3.75 lb or $3\frac{3}{4}$ lb.

**Method 2** Multiply to Convert

- Identify the conversion factor.
  1 lb : 16 oz
- Multiply by the conversion factor.

$$60 \text{ oz} = \frac{\overset{3.75}{\cancel{60}}\ \cancel{\text{oz}}}{1} \bullet \underbrace{\frac{1 \text{ lb}}{\underset{1}{\cancel{16}}\ \cancel{\text{oz}}}}_{\text{conversion factor}}$$

$= 3.75$ lb or $3\frac{3}{4}$ lb

► You can use dimensional analysis to convert between two different currencies.

If \$1 = 0.76 Euro, how many Euros would you get if you were to exchange \$25?

To convert from one currency to another, multiply one currency by the exchange rate, the conversion factor.

Currency exchange rates vary over time.

$$\$25 = \frac{\$25}{1} \bullet \underbrace{\frac{0.76 \text{ Euro}}{\$1}}_{\text{conversion factor}}$$ ← Multiply \$25 by the number of Euros in \$1.

$$= \frac{\cancel{\$}25}{1} \bullet \frac{0.76 \text{ Euro}}{\cancel{\$}1}$$ ← Cross off common units.

$= 19$ Euros

So you would get 19 Euros for \$25.

## Try These

**For exercises 5 and 6, use dimensional analysis to make each conversion. Identify the conversion factor for each.**

**1.** 4 yards to feet **2.** 36 fl oz to cups **3.** 2640 ft to miles **4.** 24 pints to gallons

**Use dimensional analysis to determine the unit price.**

**5.** 64 fl oz for \$2.90; Find the price per gallon.

**6.** 2 tons for \$840; Find the price per pound.

**7.** If \$1 = 0.76 Euro, how many Euros would you get if you were to exchange \$80?

**8.** If \$1 = 0.76 Euro, how many U.S. dollars would you get if you were to exchange 80 Euros?

**9.** **Discuss and Write** Explain how you would use dimensional analysis to convert 1 foot to centimeters using $\frac{2.54 \text{ cm}}{1 \text{ in.}}$ as the conversion factor.

Go to PRACTICE BOOK Lesson 6-10 for exercise sets.

# 6-11 Problem-Solving Strategy: Solve a Simpler Problem

**Objective** To solve problems using the strategy *Solve a Simpler Problem*

**Problem 1:** There are six steps at the entrance to Shirley's school. She likes to walk up the steps in different ways, but she always goes up either one or two steps at a time. In how many different ways can Shirley go up the steps in this manner?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Shirley takes only one or two steps at a time.

**Question:** In how many different ways can Shirley go up the steps?

**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. **Solve a Simpler Problem**
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**
You can use the strategy *Solve a Simpler Problem* and find the solution for Steps 1, 2, and 3.

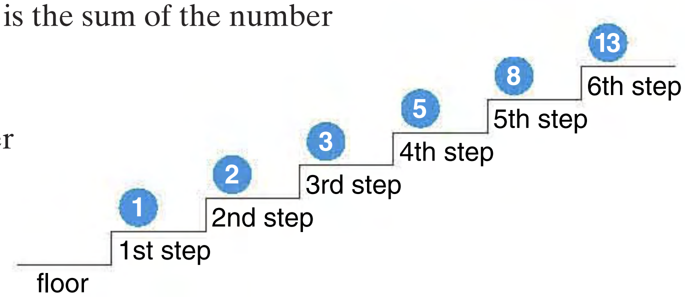
**Solve** **Apply the strategy.**
For the first step, there is only one way Shirley can climb it.

For the second step, there are two possibilities: She could get to the second step from the floor, or she could get to the second step from the first step.

Consider the third step. She could get to the third step from the first step or from the second step. Because there is 1 way to get to the first step and because there are 2 ways to get to the second step, there are $1 + 2$, or 3, ways to get to the third step.

Continue this line of thinking for the subsequent steps. Shirley can arrive at any step only from *two* steps lower or from *one* step lower. So the number of ways to get to a step is the sum of the number of ways to get to the two steps below.

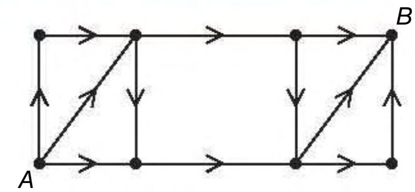
This leads to the diagram at the right. Each circled number shows the number of different ways Shirley can get to that step. In each case, this number is the sum of the numbers on the two steps below.




So Shirley can go up the six steps in 13 different ways.

**Check** **Check to make sure your answer makes sense.**
Here are the 3 ways Shirley could use to get to Step 3: 1,1,1; 1,2; 2,1.
Here are the 5 ways Shirley could use to get to Step 4: 1,1,1,1; 1,1,2; 1,2,1; 2,1,1; 2,2.
These ways to arrive at Steps 3 and 4 form the 8 ways to arrive at Step 5.
The 13 ways to arrive at Step 6 are the 5 ways to get to Step 4 plus the 8 ways to get to Step 5.

**Problem 2:** Suppose an ant is at point $A$ of the figure at the right and can walk only on the lines in the directions indicated by the arrows. In how many different ways can this ant walk from point $A$ to point $B$?




**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** An ant travels from $A$ to $B$. It can travel only on the line segments and in the directions indicated by the arrows when walking between the "nodes" (points where the line segments are joined).

**Question:** In how many different ways can this be done?

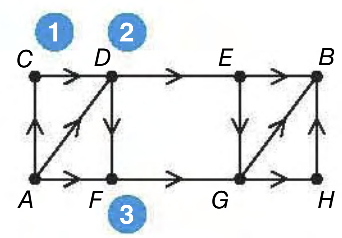
**Plan** **Select a strategy.**

By solving the simpler problem of finding how many ways the ant can get to the closer nodes, you might see how to find the number of ways it can travel to reach the farther nodes.

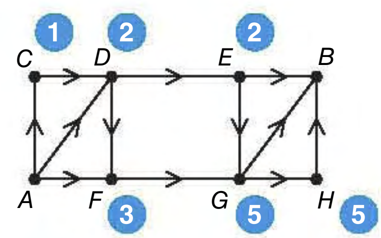
**Solve** **Apply the strategy.**

In the figure below, each node is labeled with a letter.
The circled numbers indicate how many paths lead to each node from A.

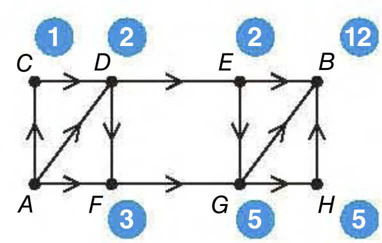
Node $D$ can be reached in one of two ways: from node $C$ or directly from node $A$. Node $F$ can be reached in three ways: directly from node $A$ or from either of the two ways through node $D$.




Continuing in this manner, you see that $E$ can be reached only from $D$ (2 ways), but $G$ can be reached from $F$ (3 ways) or from $E$ (2 ways), for a total of 5 ways. Node $H$ can only be reached from node $G$ (5 ways).




Finally, node $B$ can be reached 12 different ways from any one of the following routes: from node $E$ (2 ways), node $G$ (5 ways), or node $H$ (5 ways).




**Check** **Check to make sure your answer makes sense.**

There are 2 paths to $E$: $ACDE$, $ADE$.
There are 5 paths to $G$: $ACDEG$, $ADEG$, $ACDFG$, $ADFG$, $AFG$.
There are 5 paths to $H$: $ACDEGH$, $ADEGH$, $ACDFGH$, $ADFGH$, $AFGH$.
Finally, the 12 paths to $B$ are precisely these 12 paths, with a $B$ attached to their ends.

Go to PRACTICE BOOK Lesson 6-11 for exercise sets.

# Enrichment: Bicycle-Gear Math

**Objective** **To investigate the mathematics involved in bicycle gears**

An 18-speed bicycle has nine sprockets on its rear wheel and two sprockets attached to the pedals. A *sprocket* is a wheel with teeth on which a chain can move. A *bicycle gear* is a combination of a front sprocket and a rear sprocket, connected by a chain. So an 18-speed bicycle has 2 • 9, or 18, gears because it has 2 front sprockets and 9 rear sprockets.

► The ratio of the number of teeth on the front sprocket to the number of teeth on the rear sprocket is called the *gear ratio*. The gear ratio tells you the number of times the rear sprocket turns each time the front sprocket turns once.

Suppose a bicycle with 28-inch diameter wheels has been put in a gear for which the front sprocket has 38 teeth and the rear sprocket has 19 teeth. Find its gear ratio.

$$\text{gear ratio} = \frac{\text{front teeth}}{\text{rear teeth}} = \frac{38}{19} = 2$$

► If you know the gear ratio and the diameter of the wheels, you can figure out how far a bicycle travels with each pedal stroke.

The circumference of a bicycle wheel with a 28-in. diameter is $28\pi$ in. How far does the bicycle travel with each pedal stroke?

Let $d$ = distance traveled by the bicycle with each pedal stroke

$d$ = gear ratio • circumference = $2 \bullet 28\pi$

$\approx 2 \bullet 28 \bullet 3.14 = 175.84 \approx 176$

**Think**
Because the gear ratio is 2, the wheel turns twice with each pedal stroke.

The bicycle travels about 176 in. with each pedal stroke.

As the gear ratio increases, the amount of work needed to move the bicycle increases. It is harder to pedal the bicycle at the higher gear ratios, but a bicyclist uses a higher gear to be able to travel farther on fewer pedal strokes.

## Try These

**1. Complete the table for this 18-speed bicycle with 27-in. wheels.**

| Front Teeth | Rear Teeth | Gear Ratio | Distance for 1 Pedal Stroke (to nearest inch) |
|---|---|---|---|
| 48 | 12 | | |
| 48 | 16 | | |
| 48 | 18 | | |
| 54 | 12 | | |
| 54 | 16 | | |
| 54 | 18 | | |

**2. Discuss and Write** As the table shows, sometimes two different combinations of sprockets give the same gear ratio. Design a gear system for an 18-speed bicycle that has no duplicate gear ratios. Tell how you choose the numbers you use.

Go to PRACTICE BOOK page 189 for exercise sets.

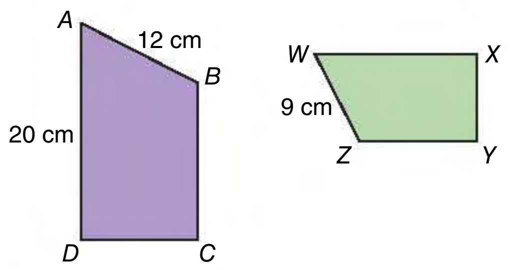
# Test Prep: Extended-Response Questions

## Strategy: Organize Information

For some test questions, you can *use a diagram* to organize your thinking. If a diagram is provided, pay close attention to labels and scales.

**Sample Test Item**

Trapezoid $ABCD$ is similar to trapezoid $WZYX$.




**Part A**
Write a proportion to find the length of $\overline{WX}$.

**Part B**
What is the length of $\overline{WX}$?
***Show all your work.***

**Read the whole test item carefully.**

- Reread the test item. Identify and summarize the information you will need to solve the problem.
- Use the diagram and list what you know.
  1. The trapezoids are similar.
  2. $AB = 12$ cm, $AD = 20$ cm, and $WZ = 9$ cm

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply an appropriate strategy.

  Use the information in the drawing.
  To solve **Part A**, determine the corresponding sides to use in order to write a proportion.
  $\overline{AB}$ corresponds to $\overline{WZ}$
  $\overline{AD}$ corresponds to $\overline{WX}$
  $$\frac{AB}{WZ} = \frac{AD}{WX}$$
  Substitute the lengths shown in the diagram.
  Let $x = WX$
  **Answer:** $\frac{12}{9} = \frac{20}{x}$

To answer **Part B**, solve the proportion.

$$\frac{12}{9} = \frac{20}{x}$$
$12x = 9 \bullet 20$ ← Cross multiply.
$12x = 180$ ← Divide both sides by 12 to isolate $x$.
$x = 15$ ← Simplify.

**Answer:** $\overline{WX}$ is 15 cm long.

**Item Analysis**

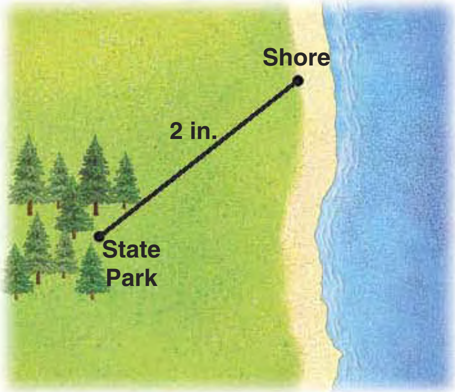
**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Do they make sense?
  Write another proportion that relates the side lengths: $\frac{AB}{AD} = \frac{WZ}{WX}$
  $$\frac{12}{20} = \frac{9}{x}$$
  $12x = 20 \bullet 9$
  $x = 15$ ✓

**Try These** Item 1 is partially worked out for you.

**Solve. Use a diagram.**

1. A bird flies from the state park to a point on the shore. Use the map at the right to find how many miles the bird flies. Explain how to use the scale of the map to solve the problem. ***Show all your work.***




Scale
$\frac{1}{4}$ inch = 75 miles

**Read the test item for a general idea of the problem.**

- Reread the test item carefully. Identify and summarize the information you will need to solve the problem.
- Use the information in the diagram.
  1. The map is a scale drawing that shows the distance the bird flies.
  2. The scale of the map relates lengths on the map to actual distances.

**Solve the problem.**

- Apply an appropriate strategy.
  Use the scale to find the actual distance the bird flies.
  To help explain how to use the scale of the map, think about how the scale relates the map length to the actual distance.

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

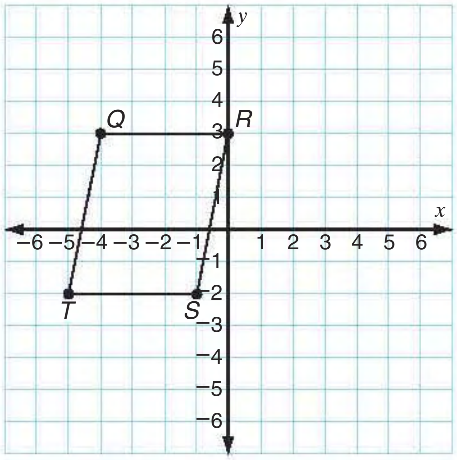
**Think**
The distances on the map are proportional to the actual distances.

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Do they make sense?
  Work backward. Use the actual distance to find the distance on the map.

2. Use parallelogram $QRST$ shown on the coordinate grid below.




Scale: 1 unit = 8 feet

**Part A**
What is the actual length of side $QR$? Explain how you found your answer.

**Part B**
If the parallelogram is enlarged so that the actual length of side $ST$ is now 40 feet, how many units long is it on the coordinate grid? ***Show all your work.***

Go to PRACTICE BOOK pages 191–192 for exercise sets.

# Percent and Consumer Applications

CHAPTER 7

## In This Chapter You Will:

- Write fractions, decimals, and ratios as percents
- Find a percentage, a percent, and an original number (or base)
- Write fractions and decimals greater than 1 or less than 0.01 as percents
- Find a percent increase or percent decrease
- Calculate sales tax, tips, discounts, markups, profit, loss, sale prices, and commissions
- Use a formula to find simple interest
- Use a table to find compound interest
- Apply the strategy: *Reason Logically*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- A ratio is a comparison of two numbers, or like quantities, by division.
- A rate is a ratio that compares two unlike quantities, such as miles per hour or dollars per week.
- A proportion is an equation stating that two ratios are equivalent.
- The Cross-Products Rule states that, in a proportion, the product of the means equals the product of the extremes. If $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$ and $d \neq 0$, then $ad = bc$.

**For Practice Exercises:**

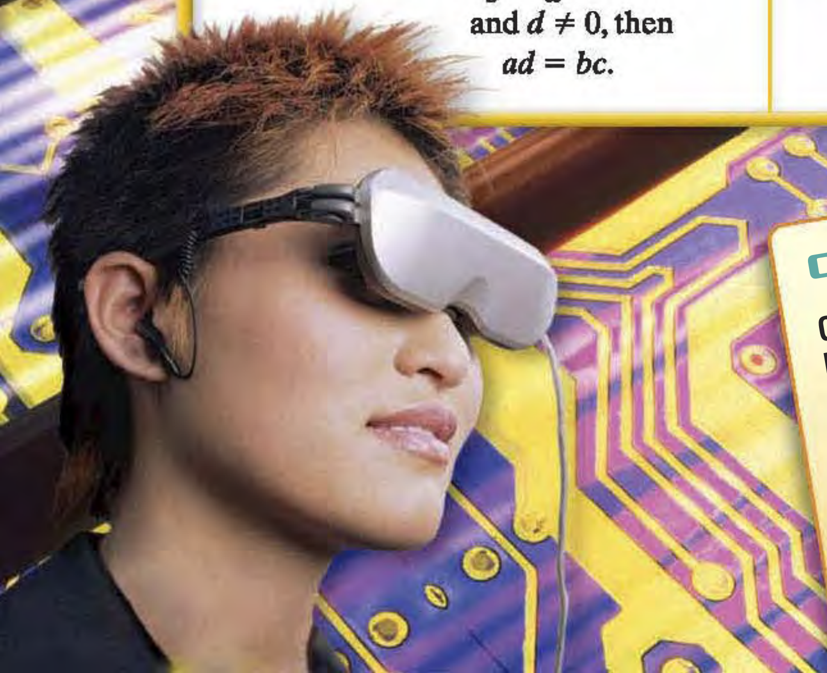
Go to PRACTICE BOOK, pp. 197–234

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES



## Critical Thinking

Game Connection was selling virtual reality headgear for $\frac{1}{5}$ off the usual price of \$1500. Murphy's Electronics was selling the same headgear for 0.75 of their usual price of \$1600. Yukiko bought the headgear at Game Connection. Her friend Malik bought the headgear at Murphy's Electronics. Who spent more money on the headgear, Yukiko or Malik?

Update your skills. See page 410.

# Percents

**Objective** To model percents • To write percents as equivalent ratios and to write ratios as equivalent percents

Some turtles lay their eggs on beaches and bury them in the sand to protect them from predators. A nest of turtle eggs may contain as many as 100 eggs. If 48 out of 100 turtle eggs in a nest produce hatchlings, what percent of the eggs in the nest produce hatchlings?

► To find the percent of the eggs in the nest that produce hatchlings, express the ratio 48 : 100 as a percent.

**Key Concept**

**Percent**

A percent (%) is a ratio or comparison of a quantity to 100.

$\frac{\text{part}}{\text{whole}} = \frac{n}{100} = n\%$

$n$ per hundred or $n$ to 100

part whole

$48$ out of $100 = 48 : 100$

$= \frac{48}{100}$ ← Write as a fraction with a denominator of 100.

$= 48\%$ ← Write as a percent.

So 48% of the turtle eggs produce hatchlings.

► When the second term of a ratio is 10 or 1000, you can use mental math to find an equivalent fraction with a denominator of 100 and then rename that fraction as a percent.

**Remember:** $\frac{n}{100} = n\%$

## Examples

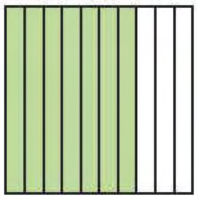
Write each as a percent.

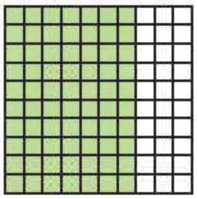
**1** $\frac{7}{10}$

$\frac{7}{10} = \frac{7 \bullet 10}{10 \bullet 10} = \frac{70}{100}$

**Think** $10 \bullet 10 = 100$

So $\frac{7}{10} = 70\%$.





**2** $\frac{70}{1000}$

$\frac{70}{1000} = \frac{70 \div 10}{1000 \div 10} = \frac{7}{100}$

**Think** $1000 \div 10 = 100$

So $\frac{70}{1000} = 7\%$.

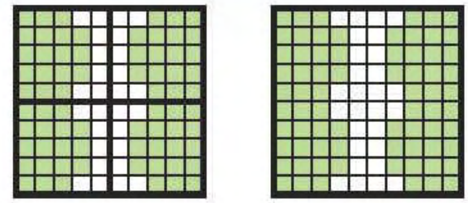
**3** $\frac{87}{1000}$

$\frac{87}{1000} = \frac{87 \div 10}{1000 \div 10} = \frac{8.7}{100}$

So $\frac{87}{1000} = 8.7\%$.

► When the second term of a ratio is a factor of 100, multiply both terms to rename the ratio as a percent.

Serena took a mathematics test that had 25 questions. She had 18 correct answers. What percent of Serena's answers were correct?



$\frac{18 \bullet 4}{25 \bullet 4} \rightarrow \frac{72}{100}$

part : whole $= \frac{\text{number of correct answers}}{\text{total number of answers}} = \%$

$18 : 25 = \frac{18}{25} = \frac{18 \bullet 4}{25 \bullet 4} = \frac{72}{100} = 72\%$

So 72% of Serena's answers were correct.

**Think**
25 • 4 = 100, so multiply both terms by 4.

## Examples

**1** Rename $\frac{3}{20}$ as a percent.

$\frac{3}{20} = \frac{3 \bullet 5}{20 \bullet 5} = \frac{15}{100} = 15\%$

So $\frac{3}{20} = 15\%$.

**2** Rename $\frac{12}{50}$ as a percent.

$\frac{12}{50} = \frac{12 \bullet 2}{50 \bullet 2} = \frac{24}{100} = 24\%$

So $\frac{12}{50} = 24\%$.

► You can rename a percent as a ratio.

## Examples

Write each percent as a ratio in simplest form.

**1** 65%

$65\% = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

**Think**
The GCF of 65 and 100 is 5.

**2** 8%

$8\% = \frac{8}{100} = \frac{8 \div 4}{100 \div 4} = \frac{2}{25}$

**Think**
The GCF of 8 and 100 is 4.

**3** 14.2%

$14.2\% = \frac{14.2}{100} = \frac{142}{1000} = \frac{71}{500}$

**Think**
The GCF of 142 and 1000 is 2.

## Try These

**Rename each ratio as a percent.**

**1.** $\frac{50}{100}$ **2.** 3 : 10 **3.** 60 to 1000 **4.** $\frac{9}{20}$ **5.** $\frac{4}{5}$ **6.** $\frac{7}{50}$

**Rename each percent as a ratio in simplest form.**

**7.** 25% **8.** 35% **9.** 60% **10.** 84% **11.** 4% **12.** 13.5%

**13.** Jeff won 6 out of the 20 games he played with his sister. What percent of the games played did Jeff win?

**14. Discuss and Write** Explain why $\frac{1}{10}$ and $\frac{10}{100}$ both represent 10%. Use models to justify your answer.

Go to PRACTICE BOOK Lesson 7-1 for exercise sets.

# 7-2 Fractions, Decimals, Percents

**Objective** To write percents as fractions and decimals • To write fractions and decimals as percents • To compare fractions, decimals, and percents

In Nina's collection of half dollars, 25% are Walking Liberty, 30% are Benjamin Franklin, and the rest are Kennedy half dollars. What fraction and what decimal represent the percent that are Kennedy half dollars?


► To answer the question, first add the given percents (the parts), and then subtract their sum from 100% (the whole).

$25\% + 30\% = 55\%$ and $100\% - 55\% = 45\%$

So rename 45% as a fraction and as a decimal.

► You can rename a percent as a fraction or as a decimal.

To rename a percent as a fraction:

- Write the percent as a fraction with a denominator of 100.

$$45\% = \frac{45}{100}$$

- Divide both terms by their GCF.

$$\frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$$

To rename a percent as a decimal:

- Remove the percent symbol. To divide by 100, move the decimal point *two* places to the *left*.

$$45\% = 0.45 = 0.45$$

So $\frac{9}{20}$ and 0.45 represent the percent of Nina's collection that are Kennedy half dollars.

## Examples

**1** Rename $12\frac{1}{2}\%$ as a fraction.

$$12\frac{1}{2}\% = \frac{12.5}{100} \longrightarrow \frac{12.5 \bullet 10}{100 \bullet 10} = \frac{125}{1000}$$

$$= \frac{125}{1000} \longrightarrow \frac{125 \div 125}{1000 \div 125} = \frac{1}{8}$$

So $12\frac{1}{2}\% = \frac{1}{8}$.

**2** Rename 14.5% as a decimal.

$14.5\% \longrightarrow 0.145$

So $14.5\% = 0.145$

► You can rename a decimal as a percent.

- Move the decimal point *two* places to the *right* to multiply by 100, and write the % sign.

*or*

- Write the decimal as a fraction that has a denominator of 100. Then write the numerator with a % sign.

Rename 0.2 as a percent.

$0.2 \longrightarrow 0.20 = 20\%$

Write zeros as needed.

*or*

$$0.2 = \frac{2}{10} = \frac{2 \bullet 10}{10 \bullet 10} = \frac{20}{100} = 20\%$$

Write 0.5638 as a percent. Round to the nearest tenth of a percent.

$0.5638 = 0.5638\%$ ← Multiply by 100.

$\approx 56.4\%$ ← Round.

**Think**

$$\frac{5638}{10{,}000} = \frac{56.38}{100} = 56.38\%$$

► You can rename a fraction as a percent.

Rename $\frac{3}{8}$ as a percent.

**Method 1** Write as a Decimal
- Divide the numerator by the denominator.
- Write the decimal as a percent. Move the decimal point *two* places to the *right*.
- Write the percent symbol.

$\frac{3}{8} = 3 \div 8 = 0.375 \longrightarrow 0.375$

$= 37.5\%$ or $37\frac{1}{2}\%$

**Method 2** Use a Proportion
- Write $x : 100$ as one ratio and the fraction as the other.
- Solve the proportion.
- Write the percent symbol.

$\frac{x}{100} = \frac{3}{8}$ ← part / ← whole

$8x = 300$ ← Cross multiply.

$\frac{8x}{8} = \frac{300}{8}$ ← Divide both sides by 8.

$x = 37.5$ or $37\frac{1}{2} \longrightarrow \frac{3}{8} = 37.5\%$ or $37\frac{1}{2}\%$

## Examples

**1** Write $\frac{1}{3}$ as a percent by using a proportion.

$\frac{n}{100} = \frac{1}{3}$ ← part / ← whole

$3n = 100$ ← Cross multiply.

$\frac{3n}{3} = \frac{100}{3}$ ← Divide both sides by 3.

$n = 33.\overline{3}$

So $\frac{1}{3} = 33.\overline{3}\%$ or $33\frac{1}{3}\%$.

**2** Write $\frac{7}{8}$ as a percent by writing it as a decimal.

$\frac{7}{8} = 7 \div 8$ ← Divide the numerator by the denominator.

$= 0.875 \longrightarrow 0.875$ ← To multiply by 100, move the decimal point 2 places to the right.

$= 87.5\%$ or $87\frac{1}{2}\%$

► To compare fractions, decimals, and percents, first rename them to be in like form.

Compare: $\frac{9}{10}$ __?__ 90% $\longrightarrow \frac{9}{10} = \frac{90}{100}$ and $90\% = \frac{90}{100}$, so $\frac{9}{10} = 90\%$

Compare: $\frac{3}{8}$ __?__ 40% $\longrightarrow \frac{3}{8} = 37\frac{1}{2}\%$ and $37\frac{1}{2}\% < 40\%$, so $\frac{3}{8} < 40\%$

Compare: $\frac{0.04}{0.16}$ __?__ 25% $\longrightarrow 0.04 \div 0.16 = 0.25 = \frac{25}{100} = 25\%$, so $\frac{0.04}{0.16} = 25\%$

## Try These

**Write each fraction or decimal as a percent.**

**1.** $\frac{4}{10}$  **2.** 0.38  **3.** $\frac{5}{8}$  **4.** 0.925

**Write each percent as a decimal and as a fraction in simplest form.**

**5.** 70%  **6.** 35.5%  **7.** 68.67%  **8.** $18\frac{3}{4}\%$

**Compare. Write <, =, or >.**

**9.** $\frac{4}{10}$ __?__ 40%  **10.** $\frac{5}{8}$ __?__ 60%  **11.** $\frac{2}{5}$ __?__ 35%  **12.** $\frac{0.03}{0.12}$ __?__ 20%  **13.** $\frac{0.05}{0.25}$ __?__ 25%

**14. Discuss and Write** Explain how you would order 10.2%, 0.1, and $\frac{1}{5}$ from least to greatest.

Go to PRACTICE BOOK Lesson 7-2 for exercise sets.

# 7-3 Percents Greater Than 100%/ Less Than 1%

**Objective** To write fractions and decimals greater than 1 and less than 1 hundredth as percents • To write percents greater than 100% and less than 1% as fractions and decimals

If the attendance at today's game is 1.5 times greater than expected, what percent of the expected attendance is the actual attendance?

To find the percent, rename the decimal 1.5 as a percent.

▶ You can rename decimals greater than 1 as percents.

To multiply by 100, move the decimal point *two* places to the *right*. Write the percent symbol.

$1.5 \rightarrow 1.50 = 150\%$

The decimal 1.5 is equivalent to 150%, so the actual attendance is 150% of the expected attendance.

**Key Concept**

**Percents <, =, or > 100%**

$100\% = \frac{100}{100} = 1,$

so a percent greater than 100% is greater than 1, and a percent less than 100% is less than 1.

▶ You can rename fractions greater than 1 as percents.

Rename $\frac{5}{4}$ as a percent.

**Method 1** Write as an Equivalent Decimal

- Divide by the denominator to write the equivalent decimal.
- Multiply by 100 to write as a percent.

$\frac{5}{4} = 5 \div 4 = 1.25 \rightarrow 1.25 = 125\%$

**Method 2** Write a Proportion

$\frac{5}{4} = \frac{n}{100}$

$500 = 4n$ ← Cross multiply.

$\frac{500}{4} = \frac{4n}{4}$ ← Divide both sides by 4.

$125 = n \rightarrow 125\%$

▶ You can rename percents greater than 100% as decimals or fractions.

Rename 225% as a decimal.

To divide by 100, move the decimal point *two* places to the *left*. Remove the % sign.

$225\% = 2.25$

So $225\% = 2.25$.

Rename 145% as a mixed number.

Write the percent as a fraction with a denominator of 100. Simplify.

$145\% = \frac{145}{100} = \frac{145 \div 5}{100 \div 5} \quad \frac{29}{20}$

So $145\% = \frac{29}{20}$, or $1\frac{9}{20}$.

▶ You can rename some percents as mixed numbers by breaking them into parts.

$116\frac{2}{3}\% \rightarrow 100\% + 16\frac{2}{3}\% = 1 + \frac{50}{3}\%$ ← Rename $16\frac{2}{3}$ as a fraction greater than 1.

$= 1 + \frac{50}{3} \bullet \frac{1}{100}$ ← To rename $\frac{50}{3}\%$ as a fraction, divide by 100. This is the same as multiplying by $\frac{1}{100}$.

$= 1 + \frac{1}{6} = 1\frac{1}{6}$

So $116\frac{2}{3}\% = 1\frac{1}{6}$.

► You can rename decimals less than 0.01 as percents.

Rename 0.007 as a percent.

**Method 1** Multiply by 100

- To multiply by 100, move the decimal point *two* places to the *right*.
- Write the percent symbol.

$0.007 \rightarrow 0.007 = 0.7\%$

So 0.007 is equivalent to 0.7%.

**Method 2** Rename as a Fraction

$0.007 = \frac{7}{1000} = \frac{0.7}{100} = 0.7\%$

**Key Concept**

**Percents < 1%**

$1\% = \frac{1}{100} = 0.01$,

so decimals less than 0.01 and fractions less than $\frac{1}{100}$ are less than 1%.

► You can rename fractions less than $\frac{1}{100}$ as percents.

Rename $\frac{1}{200}$ as a percent.

**Method 1** Rename as a Decimal

- Divide by the denominator to write the equivalent decimal.
- Write the decimal as a percent.

$\frac{1}{200} = 1 \div 200 = 0.005 \rightarrow 0.005 = 0.5\%$

**Method 2** Write a Proportion

$\frac{n}{100} = \frac{1}{200}$

$200n = 100$ ← Cross multiply.

$\frac{200n}{200} = \frac{100}{200}$ ← Divide both sides by 200.

$n = 0.5$, so $\frac{n}{100} = \frac{0.5}{100} = 0.5\%$

► You can rename percents less than 1 as decimals or fractions.

Rename 0.85% as a decimal.

- To divide by 100, move the decimal point *two* places to the *left*.
- Remove the percent symbol.

$0.85\% = 0.0085$

So $0.85\% = 0.0085$.

Rename 0.2% as a fraction.

- Write as a fraction with a denominator of 100.
- Rewrite as an equivalent fraction with a whole-number numerator.

$0.2\% = \frac{0.2}{100} = \frac{0.2 \bullet 10}{100 \bullet 10} = \frac{2}{1000} = \frac{1}{500}$

So $0.2\% = \frac{2}{1000} = \frac{1}{500}$.

## Try These

**Write each fraction or decimal as a percent.**

**1.** $\frac{4}{1}$ **2.** 0.003 **3.** $\frac{8}{5}$ **4.** 0.0092 **5.** $\frac{1}{400}$

**Write each percent as a decimal and as a fraction in simplest form.**

**6.** 750% **7.** 0.25% **8.** 185% **9.** 0.004% **10.** 150.5%

**11. Discuss and Write** What percent is equal to the fraction $\frac{3}{2}$? What percent is equal to the fraction $\frac{1}{500}$? Explain.

Go to PRACTICE BOOK Lesson 7-3 for exercise sets.

# 7-4

Update your skills. See page 412 XIII.

# Find a Percentage of a Number

**Objective** To use the percent formula to find a percentage of a number • To use a percent proportion to find a percentage of a number

A school poll showed that 15% of the 180 students who responded wanted to lengthen the school year. How many students wanted a longer school year?


**Key Concept**

**Percent Formula**

$r \bullet b = p$

rate ($r$) • base ($b$) = percentage ($p$)

rate → percent; base → number (whole); percentage → number (part)

To find the number of students, find 15% of 180.

► Here are two ways to find a *percentage* of a number:

**Method 1** Use the Percent Formula
You can write the percent as a decimal or a fraction.

• Write the percent as a *decimal*, then multiply.

$r \bullet b = p$

$15\% \bullet 180 = p$ ← Substitute for $r$ and $b$.

$0.15 \bullet 180 = p$ ← Simplify.

$27 = p$

• Write the percent as a *fraction*, then multiply.

$r \bullet b = p$

$15\% \bullet 180 = p$

**Think** $15\% = \frac{15}{100} = \frac{3}{20}$

$\frac{3}{\cancel{20}_1} \bullet \frac{\cancel{180}^9}{1} = p$ ← Simplify. Use the GCF.

$27 = p$

**Method 2** Use the Percent Proportion

**Solve:** $\frac{p}{180} = \frac{15}{100}$ ← Substitute 180 for the base and 15 for the percent.

$100p = 180 \bullet 15$ ← Cross multiply.

$\frac{100p}{100} = \frac{2700}{100}$ ← Simplify. Divide both sides by 100 to isolate $p$.

$p = 27$

**Check:** $\frac{27}{180} \stackrel{?}{=} \frac{15}{100}$ ← Substitute 27 for $p$. Cross multiply.

$2700 = 2700$ True

**Key Concept**

**Percent Proportion**

In a percent proportion, one number, the part (called the percentage), is being compared to the whole quantity (called the base). The other ratio is the percent written as a fraction, where the whole is 100.

$$\frac{\text{part} \rightarrow \text{percentage } (p)}{\text{whole} \rightarrow \text{base } (b)} = \frac{\text{percent } (\%)}{100}$$

So 15% of 180 is 27. This means that 27 students are in favor of a longer school year.

## Examples

**1** Find $37\frac{1}{2}\%$ of 80 using the percent formula.

$r \bullet b = p$

$37\frac{1}{2}\% \bullet 80 = p$ ← Substitute for $r$ and $b$.

$0.375 \bullet 80 = p$ ← Write $37\frac{1}{2}\%$ as a decimal.

$30 = p$ ← Multiply.

So $37\frac{1}{2}\%$ of $80 = 30$.

**2** Find 15.2% of 81 using a percent proportion.

$\frac{p}{81} = \frac{15.2}{100}$ ← Write the proportion.

$100p = 81 \bullet 15.2$ ← Cross multiply.

$100p = 1231.2$ ← Simplify. Divide both sides by 100 to isolate $p$.

$p = 12.312$

So 15.2% of $81 = 12.312$.

► You can use the percent formula or a percent proportion to solve measurement problems.

## Examples

**1** How many degrees are in a section of a circle that is 24% of the whole circle?

Find 24% of 360°.

**Hint** A circle measures 360°.

$r \bullet b = p$

$24\% \text{ of } 360 = 0.24 \bullet 360$

$= 86.4$ ← Simplify.

The section of the circle measures 86.4°.

**2** The mass of Earth expressed in scientific notation is $5.974 \times 10^{24}$ kg. What is 50% of Earth's mass?

$r \bullet b = p$

$\frac{1}{2} \times 5.974 \times 10^{24} = p$ ← $50\% = \frac{1}{2}$, so substitute $\frac{1}{2}$ for $r$.

$\frac{5.974}{2} \times 10^{24} = p$ ← Divide like terms.

$2.987 \times 10^{24} = p$ ← Write in scientific notation.

So 50% of Earth's mass is $2.987 \times 10^{24}$ kg.

## Try These

**Use the percent formula to find each percentage.**
**Rename the percent as a decimal or fraction.**

**1.** 40% of 45 **2.** 3% of 80 **3.** 111% of 32 **4.** 0.6% of 30

**5.** Erica took a 19-hour plane ride. She slept for about 30% of the time. For about how long was she asleep?

**Use a percent proportion to find each percentage.**

**6.** 60% of 40 **7.** 115% of 90 **8.** 8% of 260 **9.** 0.4% of 50

**10.** Lake Superior is 350 miles in length. Avery kayaked 35% of the distance. How many miles did Avery travel in his kayak?

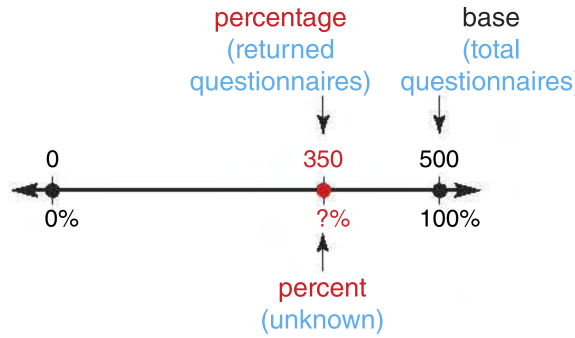
**11.** **Discuss and Write** When finding a percentage of a number, when might you prefer to express the percent as a fraction rather than as a decimal? Provide an example to support your reasoning.

Go to PRACTICE BOOK Lesson 7-4 for exercise sets.

# 7-5 Find a Percent

**Objective** To find what percent one number is of another using the percent formula or a percent proportion

A research group sent out questionnaires to determine how many hours teenagers typically spend each week playing electronic games. Of the 500 questionnaires sent out, 350 were answered and returned. What percent of the questionnaires sent out were answered and returned?




To find the percent, find what percent 350 is of 500.

► Here are two ways to find the percent one number is of another:

**Method 1** Use the Percent Formula

$r \bullet b = p$

$r \bullet 500 = 350$ ← Substitute 350 for the percentage, $p$, and 500 for the base, $b$.

$\frac{r \bullet 500}{500} = \frac{350}{500}$ ← Divide both sides by 500 to isolate $r$.

$\frac{r \bullet \cancel{500}}{\cancel{500}} = \frac{\cancel{350}^{70}}{\cancel{500}_{100}}$ ← Simplify.

$r = \frac{70}{100} = 70\%$

> **Remember:**
> **Percent Formula**
> $r \bullet b = p$
> rate ($r$) • base ($b$) = percentage ($p$)

**Method 2** Use a Percent Proportion

Let $n$ represent the unknown percent.

**Solve:** $\frac{350}{500} = \frac{n}{100}$ ← Substitute 350 for the percentage, $p$, and 500 for the base, $b$.

$350 \bullet 100 = 500n$ ← Cross multiply.

$\frac{350 \bullet 100}{500} = \frac{500n}{500}$ ← Divide both sides by 500 to isolate $n$.

$\frac{\cancel{350}^{70} \bullet \cancel{100}^{1}}{\cancel{500}_{\cancel{5}_{1}}} = \frac{\cancel{500}^{1}n}{\cancel{500}_{1}}$ ← Simplify.

$70 = n$

So $\frac{n}{100} = \frac{70}{100} = 70\%$.

**Check:** $\frac{350}{500} \stackrel{?}{=} \frac{70}{100}$ ← Substitute 70 for $n$, and cross multiply.

$35{,}000 = 35{,}000$ True

> **Remember:**
> **Percent Proportion**
> part → whole → $\frac{\text{percentage } (p)}{\text{base } (b)} = \frac{\text{percent } (\%)}{100}$

Both methods show that 70% of the questionnaires sent out were answered and returned.

## Examples

**1** What percent of 20 is 0.5? Use the formula.

$r \bullet b = p$

$r \bullet 20 = 0.5$ ← Substitute 20 for $b$ and 0.5 for $p$.

$\frac{r \bullet 20}{20} = \frac{0.5}{20}$ ← Divide both sides by 20 to isolate $r$.

$r = \frac{0.5}{20}$ ← Simplify.

$r = 0.025 = 2.5\%$ ← Divide. Rename the decimal as a percent.

So 0.5 is 2.5% of 20.

**2** 30 is what percent of 8? Use a proportion. Let $n$ = the unknown percent.

$\frac{30}{8} = \frac{n}{100}$ ← Write the proportion.

$30 \bullet 100 = 8n$ ← Cross multiply.

$\frac{3000}{8} = \frac{8n}{8}$ ← Divide both sides by 8 to isolate $n$.

$375 = n$

So 30 is 375% of 8.

**3** A circle graph shows how Earl's cat spends a 24-hour day. A 45° angle forms the section of the circle representing the time the cat spends eating. What percent of a day does the cat spend eating? Use a proportion. Let $n$ = the unknown percent.

$\frac{45}{360} = \frac{n}{100}$ ← Write a proportion.

$45 \bullet 100 = 360n$ ← Cross multiply.

$\frac{4500}{360} = \frac{360n}{360}$ ← Divide both sides by 360 to isolate $n$.

$12.5 = n$

Earl's cat spends 12.5% of a day eating.

**4** What percent of 10 is $\frac{1}{2}$? Use the formula.

$r \bullet b = p$

$r \bullet 10 = \frac{1}{2}$ ← Substitute $\frac{1}{2}$ for $p$ and 10 for $b$.

$\frac{r \bullet 10}{10} = \frac{\frac{1}{2}}{10}$ ← Divide both sides by 10 to isolate $r$.

$r = \frac{1}{2} \bullet \frac{1}{10}$ ← Multiply by the reciprocal of 10.

$r = \frac{1}{20} = \frac{5}{100} = 5\%$ ← Rename the fraction as a percent.

So $\frac{1}{2}$ is 5% of 10.

## Try These

**Use the percent formula to find each percent.**

1. What percent of 200 is 11?
2. 0.3 is what percent of 0.5?
3. 125 is what percent of 25?
4. For an important test, Juan studied for 2 h 20 min, while Ken studied for two thirds of an hour. What percent of Ken's study time was Juan's?

**Use a proportion to find each percent.**

5. What percent of 2.5 is 2?
6. 4.6 is what percent of 25?
7. What percent is $\frac{1}{5}$ of 10?
8. Ralph made 12 of his 25 free-throw attempts, while Maria made 20 of 50 attempts. Who made the higher percent of free throws? How much higher?
9. **Discuss and Write** Show how you could use the fraction $\frac{350}{500}$ to solve the opening problem about questionnaires. Tell which approach you prefer, and explain why you prefer it.

Go to PRACTICE BOOK Lesson 7-5 for exercise sets.

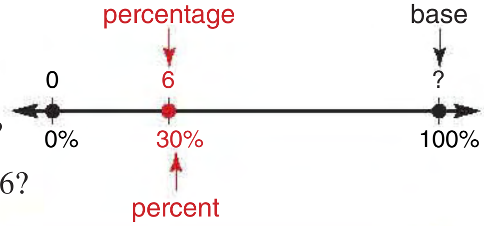
7-6

Update your skills. See page 413 XIV.

# Find the Original Number or the Base

**Objective** To find a number when a percent of it is known, using the percent formula or a percent proportion • To determine whether the percentage, base, or percent is missing in a percent problem and solve the problem

Six of Greene County's middle schools sent bands to the Earth Day Festival. If 30% of the county's middle schools sent bands to the event, how many middle schools are in the county?




To find the total number, or *base*, solve: 30% of *what number* is 6?

▶ Use the percent formula or a proportion to find the base.

**Remember:**
**Percent Formula**
$r \bullet b = p$
rate ($r$) • base ($b$) = percentage ($p$)

**Method 1** Use the Percent Formula: $r \bullet b = p$

Let $b$ = the total number of middle schools in the county.

$30\% \bullet b = 6$ ← Substitute 30% for $r$ and 6 for $p$.

$0.3b = 6$ ← Write the percent as a decimal.

$\frac{0.3b}{0.3} = \frac{6}{0.3}$ ← Divide both sides by 0.3 to isolate $b$.

$b = 20$

**Remember:**
**Percent Proportion**
part → whole → $\frac{\text{percentage } (p)}{\text{base } (b)} = \frac{\text{percent } (\%)}{100}$

**Method 2** Use a Percent Proportion

Let $b$ = the total number of middle schools in the county.

**Solve:** $\frac{6}{b} = \frac{30}{100}$ ← Substitute 6 for $p$ and 30 for the percent.

$6 \bullet 100 = 30b$ ← Cross multiply.

$600 = 30b$ ← Simplify.

$\frac{600}{30} = \frac{30b}{30}$ ← Divide both sides by 30 to isolate $b$.

$20 = b$

**Check:** $\frac{6}{20} \stackrel{?}{=} \frac{30}{100}$ ← Substitute 20 for $b$ and cross multiply.

$600 = 600$ True

So both methods show that there are 20 middle schools in the county.

## Examples

**1** 60% of what number is 12?
Use the percent formula: $r \bullet b = p$.

$60\% \bullet b = 12$ ← Substitute 60% for $r$ and 12 for $p$.

$0.6 \bullet b = 12$ ← Rename 60% as a decimal.

$\frac{0.6b}{0.6} = \frac{12}{0.6}$ ← Divide both sides by 0.6 to isolate $b$.

$b = 20$

So 60% of 20 is 12.

**2** 120% of what number is 24?
Use a percent proportion.

$\frac{24}{b} = \frac{120}{100}$

$24 \bullet 100 = 120b$ ← Cross multiply.

$\frac{2400}{120} = \frac{120b}{120}$ ← Divide both sides by 120 to isolate $b$.

$20 = b$

So 120% of 20 is 24.

► You can use the percent formula or a proportion to solve three types of problems.

| Problem | Represented by the Variable | Percent Formula | Proportion |
|---|---|---|---|
| 5 is what percent of 20? | percent | $r \bullet 20 = 5$ | $\frac{5}{20} = \frac{n}{100}$ |
| 5 is 25% of what number? | base | $25\% \bullet b = 5$ | $\frac{5}{n} = \frac{25}{100}$ |
| What number is 25% of 20? | percentage | $25\% \bullet 20 = p$ | $\frac{n}{20} = \frac{25}{100}$ |

## Examples

**1** Of the bands in a festival, 40% are performing for the first time. If 6 bands are performing for the first time, how many bands in all are performing?

Let $n$ = the total number of bands that are performing.

**Solve:** $\frac{6}{n} = \frac{40}{100}$

$600 = 40n$ ← Cross multiply.

$\frac{600}{40} = \frac{40n}{40}$ ← Divide both sides by 40 to isolate $n$.

$15 = n$

**Check:** $\frac{6}{15} \stackrel{?}{=} \frac{40}{100}$

$600 = 600$ True

So 15 bands in all are performing.

**2** Chan spent an hour working on his essay. He spent 24 minutes outlining and the rest of the hour writing. What percent of an hour did Chan spend writing?

Use the percent formula: $r \bullet b = p$

$r \bullet 60 = 24$ ← Substitute 60 for $b$ and 24 for $p$.

$\frac{r \bullet 60}{60} = \frac{24}{60}$ ← Divide both sides by 60 to isolate $r$.

$r = \frac{24 \div 12}{60 \div 12} = \frac{2}{5}$ ← Divide by the GCF, 12.

$\frac{2}{5} = \frac{40}{100} = 40\%$ ← time spent outlining

$100\% - 40\% = 60\%$ ← time spent writing

So Chan spent 60% of an hour writing.

## Try These

**Solve for $b$, the original number.**

**1.** 50% of $b$ is 11.5.

**2.** 37.5% of $b$ is 48.

**3.** 125% of $b$ is 30.5.

**Solve for the unknown percentage, base, or percent.**

**4.** What percent of 25 is 30?

**5.** Find 250% of 250.

**6.** $12\frac{1}{2}\%$ of $b$ is 0.4.

**7.** If 130% of a number is 26, is the number greater or less than 26? If 50% of a number is 26, is the number greater or less than 26? Explain.

**8.** **Discuss and Write** Lamar's survey of the seventh graders in his school showed that 25% of them would like to be in a rock band. The rest of the seventh graders, 54 students, said that they would *not* care to join a rock band. Explain how you would use this information to find how many seventh graders there are in Lamar's school.

Go to PRACTICE BOOK Lesson 7-6 for exercise sets.

Algebra

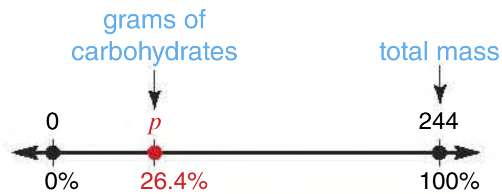
# 7-7

*Update your skills. See page 411 X.*

# Estimate with Percents

**Objective** **To estimate to find percent • To estimate a percent from a model that is not 100 units • To estimate a percent from a three-dimensional model**

If a snack pack has a mass of 244 grams and the percent of carbohydrates is 26.4%, about how many grams of carbohydrates are in the snack pack?




► To find about how many grams, estimate.

$r \bullet b = p$ ← Use the percent formula.

$26.4\% \bullet 244 = p$ ← Substitute known values into the formula.

$26.4\% \approx 25\% = \frac{1}{4}$ ← Change the percent, $r$, to a common percent.

$244 \approx 240$ ← Change the base, $b$, to a compatible number.

$\frac{1}{4} \bullet 240 = 60$ ← Compute mentally.

So the percentage of carbohydrates in the snack pack is about 60 grams.

> **Remember:**
> *Compatible numbers* are numbers that are easy to compute mentally.

► You can also use compatible numbers to estimate the rate and the base.

## Examples

**1** About what percent of 237 is 177?

$\frac{177}{237}$ ← Write the numbers in ratio form.

$\frac{177}{237} \approx \frac{180}{240}$ ← Choose compatible numbers.

$\frac{180}{240} = \frac{3}{4}$ ← Simplify.

$\frac{3}{4} = 75\%$

So 177 is about 75% of 237.

**2** Estimate the base if 38.6 is 18.8% of it.
Use the percent formula, and solve for $b$.

$r \bullet b = p$

$18.8\% \bullet b = 38.6$ ← Substitute known values for $r$ and $p$.

$18.8\% \approx 20\%$ ← Change 18.8% to a common percent.

$38.6 \approx 40$ ← Change the percentage to a compatible number.

$20\% \bullet b = 40$ ← Substitute 20% for $r$ and 40 for $p$.

$\frac{1}{5}b = 40$ ← Use $20\% = \frac{1}{5}$.

$\frac{1}{5}b \div \frac{1}{5} = 40 \div \frac{1}{5}$ ← Divide both sides by $\frac{1}{5}$.

$b = 200$

So the base is about 200.

► Memorizing *benchmark* percents can help you use mental math and estimate.

| Benchmark Percents | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Percent** | 1% | 10% | $12\frac{1}{2}$% | 20% | 25% | $33\frac{1}{3}$% | 50% | $66\frac{2}{3}$% | 75% |
| **Fraction** | $\frac{1}{100}$ | $\frac{1}{10}$ | $\frac{1}{8}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ |
| **Decimal** | 0.01 | 0.1 | 0.125 | 0.2 | 0.25 | $0.\overline{3}$ | 0.5 | $0.\overline{6}$ | 0.75 |

► A quick, reliable way to estimate with percents is to use benchmark percents, such as 1%, 10%, 25%, 50%, and 75%.

- Estimate 6% of 39.

  $39 \approx 40$

  $1\%$ of $40 = 0.01 \bullet 40 = 0.4$ ← 1% = 0.01. Multiply by 0.01.

  $6\%$ of $40 = 6 \bullet 0.4 = 2.4$ ← 6% = 6 • 1%. Multiply 0.4 by 6.

  So 6% of 39 is about 2.4.

- Estimate 15% of \$32.05.

  $\$32.05 \approx \$30$

  $15\% = 10\% + 5\%$ ← Use 10% benchmark.

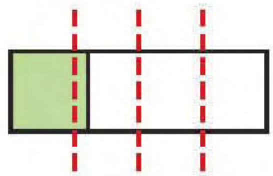
  $10\%$ of $\$30 = 0.1 \bullet \$30 = \$3.00$ ← Multiply mentally by 0.1.

  $5\%$ of $\$30 = \$3.00 \div 2 = \$1.50$ ← 5 is half of 10, so divide 10% of \$30 by 2.

  $\$3.00 + \$1.50 = \$4.50$ ← Add the estimates for 10% and 5%.

  So 15% of \$32.05 is about \$4.50.

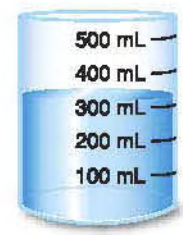
- About what percent of the bar is shaded?



  Think of dividing the bar into four equal parts. Then find a benchmark percent that is closest to the length of the shaded part.

  The shaded part is between 25% and 50%, but it is much closer to 25%. It is about 30% shaded.

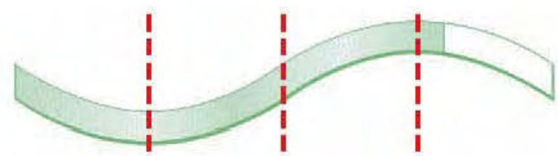
- About what percent of the container is full?




  The container is filled to about 350 mL, and $\frac{350}{500} = 0.7 = 70\%$.

  The container is about 70% full.

- About what percent of the figure is shaded?



  Think of dividing the length into equal parts. Then use benchmarks to estimate the percent shaded. The figure is about 80% shaded.

## Try These

**Estimate.**

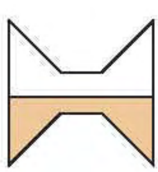
**1.** Estimate 53% of 67.

**2.** About what percent of 396 is 47?

**3.** 3.9 is about 23.7% of what number?

**Estimate the percent of each figure that is shaded.**

**4.**



**5.**

**6. Discuss and Write** Explain how you made your estimates for exercises 4 and 5.

Go to PRACTICE BOOK Lesson 7-7 for exercise sets.

Algebra

# 7-8 Percent Increase

Update your skills. See page 408 III.

**Objective** To find the percent increase • To find profit • To find the selling price for an item sold at a profit

According to the United Nations Department of Public Information, world exports had a total value of \$1.9 trillion in 1985. Fifteen years later, world exports had a total value of \$6.3 trillion. About what percent increase was there in the value of world exports from 1985 to 2000?

To find the percent increase in the value of world exports, compare the amount of increase to the original amount.

**Key Concept**

**Percent Change**

Percent change is the ratio comparing a change in a quantity to the original amount. A change in a number or a quantity is an increase or a decrease.

$$\text{percent change} = \frac{\text{amount of change}}{\text{original amount}}$$

► You can use the formula or a proportion to find the percent increase. Notice that since both amounts in the problem are in trillions, the nonzero digits can be used to calculate the percent increase.

**Method 1** Write an Equation

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

Let $R_I$ = the percent increase.

$R_I = \frac{6.3 - 1.9}{1.9}$ ← Subtract the original amount from the new amount to find the amount of increase. Substitute 6.3 for the new amount and 1.9 for the original amount.

$R_I = \frac{4.4}{1.9}$ ← Divide to simplify.

$R_I \approx 2.32$ ← Round the decimal to the nearest hundredth.

$R_I \approx 232\%$ ← Multiply the decimal by 100 to write as a percent. Write the % sign.

**Method 2** Write and Solve a Proportion

Let $n$ = the unknown percent increase.

$6.3 - 1.9 = 4.4$ ← Subtract the original amount from the new amount to find the amount of increase.

$$\begin{array}{l} \text{part} \rightarrow \\ \text{whole} \rightarrow \end{array} \frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{percent increase (\%)}}{100}$$

$\frac{4.4}{1.9} = \frac{n}{100}$ ← Substitute 4.4 for the amount of increase and 1.9 for the original amount. Cross multiply.

$440 = 1.9n$

$\frac{440}{1.9} = \frac{1.9n}{1.9}$ ← Divide both sides by 1.9. Round to the nearest one.

$232 \approx n$

So the percent increase in the value of world exports from 1985 to 2000 was about 232%.

► To find the amount of profit, multiply the original cost by the percent profit.

- The **cost** of an item is the original amount spent for it.
- **Profit** is the money gained when an item is sold above the cost.

A company buys microwave ovens for \$200 each and then sells each at a gain of 15%. What is the company's profit from buying and selling each microwave oven?

**Method 1** Write an Equation

Let $P$ = the amount of profit.

$P = 15\% \bullet \$200$ ← Multiply the original cost by the percent of profit.

$P = 0.15 \bullet \$200$ ← Write 15% as a decimal, then multiply.

$P = \$30.00$

**Method 2** Write and Solve a Proportion

$$\frac{\text{profit } (P)}{\text{original cost}} = \frac{\text{percent profit } (\%)}{100}$$

$\frac{P}{200} = \frac{15}{100}$ ← Substitute \$200 for the cost and 15 for the percent profit.

$100P = 200 \bullet 15$ ← Cross multiply.

$\frac{100P}{100} = \frac{3000}{100}$ ← Divide both sides by 100.

$P = 30$

So the profit per microwave oven is \$30.

► The **selling price ($SP$)** of an item is the amount for which the item is sold. When an item is sold at a profit, the selling price is *more* than the cost. Add the profit to the cost to find the selling price.


A video dealer imports large flat screen TVs for \$1200 each. The dealer then sells each TV at a profit of 30%. Find the selling price per unit.

$SP = \$1200 + (30\% \text{ of } \$1200)$ ← The cost is \$1200, and the profit is 30% of \$1200.

$SP = \$1200 + (0.30 \bullet \$1200)$ ← Write 30% as the decimal 0.30, then multiply 1200 by 0.30.

$SP = \$1200 + \$360$ ← The profit is \$360, so add \$360 to the cost, \$1200.

$SP = \$1560$

So the selling price for each flat screen TV is \$1560.

## Try These

**Find the percent increase. If necessary, round to the nearest tenth of a percent.**

**1.** from 12 to 15
**2.** from 0.3 to 0.66
**3.** from 40 to 45

**Find the profit to the nearest cent.**

**4.** Cost: \$48.60
Percent profit: 25%

**5.** Cost: \$124
Percent profit: 6.25%

**6.** Cost: \$92.50
Percent profit: 9.2%

**7.** An antique car dealer made a profit of 18% on a car that cost \$40,000. For how much did he sell the car?

**8.** **Discuss and Write** Explain how you find the selling price of an item when you know the original price and the percent profit.

Go to PRACTICE BOOK Lesson 7-8 for exercise sets.

Algebra

# 7-9 Percent Decrease

**Objective** To find the percent decrease • To find the selling price for an item sold at a loss

In the first modern Olympic Games in 1896, the winning time in the men's 100-meter sprint was 12 seconds. By the 2004 games, the winning time had dropped to 9.85 seconds. To the nearest tenth of a percent, what is the percent decrease in the winning time from 1896 to 2004?

To find the percent decrease for the winning time from 1896 to 2004, compare the amount of decrease to the original amount.

► You can use the formula for rate of decrease, or you can use a proportion to find the percent decrease.

**Remember:**
percent change $= \dfrac{\text{amount of change}}{\text{original amount}}$

**Method 1** Write an Equation

percent decrease $= \dfrac{\text{amount of decrease}}{\text{original amount}}$

Let $R_D$ = the percent decrease.

$R_D = \dfrac{12 - 9.85}{12}$ ← Subtract the new amount from the original amount to find the amount of decrease. Substitute 12 for the original amount and 9.85 for the new amount.

$R_D = \dfrac{2.15}{12}$ ← Divide to simplify.

$R_D = 0.1791\overline{6}$ ← Multiply by 100 to rename the decimal as a percent.

$R_D \approx 17.9\%$ ← Round to the nearest tenth of a percent.

**Method 2** Write and Solve a Proportion

Let $n$ = the unknown percent decrease.

$12 - 9.85 = 2.15$ ← Subtract the original amount from the new amount to find the amount of decrease.

part → $\dfrac{\text{amount of decrease}}{\text{original amount}} = \dfrac{\text{percent decrease (\%)}}{100}$
whole →

$\dfrac{2.15}{12} = \dfrac{n}{100}$ ← Substitute 2.15 for the amount of decrease and 12 for the original amount, then cross multiply.

$2.15 \bullet 100 = 12n$

$\dfrac{215}{12} = \dfrac{12n}{12}$ ← Divide both sides by 12

$n = 17.91\overline{6}$

$n \approx 17.9$ ← Round to the nearest tenth.

So the percent decrease in the winning time for the men's 100-meter sprint from 1896 to 2004 is approximately 17.9%.

▶ Merchants sometimes suffer a *loss* on the sale of items. When there is a loss of money, the item is sold *below* its cost.

Finding the amount of loss is similar to finding the amount of profit: Multiply the original cost by the percent loss.

A nursery bought spruce trees for $80 each. After the holidays, the nursery sold the trees that were left at a loss of 20%. What was the loss ($L$) per tree on the trees sold after the holidays?

**Method 1** Write an Equation

Let $L$ = the amount of loss.

$L = 20\% \bullet \$80$ ← Multiply the original cost, $80, by the percent loss, 20%.

$L = 0.20 \bullet \$80$ ← Write 20% as a decimal, then multiply.

$L = \$16.00$

**Method 2** Write and Solve a Proportion

$$\frac{\text{loss } (L)}{\text{cost } (C)} = \frac{\text{percent loss } (\%)}{100}$$

$\frac{L}{80} \quad \frac{20}{100}$ ← Substitute 80 for the cost ($C$) and 20 for the percent loss. Cross multiply.

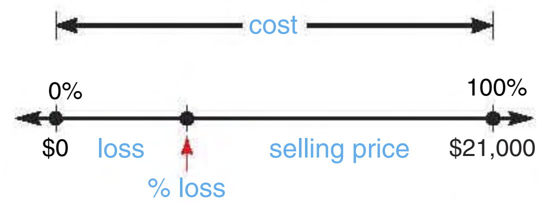
$100L = 80 \bullet 20$

$\frac{100L}{100} = \frac{1600}{100}$ ← Divide both sides by 100.

$L = 16$

So the loss on each tree was $16.

▶ When an item is sold at a *loss*, the selling price is *less* than the cost. Subtract the loss from the cost to find the selling price.




A family bought a car for $21,000 and then sold it at a loss of 30%. Find the selling price.

$SP = \$21{,}000 - (30\% \text{ of } \$21{,}000)$ ← The cost is $21,000, and the loss is 30% of $21,000.

$SP = \$21{,}000 - (0.3 \bullet \$21{,}000)$ ← Write 30% as the decimal 0.3, and multiply 21,000 by 0.3.

$SP = \$21{,}000 - \$6300$ ← The amount of loss is $6300, so subtract $6300 from the cost, $21,000.

$SP = \$14{,}700$

So the selling price is $14,700.

## Try These

**Find the percent decrease.**

**1.** from $5 to $4 **2.** from 0.7 to 0.07 **3.** from 90.6 to 35.2

**Find the loss to the nearest cent.**

**4.** Cost: $385.20
Percent loss: 3.5%

**5.** Cost: $79.25
Percent loss: 3.4%

**6. Discuss and Write** How is finding the percent decrease like finding the percent increase? How is it different?

Go to PRACTICE BOOK Lesson 7-9 for exercise sets.

# 7-10 Sales Tax and Tips

**Objective** **To calculate sales tax and total cost • To read and use a tax table • To calculate a tip and total cost**

A calculator is priced at \$29.95 with a sales tax of 6%. What is the amount of sales tax to the nearest cent? What is the total cost?

To find the total cost of the calculator, find the amount of sales tax and add it to the marked price.

Sales tax is the amount of tax added to the marked price of an item by a state or local government. The sales tax rate is the ratio of the amount of sales tax to the marked price expressed as a percent. The total cost of an item is the sum of its marked price and the amount of sales tax.


**Key Concept**

**Sales Tax**

sales tax = sales tax rate • marked price

► You can write and solve an equation or a proportion to find the amount of sales tax.

**Method 1** Write and Solve an Equation

amount of sales tax = sales tax rate • marked price

Let $T$ = the amount of sales tax.

$T = 6\% \bullet \$29.95$ ← Multiply the marked price, \$29.95, by the sales tax rate, 6%.

$T = 0.06 \bullet \$29.95$ ← Write 6% as a decimal, then multiply.

$T = \$1.7970 \rightarrow \$1.80$ ← Round to the nearest cent.

**Method 2** Write and Solve a Proportion

$$\frac{\text{sales tax } (T)}{\text{marked price } (MP)} = \frac{\text{sales tax rate } (\%)}{100} \begin{matrix} \leftarrow \text{part} \\ \leftarrow \text{whole} \end{matrix}$$

$\frac{T}{29.95} = \frac{6}{100}$ ← Substitute 29.95 for $MP$ and 6 for the sales tax rate. Cross multiply.

$100T = 6 \bullet 29.95$

$\frac{100T}{100} = \frac{179.70}{100}$ ← Divide both sides by 100.

$T = 1.797 \rightarrow \$1.80$ ← Round to the nearest cent.

Using either method, you find that the amount of sales tax is \$1.80.

Add \$1.80 to the marked price to find the total cost.

$\$29.95 + \$1.80 = \$31.75$.

So the total cost of the calculator is \$31.75.

► You can use a Sales Tax Table to compute the amount of sales tax and the total cost.

Use the 6% sales tax table at the right to find the amount of sales tax and the total cost of a printer cartridge marked at $20.95.

1. Find $20.95 in the range of prices in the table.
   $20.95 is in the row for $20.92–$21.08.
2. Look across the row.
   The tax for prices in that range is $1.26
3. Add the sales tax to the marked price to find the total cost.
   $20.95 + $1.26 = $22.21

| Sale | Tax |
|---|---|
| $0.00–$0.08 | $0.00 |
| 0.09– 0.24 | .01 |
| 0.25– 0.41 | .02 |
| 0.42– 0.58 | .03 |
| 0.59– 0.74 | .04 |
| 0.75– 0.91 | .05 |
| 0.92– 1.08 | .06 |

| Sale | Tax |
|---|---|
| $20.09–$20.24 | $1.21 |
| 20.25– 20.41 | 1.22 |
| 20.42– 20.58 | 1.23 |
| 20.59– 20.74 | 1.24 |
| 20.75– 20.91 | 1.25 |
| 20.92– 21.08 | 1.26 |
| 21.09– 21.24 | 1.27 |

The total cost of the printer cartridge is $22.21.

► A common application of percents is leaving a tip, or gratuity, for service provided in places such as restaurants or hotels.

You can use rounding, benchmark percents, and mental math to calculate a tip.

Calculate the tip and total cost for a dinner that costs $31.50.
Round the amount up to the next dollar: $31.50 ≈ $32.

**Calculate a 10% tip.**
- Multiply $32 by 10%.
  10% of $32 = 0.1 • $32
  Move the decimal point one place to the left.
  Tip: $3.20
- Add: $31.50 + $3.20
  Total cost: $34.70

**Calculate a 15% tip.**
- 15% = 10% + 5%
- 10% = $3.20
- 5% = half of 10%, and half of $3.20 = $1.60
- Add: $3.20 + $1.60 = $4.80
  Tip: $4.80
- Add: $31.50 + $4.80
  Total cost: $36.30

**Calculate a 20% tip.**
- 20% = 10% + 10%
- 10% = $3.20
- 20% = 2 • $3.20 = $6.40
  Tip: $6.40
- Add: $31.50 + $6.40
  Total cost: $37.90

## Try These

**Find the amount of sales tax and the total cost.**

**1.** a $450 rug with $5\frac{1}{2}$% sales tax rate

**2.** a $926 dresser with a 3.5% sales tax rate

**Find the tip and the total cost. Round and use mental math.**

**3.** a $48.75 meal with a 10% tip

**4.** a $31 food delivery with a 5% tip

**5.** a $39.85 dinner with a 20% tip

**6.** a $9.75 taxi ride with a 15% tip

**7. Discuss and Write** How can you estimate the amount of sales tax and the total cost of an item? Support your explanation with an example from this lesson.

Go to PRACTICE BOOK Lesson 7-10 for exercise sets.

# 7-11 Discount and Markup

**Objective** To find the amount of discount • To find the sale price • To find the discount rate • To find the amount of markup • To find the markup rate

PROBLEM-SOLVING STRATEGY 7

A store sells game tables for $188 each. This week, every game table has a 25% discount. What is the sale price of each game table?

► To find the *sale price*, find the amount of the 25% discount, and then subtract that amount from the original price.

A **discount** is the amount by which the regular or **list price** of an item is reduced. The **discount rate** is the percent decrease in the list price. The **sale price** of an item is the difference between its list price and the amount of discount.

► Use an equation or a proportion to find the amount of discount.

**Method 1** Write and Solve an Equation

discount = discount rate • list price

Let $D$ = the amount of discount.

$D = 25\% \bullet \$188$ ← Multiply the list price, $188, by the discount rate, 25%.

$D = 0.25 \bullet \$188$ ← Write 25% as a decimal, then multiply.

$D = \$47$

**Method 2** Write and Solve a Proportion

$\frac{\text{discount } (D)}{\text{list price } (LP)} = \frac{\text{discount rate } (\%)}{100}$ ← part ← whole

$\frac{D}{\$188} = \frac{25}{100}$ ← Substitute the known values, then cross multiply.

$100D = 25 \bullet \$188$

$\frac{100D}{100} = \frac{4700}{100}$ ← Divide both sides by 100.

$D = \$47$

Using either method, you find that the amount of discount is $47.

To find the sale price, subtract the amount of discount from the list price: $188 − $47 = $141

So the sale price for each game table is $141.

**Key Concept**

**Discount and Sale Price**

discount = discount rate • list price

sale price = list price – discount

► You can find the discount rate of an item when you know the list price and the sale price of that item.

The list price of racquetball rackets is $39. Now they are on sale for $31.20. What is the discount rate of a racket?

$\text{discount rate} = \frac{\text{amount of discount}}{\text{list price}}$

Let $R$ = the discount rate.

$R = \frac{\$39.00 - \$31.20}{\$39.00}$ ← Subtract the sale price from the original price to find the amount of discount.

$R = \frac{\$7.80}{\$39.00}$ ← Simplify. Divide the amount of discount, $7.80, by the list price, $39.

$R = 0.2 = 20\%$ ← Write the decimal as a percent.

So the discount rate for each racquetball racket is 20%.

**Key Concept**

**Discount Rate**

$\text{discount rate} = \frac{\text{amount of discount}}{\text{list price}}$

Stores make a profit by selling items for a price higher than the *wholesale price*. The wholesale price is the lower price that stores pay for the items. The difference between the wholesale price and the list price is called the markup. To find the markup rate, which is the percent increase in the wholesale price, find the amount of markup.

**Key Concept**

**Markup and Markup Rate**

markup = list price − wholesale price

$\text{markup rate} = \dfrac{\text{markup}}{\text{wholesale price}}$

The markup rate is expressed as a percent.

A store buys golf umbrellas for $40 each and sells them for $56. You can write and solve an equation to find the markup rate.

$\text{markup rate} = \dfrac{\text{amount of markup}}{\text{wholesale price}}$

Let $R$ = the markup rate.

$R = \dfrac{\$56 - \$40}{\$40}$ ← Subtract the wholesale price from the list price to find the amount of markup.

$R = \dfrac{\$16}{\$40}$ ← Simplify. Divide the amount of markup by the wholesale price.

$R = 0.40$

$R = 40\%$ ← Write the decimal as a percent.

So the markup rate for each golf umbrella is 40%.

## Try These

**Find the amount of the discount and the sale price to the nearest cent.**

**1.** 15% discount rate on a $400 stereo system

**2.** a $45 headset at a 25% discount rate

**3.** 20% discount rate on a $6800 used car

**4.** 30% discount rate on a $250 DVD player

**Find the discount rate.**

**5.** a $3000 motorcycle on sale for $2400

**6.** a $64 phone on sale for $58.88

**7.** a $300 digital camera on sale for $250

**8.** an $850 notebook computer on sale for $650

**Find the markup rate.**

**9.** a CD bought for $24 and sold for $25.80

**10.** wholesale price: $48; list price: $58.56

**11. Discuss and Write** Explain how you could find (a) the list price, given the wholesale price and the markup rate, and (b) the wholesale price, given the list price and the markup rate.

Go to PRACTICE BOOK Lesson 7-11 for exercise sets.

# 7-12 Commission

**Objective** To find the amount of commission • To find the commission rate • To find the total sales • To compare commissions when the total sales and commission rates are given

A salesperson sells a computer system listed at \$1500. If his rate of commission is 7%, how much commission does he earn on the sale?

► To find the commission the salesperson will earn, multiply the total sales and the rate of commission.

Commission is the amount of money earned for selling goods or services. The commission rate is the percent of the total amount of goods or services sold that is earned by the seller. The total sales refers to the total amount of goods or services sold.

You can write and solve an equation or a proportion to find the amount of commission when you know the total sales and the commission rate.

**Key Concept**

**Commission**

commission = commission rate • total sales

commission rate = $\frac{\text{commission}}{\text{total sales}}$

total sales = $\frac{\text{commission}}{\text{commission rate}}$

**Method 1** Write and Solve an Equation

commission = commission rate • total sales

Let $C$ = the amount of commission.

$C = 7\% \bullet \$1500$ ← Multiply the total sales, \$1500, by the rate of commission, 7%.

$C = 0.07 \bullet \$1500$ ← Write 7% as a decimal, then multiply.

$C = \$105.00$

**Method 2** Write and Solve a Proportion

$\frac{\text{commission } (C)}{\text{total sales}} = \frac{\text{commission rate } (\%)}{100}$

$\frac{C}{\$1500} = \frac{7}{100}$ ← Substitute the known values, then cross multiply.

$100C = 7 \bullet \$1500$

$\frac{100C}{100} = \frac{\$10{,}500}{100}$ ← Divide both sides by 100.

$C = \$105.00$

So the salesperson earns a commission of \$105.

► You can write and solve an equation or a proportion to find the commission rate when you know the total sales (\$1500) and the commission (\$105).

**Method 1** Write and Solve an Equation

commission rate = $\frac{\text{commission}}{\text{total sales}}$

Let $R$ = the commission rate.

$R = \frac{105}{1500}$ ← Substitute the known values.

$R = 0.07$ ← Divide to simplify.

$R = 7\%$ ← Write the decimal as a percent.

**Method 2** Write and Solve a Proportion

$\frac{\text{commission}}{\text{total sales}} = \frac{\text{commission rate } (\%)}{100}$

Let $n$ = the unknown percent.

$\frac{105}{1500} = \frac{n}{100}$ ← Substitute the known values, then cross multiply.

$105 \bullet 100 = 1500n$

$\frac{10{,}500}{1500} = \frac{1500n}{1500}$ ← Divide both sides by 1500.

$7 = n$

So $\frac{n}{100} = 7\%$.

So the commission rate is 7%.

▶ You can find the total sales when you know the commission (\$105) and the commission rate (7%).

**Method 1** Write and Solve an Equation

total sales $= \dfrac{\text{commission}}{\text{commission rate}}$

Let $TS$ = the total sales.

$TS = \dfrac{\$105}{7\%}$ ← Substitute the known values.

$TS = \dfrac{\$105}{0.07}$ ← Write 7% as a decimal, then divide.

$TS = \$1500$

**Method 2** Write and Solve a Proportion

$\dfrac{\text{commission}}{\text{total sales }(TS)} = \dfrac{\text{commission rate }(\%)}{100}$

$\dfrac{105}{TS} = \dfrac{7}{100}$ ← Substitute the known values, then cross multiply.

$105 \bullet 100 = 7TS$

$\dfrac{10{,}500}{7} = \dfrac{7TS}{7}$ ← Divide both sides by 7.

$1500 = TS$

So the amount of total sales is \$1500.

▶ You can write and solve an equation to compare commissions.

Ms. Spann sold a condominium for \$380,000 at a commission rate of 9%.
Ms. Lee sold a condominium for \$440,000 at a commission rate of 7.5%.
Which real estate agent earned a higher commission? How much higher?

Ms. Spann's commission:

commission = commission rate • total sales

Let $C$ = the amount of commission.

$C = 9\% \bullet \$380{,}000$ ← Substitute the known values.

$C = 0.09 \bullet \$380{,}000$ ← Write 9% as a decimal, then multiply.

$C = \$34{,}200$

Ms. Lee's commission:

commission = commission rate • total sales

Let $C$ = the amount of commission.

$C = 7.5\% \bullet \$440{,}000$ ← Substitute the known values.

$C = 0.075 \bullet \$440{,}000$ ← Write 7.5% as a decimal, then multiply.

$C = \$33{,}000$

Subtract to find the difference in the two commissions: $\$34{,}200 - \$33{,}000 = \$1{,}200$.

So Ms. Spann's commission was \$1200 greater than Ms. Lee's.

## Try These

**Find the commission.**

1. 6% commission on \$25,000 in sales
2. 6.2% commission on \$48,200 in sales

**Find the rate of commission.**

3. \$1755 earned on \$92,000 in sales
4. \$4936 earned on \$36,000 in sales

**Find the total sales.**

5. \$3480 earned with 10% commission rate
6. \$720.76 earned with 3.7% commission rate

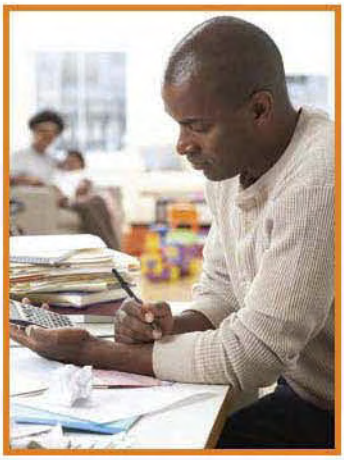
7. **Discuss and Write** Explain how you would find a 3% commission for an item sold at \$7800. Then solve.

Go to PRACTICE BOOK Lesson 7-12 for exercise sets.

# 7-13 Simple Interest

**Objective** To find simple interest • To find the total amount earned or due • To find the rate of interest • To find the time that principal is left on deposit • To use spreadsheet software to compute simple interest for different principals, rates, and lengths of time

Mr. Floyd borrowed \$2500 from his brother to be paid back in 3 years. He agreed to repay the money with interest at a rate of 5.5% per year. How much interest will Mr. Floyd pay at the end of the 3 years? What is the total amount or *balance* due at the end of the 3 years?



Principal ($p$) is the amount of money borrowed or deposited. Interest ($I$) is the amount earned or paid in exchange for the use of money. Simple interest is interest earned or paid *only* on the principal for a stated period of time. The rate of interest ($r$) is the percent of interest earned or paid. Time ($t$) represents how long, in years, the principal is borrowed or left on deposit. Time must be in years or a fractional part of a year.

**Key Concept**

**Simple Interest**

Interest = principal • rate • time

$I = prt$

Time must be expressed in years or a fractional part of a year.

► To find the amount of interest Mr. Floyd will pay, multiply the principal, the rate of interest, and the time, in years. To find the total amount, or *balance due,* add the interest to the principal.

1. You can use a formula to find the interest.

   $I = prt$

   $I = \$2500 \cdot 5.5\% \cdot 3$ ← Substitute \$2500 for $p$, 5.5% for $r$, and 3 for $t$.

   $I = \$2500 \cdot 0.055 \cdot 3$ ← Write 5.5% as a decimal, then multiply.

   $I = \$412.50$

2. Add the interest to the principal to find the total amount or balance due.

   $\$2500 + \$412.50 = \$2912.50$

So Mr. Floyd will pay \$412.50 in interest. The balance due will be \$2912.50.

► You can find the annual rate of interest when you know the principal, the time, and the interest amount.

Lupe has \$365. If she deposits that money for 24 months and earns \$36.50, what is the annual rate of interest earned?

You can use the interest formula to find the rate of interest.

$I = prt$

$36.50 = 365 \cdot r \cdot 2$ ← Substitute 36.50 for $I$, 365 for $p$, and 2 for $t$.

$36.50 = 730r$ ← Multiply to simplify.

$\frac{36.50}{730} = \frac{730r}{730}$ ← Divide both sides by 730.

$r = 0.05$ ← Write the decimal as a percent.

$r = 5\%$

**Think**
24 months = 2 years

So the annual rate of interest earned by Lupe's deposit is 5%.

► You can find the amount of time that interest is paid or earned when you know the principal, the annual rate of interest, and the interest amount.

Darrin borrowed $1100 at an interest rate of 6% per year. If he paid interest of $297, for how many years did he take out the loan?

To find the *time* of a loan or deposit, in years, use the interest formula to solve for $t$.

$I = prt$

$297 = 1100 \bullet 6\% \bullet t$ ← Substitute $297 for $I$, $1100 for $p$, and 6% for $r$.

$297 = 1100 \bullet 0.06 \bullet t$ ← Write 6% as a decimal.

$297 = 66.00t$ ← Multiply to simplify.

$\frac{297}{66} = \frac{66t}{66}$ ← Divide both sides by 66 to isolate $t$.

$4.5 = t$

So Darrin took out the loan for a period of 4.5 years, or 4 years and 6 months.

**Think**
Fractional parts of a year are represented as decimals:
6 months = 0.5 year
9 months = 0.75 year
2 years 6 months = 2.5 years

## Try These

**Find the simple interest, *I*, and the balance earned or due.**

1. $800 at an annual rate of 8% for 9 months
2. $720 at an annual rate of 8.5% for 6 months

**Find the annual interest rate, *r*.**

3. $200 borrowed for 2 years, $24 in interest
4. $400 deposited for 9 months, $24 in interest

**Find the time, *t*.**

5. $350 at 8% per year, $84 in interest
6. $800 at 7% per year, $42 in interest

7. **Discuss and Write** Consider the interest on a savings account and the interest on a car loan. How are they the same? How are they different?

## Technology

The spreadsheet shows the balances for five different principal amounts at an interest rate of 5% over a period of 3 years.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Principal (*P*) | Rate (*R*) | Time (*T*) | Interest (*I*) | Balance |
| 2 | | 5 | 3 | | |
| 3 | 1000 | = B2/100 | = C2 | = A3*B3*C3 | = A3+D3 |
| 4 | 1500 | = B2/100 | = C2 | = A4*B4*C4 | = A4+D4 |
| 5 | 2000 | = B2/100 | = C2 | = A5*B5*C5 | = A5+D5 |
| 6 | 2500 | = B2/100 | = C2 | = A6*B6*C6 | = A6+D6 |
| 7 | 3000 | = B2/100 | = C2 | = A7*B7*C7 | = A7+D7 |
| 8 | | | | | |

Sheet 1 Sheet 2 Sheet 3

For each principal in column A, the spreadsheet calculates simple interest for any values of rate and time entered in B2 and C2.

The spreadsheet adds simple interest to the principal.

Cell B2 is the box in column B, row 2.
Cell C2 is the box in column C. row 2.
= A3*B3*C3 means that the values in cells A3, B3, and C3 are multiplied using the formula $I = prt$.

Cell D3 $= 1000 \bullet \frac{5}{100} \bullet 3 = 150$

Cell E3 $= 1000 + 150 = 1150$

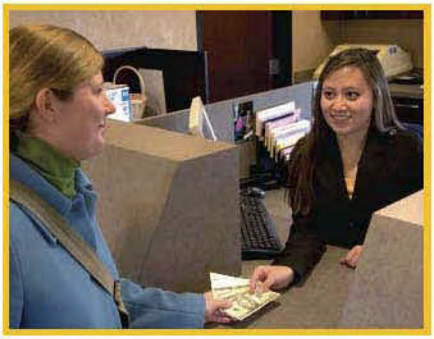
Go to PRACTICE BOOK Lesson 7-13 for exercise sets.

# 7-14

# Compound Interest

**Objective** To compute compound interest using tables

Ms. Johnson deposits $12,000 in a savings bank account that pays interest quarterly. The annual interest rate paid by the bank is 6%. How much money will she have in the bank at the end of 1 year?



► To find how much money Ms. Johnson will have in the bank at the end of 1 year, compute compound interest.

Compound interest is the interest paid on the principal and on the interest accumulated to date. The balance of an interest-bearing account is the sum of the principal plus the interest earned.

You can use a compound interest table to compute compound interest and find the balance in an account.

The compound interest table at the right shows what factor the deposit will be multiplied by to find the balance.

interest rate per period

**Compound Interest Table**

| Number of Periods | 1.5% | 2% | 2.5% | 3% | 3.5% | 4% |
|---|---|---|---|---|---|---|
| 1 | 1.0150 | 1.0200 | 1.0250 | 1.0300 | 1.0350 | 1.0400 |
| 2 | 1.0302 | 1.0404 | 1.0506 | 1.0609 | 1.0712 | 1.0816 |
| 3 | 1.0457 | 1.0612 | 1.0769 | 1.0927 | 1.1087 | 1.1248 |
| 4 | 1.0614 | 1.0824 | 1.1038 | 1.1255 | 1.1475 | 1.1699 |
| 5 | 1.0773 | 1.1041 | 1.1314 | 1.1593 | 1.1877 | 1.2167 |
| 6 | 1.0934 | 1.1262 | 1.1597 | 1.1941 | 1.2293 | 1.2653 |
| 7 | 1.1098 | 1.1487 | 1.1887 | 1.2299 | 1.2723 | 1.3159 |
| 8 | 1.1265 | 1.1717 | 1.2184 | 1.2668 | 1.3168 | 1.3686 |

factors

To find the balance in the account:

1. Find the interest rate per period.

   Number of interest periods: 4 ← 1 year has 4 quarters or periods.

   Annual rate of interest: 6%

   Quarterly rate of interest: 6% ÷ 4 = 1.5% ← Divide the annual interest rate by 4 to find the interest rate per period.

2. Find the factor.

   Look across the 4th period row and down the 1.5% interest rate column to find the factor.

3. Multiply the deposit by the factor to find the balance.

   1.0614 • $12,000 = $12,736.80

4. Subtract the principal from the balance to find how much interest is earned.

   $12,736.80 – $12,000 = $736.80

So Ms. Johnson will earn $736.80 interest, leading to a balance of $12,736.80 at the end of the year.

## Examples

**1** Mel deposits $5000 in an account. The annual interest rate is 8%, compounded semi-annually. How much interest will he earn in 1 year? What is the balance in his account after 1 year?
*Hint:* Semi-annual refers to 2 interest periods per year.

- Find the number of interest periods and the rate of interest.
  The annual interest rate is 8%, compounded semi-annually.
  8% ÷ 2 = 4% ← Divide the annual interest rate by 2 to find the interest rate per period.
- Find the factor.
  Look across the 2nd period row and down the 4% column to find the factor.
- Multiply the deposit by the factor to find the balance.
  1.0816 • $5000 = $5408
- Subtract the principal from the balance to find the amount of interest.
  $5408 − $5000 = $408

Mel will earn $408 interest, leading to a balance of $5408 after 1 year.

**2** Dylan deposits $4000 in his savings account. The annual interest rate is 12%, compounded quarterly. How much interest will he earn in 2 years? What is the balance in his account after 2 years?
*Hint:* Quarterly refers to 4 interest periods per year.

- Find the number of interest periods and the rate of interest.
  The annual interest rate is 12%, compounded quarterly.
  12% ÷ 4 = 3% ← Divide the annual interest rate by 4 to find the interest rate per period.
- Find the factor.
  Dylan will earn interest for 2 years, or 8 periods.
  Look across the 8th period row and down the 3% column to find the factor.
- Multiply the deposit by the factor to find the balance.
  1.2668 • $4000 = $5067.20
- Subtract the principal from the balance to find how much interest he earns.
  $5067.20 − $4000 = $1067.20

Dylan will earn $1067.20 interest, leading to a balance of $5067.20 after 2 years.

## Try These

**Use the compound interest table to find each balance.**

**1.** $7000 at an annual rate of 8%, compounded quarterly, for 1 year

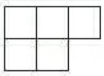
**2.** $10,000 at an annual rate of 7%, compounded semi-annually, for 2 years

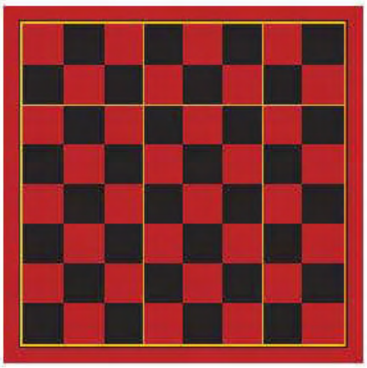
**3. Discuss and Write** How is earning an annual rate of 6% simple interest for 1 year on $2000 different from earning an annual interest rate of 6%, compounded quarterly for 1 year, on $2000? Explain.

Go to PRACTICE BOOK Lesson 7-14 for exercise sets.

# 7-15 Problem-Solving Strategy: Reason Logically

**Objective** To solve problems using the strategy *Reason Logically*.

**Problem 1:** Is it possible to cover a typical 8-by-8 checkerboard, such as the one pictured below, with tiles of this shape ?

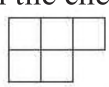


**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. **Reason Logically**
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** A checkerboard is an 8-by-8 arrangement of small squares. Each tile is made up of 5 small checkerboard squares in a fixed pattern.

**Question:** Can the checkerboard be covered exactly with tiles shaped like ? If so, how?

**Plan** **Select a strategy.**
Try using the strategy *Reason Logically*.

**Solve** **Apply the strategy.**
Suppose it is possible to cover a checkerboard with such tiles.
This would mean that 64 little squares can be covered exactly by tiles made up of 5 little squares each.
However, that would mean that 64 is a multiple of 5 (or, equivalently, that 64 is evenly divisible by 5).
You know that 64 is *not* a multiple of 5 (because $64 \div 5 = 12 \text{ R}4$).
So it is *not* possible to cover the checkerboard with the tiles.

**Check** **Check to make sure your answer makes sense.**
Twelve tiles would have $12 \cdot 5$, or 60, small squares.
This is not enough to cover the board.

Thirteen tiles have $13 \cdot 5$, or 65, small squares.
This is too many to cover the board exactly.

Therefore, since it is necessary to use a whole number of tiles, it is not possible to cover the board exactly with tiles of the given shape.

**Problem 2:** Suppose you are given a jar containing 6 blue marbles and 2 red marbles. So $\frac{6}{8}$, or 75%, of the marbles are blue.

You are challenged to reduce the percent of blue marbles in the jar to 20% by adding more blue and red marbles. The only restriction is that each time you add marbles, you must double the total number of marbles in the jar. Can this be done?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** A jar holds 8 marbles—6 blue and 2 red.
So 75% of the marbles are blue.
You can add blue and red marbles to the jar.
Whenever you add any marbles, you must double the total number of marbles in the jar.

**Question:** Can you reduce the percent of blue marbles in the jar to 20%?

**Plan** **Select a strategy.**
You could try to solve this problem by exploring what happens when you add different numbers of red marbles and blue marbles to the jar. However, this could take a very long time, and it might not lead to the answer. Since it is not immediately obvious how to approach this problem, you might first try to reason logically about what you are being asked to do.

**Solve** **Apply the strategy.**
You are starting with a jar that contains exactly 8 marbles. The first time you double the number of marbles, there will be 16 marbles in the jar. The second time you double the number of marbles, there will be 32 marbles in the jar. You know that 8, 16, and 32 are all powers of 2. Specifically, $8 = 2 \times 2 \times 2 = 2^3$, $16 = 2 \times 2 \times 2 \times 2 = 2^4$, and $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$. In fact, since you must always double the number of marbles in the jar, the number of marbles in the jar will always be a power of 2.

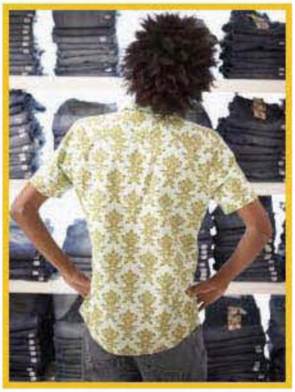
Now 20% is equivalent to $\frac{1}{5}$, so you would need $\frac{1}{5}$ of the marbles to be blue. This would mean that the number of marbles in the jar would have to be divisible by 5. However, a number that is a power of 2 is not divisible by 5 because its only prime factor is 2. So no matter how many blue and red marbles you add to the jar, it is impossible to double the total number of marbles and yet reduce the percent of blue marbles in the jar to 20%.

**Check** **Check to make sure your answer makes sense.**
Using a calculator, you can try taking 20% of various powers of 2. Each and every time, you will discover that the result is not a whole number.

Go to PRACTICE BOOK Lesson 7-15 for exercise sets.

# Enrichment:
## Successive Discounts and Increases

**Objective** **To determine the overall percent of change when more than one percent discount or percent increase is applied.**



Two stores in Diego's town are having sales.

- At Big Bill's, jeans were discounted 30% last week. This week, jeans are marked down an *additional* 20%.
- At Small Change Shop, everything is 50% off.

The original price of the jeans was $60 at both stores.
Is the sale price the same at both stores?

**Big Bill's**

- Original Price: $60
- First Markdown: 30% of $60, or $18
  First Sale Price: $60 − $18 = $42
- Second Markdown: 20% of $42, or $8.40
  Second Sale Price: $42 − $8.40 = $33.60

**Small Change Shop**

- Original Price: $60
- 50% off: 50% of $60, or $30
  Sale Price: $60 − $30 = $30

The sale prices are not the same. One discount of 50% off is a greater discount than two successive discounts of 30% and 20%. To find a single discount that is the same as a 30% discount followed by a 20% discount, follow these steps:

1. Suppose the original price of an item is $p$. If you *save* 30%, then you *pay* 70%, so the price after a 30% discount is 70% of $p$, or $0.7p$.
2. Applying the second discount, you save 20% of $0.7p$. This means you *pay* 80% of $0.7p$. So the final price is 80% of $0.7p$, or $(0.8)(0.7p)$, or $0.56p$.
3. The final price, $0.56p$, is 56% of $p$. Paying 56% of the original price is the same as saving 44%. So a 30% discount followed by a 20% discount is equivalent to a 44% discount.

## Try These

**Find a single discount equivalent to each set of successive discounts.**

**1.** 40%, 20% **2.** 25%, 25% **3.** 10%, 20%, 30%

**Find a single increase equivalent to each set of successive increases.**

**4.** 40%, 20% **5.** 20%, 40% **6.** 10%, 20%, 30%

**7. Discuss and Write** *True* or *False*: A decrease of 10% followed by an increase of 10% does not change the starting value. Explain your reasoning.



Go to PRACTICE BOOK page 227 for exercise sets.

# Test Prep: Extended-Response Questions

## Strategy: Answer All Parts

Extended-response questions often contain more than one part and may require several steps to complete each part. *Show or describe your steps* to help to demonstrate your understanding.

### Sample Test Item

April went to a shopping mall. She spent \$28.95 on a jacket, \$12.75 on a belt, and \$6.59 on socks, before tax. The tax rate was 7.5%.

**Part A**
April had a coupon for 20% off the price of the jacket. What was the discounted price of the jacket, before tax?

**Part B**
What was the total cost of April's purchases, including the coupon and tax?
***Show all your work.***

**Read the whole test item carefully.**

- Reread the test item. Use context clues to help determine the meaning of any unfamiliar words.
- Determine the steps needed to solve each part.
  1. Find the amount of discount, then find the discounted price.
  2. Find the total cost of the items before tax, then find the tax and add it to the total.

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply appropriate rules, definitions and formulas.

To solve **Part A**, find the amount of discount.

**Think**
Discount = Rate of Discount • List Price

$20\% \bullet \$28.95 = 0.20 \bullet 28.95 = \$5.79$
Subtract the discount from the original price.
$$\$28.95 - \$5.79 = \$23.16$$
**Answer:** The discounted price of the jacket was \$23.16.

To solve **Part B**, add the prices before tax.
$$\$23.16 + \$12.75 + \$6.59 = \$42.50$$
Then find the amount of tax.

**Think**
Sales Tax = Rate of Tax • Marked Price

$7.5\% \bullet \$42.50 = 0.075 \bullet 42.50 = \$3.1875 \approx \$3.19$
Add the tax to the total of the items purchased.
$$\$42.50 + \$3.19 = \$45.69.$$
**Answer:** The total cost was \$45.69.

### Item Analysis

**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Do they make sense?
  Estimate amount of discount and new price.
  $$20\% \text{ of } \$30 = \$6$$
  $$\$30 - \$6 = \$24.$$

The discounted price is reasonable.

Estimate the total price of the items, before tax.
$$\$20 + \$10 + \$10 = \$40$$
Estimate the tax and add to total.
$$10\% \text{ of } \$40 = \$4$$
$$\$40 + \$4 = \$44$$
The total cost is reasonable.

**Try These** Item 1 is partially worked out for you.

**Solve. Show or describe your steps.**

1. At 6 A.M., the temperature at the Ski Resort was −6°F. By noon, the temperature had risen 25°F. At 10 P.M., the temperature was −14°F.

**Part A**
What was the temperature at noon?
***Show all your work.***

**Part B**
What was the temperature change from noon to 10 P.M.?
***Show all your work.***

**Part C**
What was the temperature change from 6 A.M. to 10 P.M.?
***Show all your work.***

**Read the test item for a general idea of the problem.**

- Reread the test item. Use context clues to help determine the meaning of any unfamiliar words.
- Determine the steps needed to answer the problem.
  1. Find the new temperature.
  2. Determine changes in temperature.

**Solve the problem.**

- Apply appropriate rules, definitions, properties, and strategies.

To solve **Part A**, use the temperature at 6 A.M. and the amount of change to find the temperature at noon.

**Remember:** Watch for signs when adding and subtracting positive and negative numbers.

Use your answer from **Part A** to solve **Part B**.

To solve **Part C**, think about the direction of the temperature change.

### Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the items.**

- Analyze your answers. Do they make sense? Check by working backward.

2. Greg is putting a fence around a square garden with a side length of 9 feet. The fence costs $15.50 per foot.

**Part A**
What is the cost, before tax, of the fencing Greg needs for the square garden?
***Show all your work.***

**Part B**
If there is a sales tax of 6% on the fencing, what is the total cost for the fence?
***Show all your work.***

Go to PRACTICE BOOK pages 229–230 for exercise sets.

# CHAPTER 8

# Data Analysis and Statistics

## In This Chapter You Will:

- Use sampling to conduct a survey
- Find the range and the measures of central tendency of a data set
- Make and interpret graphs—including bar graphs, line graphs, histograms, stem-and-leaf plots, box-and-whisker plots, Venn diagrams, and scatter plots
- Find the line of best fit for a scatter plot
- Identify what type of correlation is shown by the data in a scatter plot
- Recognize misleading graphs and statistics
- Use spreadsheets to make graphs
- Review problem-solving strategies
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- A percent (%) is a ratio or comparison of a quantity to 100.
- The Percent Formula is: rate ($r$) • base ($b$) = percentage ($p$).
- Percent change = $\dfrac{\text{amount of change}}{\text{original amount}}$
- In an ordered pair, the first number indicates the position of a point with respect to the horizontal ($x$) axis. The second number locates that point with respect to the vertical ($y$) axis.


**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 235–270

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com


- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES


## Critical Thinking

Bolle Motors established a monthly sales goal for each district manager. In the first month, the goal for Frank's district was to sell 500 cars; his district actually sold 400 cars. Patty's goal was to sell 300 cars; her district actually sold 225 cars. Ray's goal was to sell 400 cars; his district actually sold 340 cars. Which manager's district performed best in terms of meeting the greatest percent of its sales goal? Explain your answer.

8-1

# Samples and Surveys

**Objective** **To use sampling to conduct a survey • To make and use cumulative frequency tables to organize data • To use a sample to predict data for an entire population**

Alonzo works for a city planner. He wants to find out what the city's voters think of the mayor's plan to put bicycle lanes on most streets. Since it is impractical to question every citizen, how can Alonzo gather this information?

► To gather information about the opinions of a large group of people, Alonzo can conduct a survey of a sample of that population.

A survey is an examination of public opinion, attitudes, or behavior. Alonzo's survey will consist of finding out what people think about having bicycle lanes on most streets.

A population is a group of people or things. The population for Alonzo's survey is all city residents who are registered voters. Questioning each voter would be impractical, so Alonzo needs to question a part of that larger population, called a sample. A sample is said to be a representative sample if it has characteristics similar to the entire population.

Two kinds of samples are *random samples* and *convenience samples.*

In a random sample, each member or part of the population or total group has an equally likely chance of being chosen.

Example: every third person walking along a main thoroughfare

In a convenience sample, members of a population or total group are chosen because they are readily available.

Example: all 20 people standing at a bus stop

A random sample is more likely to fairly represent a population than a convenience sample. So for Alonzo to gather the most useful information about what voters think of bicycle lanes, he should survey a random sample of that population.

► Samples may or may not be representative of a population. If a sample of a population is carefully chosen, it shares that population's characteristics. If the sample is *not* representative of a population, the conclusions drawn from it may be erroneous.

Erroneous results may derive from a bias in the survey or a flaw in the sampling procedure. A bias is anything that favors a particular outcome. Examples of bias can include the way a question is asked or the way a sample is chosen. A biased sample will not be representative of the population under study.

Alonzo's results would be biased if he surveyed only people shopping in a bicycle store. The results would also be biased if he asked a registered voter a question such as, "What would you think of devoting space to bicycle lanes on our already crowded and narrow city streets?"

► A frequency table shows the number of times each type of answer occurs. To organize data in a frequency table:

1. List all the possible responses.
2. Use tally marks to record the data.
3. Count and record the tally marks (called the frequency of each response).
4. Add the frequencies to keep a running total of the number of all the responses (called the cumulative frequency).

This frequency table shows Alonzo's findings after surveying 80 voters.

| Question: Should bicycle lanes be placed on most streets? | | | |
|---|---|---|---|
| Response | Tally | Frequency | Cumulative Frequency |
| Yes | HHT HHT HHT HHT HHT HHT HHT HHT HHT III | 48 | 48 |
| No | HHT HHT HHT HHT | 20 | 68 |
| No Opinion | HHT HHT II | 12 | 80 |

The frequency table shows that 48 of 80 people surveyed favor the Mayor's plan.

► Proportional reasoning can be used to make predictions for a population if the data are based on carefully chosen samples.

PROBLEM-SOLVING STRATEGY 6

How many "Yes" votes would you predict there would be in a voting population of 4000?

Let $n$ = the predicted number of "Yes" votes in a voting population of 4000.

$\frac{48}{80} = \frac{n}{4000}$ ← Write a proportion.

$48 \bullet 4000 = 80 \bullet n$ ← Cross multiply.

$\frac{192{,}000}{80} = \frac{80 \bullet n}{80}$ ← Divide both sides by 80 to isolate $n$.

$n = 2400$

**Think**

$\frac{\text{"Yes" votes in sample}}{\text{Size of sample}} = \frac{\text{"Yes" votes in population}}{\text{Size of population}}$

So you could predict that 2400 people in a voting population of 4000 would vote "Yes."

## Try These

**Choose the sample that is more likely to be representative of the population.**

**1.** To survey teens about their music preferences, you would question several teens:
**a.** attending a classical choir recital. **b.** as they enter a music store.

**2.** Write and solve a proportion to predict the number of "No" votes in a voting population of 500. Use the frequency table at the top of this page.

**3.** In a random sample of 60 shoppers, 12 said that they only buy products that are on sale. If there are 3000 shoppers at the mall today, about how many would you predict only buy items on sale?

**4.** **Discuss and Write** How would you sample the students in your school to accurately find out the name of their favorite professional sports team?

Go to PRACTICE BOOK Lesson 8-1 for exercise sets.

# 8-2 Measures of Central Tendency and Range

**Objective** To find median, mean, mode, and range • To choose the best measure of central tendency to represent a set of data • To find a missing value of a set of numbers, given its mean, median, or mode

The salaries of five starting players on a professional basketball team are \$650,000, \$580,000, \$710,000, \$780,000, and \$540,000. How can you best represent this set of data?

► To find how you can best represent this set of data, choose one of the *measures of central tendency* or use the *range* of that set of data.

The **measures of central tendency** are *mean*, *median*, and *mode*.

- The **mean** is a type of statistical average. To find the mean of a set of data, divide the sum of all the numbers in the set by the number of items. The mean is most useful when the data are close together.

$$\frac{\$650{,}000 + \$580{,}000 + \$710{,}000 + \$780{,}000 + \$540{,}000}{5} = \frac{\$3{,}260{,}000}{5} = \$652{,}000$$

The mean of the salaries is \$652,000.

- The **median** is the middle value or average of two middle values of a set of data that are arranged in order. To find the median, list the values in order, and identify the middle value. For a set with an even number of values, the median is the mean of the two middle numbers. The median is most useful when the data contain extremely high or extremely low values.

\$540,000 \$580,000 \$650,000 \$710,000 \$780,000 ← You can list either from least to greatest or greatest to least.

The median salary is \$650,000.

- The **mode** is the item that occurs most frequently in the data. A data set may have no mode, one mode, or more than one mode. When there is no value that repeats, there is no mode. The mode is most useful when many items in the data set have the same value. The mode is the only appropriate measure for non-numeric data—for example, when considering the votes for favorite school mascot.

- The **range** is the difference between the greatest and least values in a data set.

\$780,000 − \$540,000 = \$240,000 → The range of the salaries is \$240,000.

► Certain measures of central tendency sometimes represent the data better than others. The measure(s) closest to most of the data most accurately represent(s) that data.

- The mean, \$652,000, is \$128,000 less than the greatest salary and \$112,000 more than the least salary, so the salaries are fairly close together. The mean is appropriate in representing this data set.
- Since the values do not repeat, there is no mode.
- The range does not tell you what most of the players are being paid.
- The median, \$650,000, is close to most of the data values and is appropriate in representing the data set.

► You can use an equation to find a missing item in a data set.

**Remember:**
mean $= \frac{\text{sum of data}}{\text{number of data items}}$

PROBLEM-SOLVING STRATEGY 2

In 4 games, Neala has scored 11, 18, 22, and 10 points. To average 15 points per game, how many points does she need to score in her next game?

Write and solve an equation.
Let $n$ = points Neala needs to score in the 5th game.

**Think**
$\frac{\text{total number of points}}{\text{5 games}} = \frac{\text{15 points}}{\text{game}}$

$\frac{11 + 18 + 22 + 10 + n}{5} = 15$ ← Use the definition of *mean*.

$\frac{61 + n}{5} = 15$ ← Add the number values.

$5 \bullet \frac{(61 + n)}{5} = 5 \bullet 15$ ← Multiply both sides by 5 to isolate the term containing $n$.

$(61 + n) = 75$

$61 + n - 61 = 75 - 61$ ← Subtract 61 from both sides to isolate $n$.

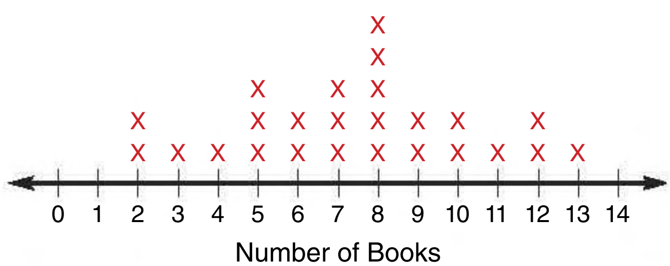
$n = 14$

So Neala needs to score 14 points in her next game to average 15 points per game.

► A line plot is especially useful for showing the mode and range of a set of data. A line plot is a graph that displays data with X marks above each data value on a number line.

This set of data is shown in the line plot at the right.
2, 8, 8, 3, 7, 13, 5, 9, 8, 6, 4, 7, 2, 6, 5, 12, 10, 12, 8, 11, 9, 10, 8, 5, 7

**Number of Books Read for Pleasure**




The greatest number of Xs is above the 8, so the mode is 8.
The least number of books read is 2, and the greatest number is 13.
So the range is 13 − 2, or 11, books.

## Try These

**Find the mean, median, mode, and range for each data set.**

**1.** 5.2, 8.4, 4.3, 6.7, 5.8

**2.** −2, −13, −13, −5, −7

**3.** 8, 6, $6\frac{1}{2}$, $8\frac{1}{2}$, $7\frac{1}{2}$, 7, 8, $5\frac{1}{2}$, 7, 8

**Solve.**

**4.** Add a number to the following data set that makes the median 17.
15, 18, 20, __?__

**5.** Remove a number from the data set so that the mode is 73.
54, 62, 73, 78, 62, 81, 73, 60

**6.** In the first 7 years of his career, a baseball player had 190, 186, 182, 190, 200, 198, and 184 hits. For the mean, median, and mode to remain the same after his 8th year, how many hits would he have to get in that year?

**7. Discuss and Write** Use the line plot in this lesson to explain how a line plot shows the frequency of data.

Go to PRACTICE BOOK Lesson 8-2 for exercise sets.

8-3

# Interpret Data

**Objective** To examine how outliers in a set of data affect the mean, median, and mode • To use range as a measure of dispersion • To use statistics to make estimates • To organize data using a spreadsheet

Six students are taking a film class. The table shows how many films each student has seen so far this year. One number greatly differs from the others. How does it affect the statistical measures you would use to describe the data set?

| Name | Number of Films Seen |
|---|---|
| Elena | 11 |
| Ray | 13 |
| James | 8 |
| Inez | 104 |
| Li | 16 |
| Lamar | 10 |

► To find how one number affects the statistical measure of a set of data, find the mean, median, mode, and range with and without that number.

► Numbers in a data set that are much greater or much less than others in the set are called outliers. An outlier is an extreme value.

The outlier for the number of films watched is 104.

Outliers affect some measures of central tendency more than others.

- Outliers have the greatest effect on the mean and range of a data set.
- The median gives the best description of a data set that contains an outlier.

**To find the mean:**

**With outlier**

$$\frac{11 + 13 + 8 + 104 + 16 + 10}{6} = \frac{162}{6} = 27$$

The mean is 27.

**Without outlier**

$$\frac{11 + 13 + 8 + 16 + 10}{5} = \frac{58}{5} = 11.6$$

The mean is 11.6.

The outlier increases the mean of the data by 15.4.

**To find the median:**

**Remember:** The median of an even set of numbers is the mean of the two middle numbers.

**With outlier**

8, 10, 11, 13, 16, 104

$$\frac{11 + 13}{2} = 12$$

The median is 12.

**Without outlier**

8, 10, 11, 13, 16

The median is 11.

The outlier increases the median of the data by 1.

If the outlier is one extreme value, it can never affect the mode because it would not occur more often than any other value in the data set. Therefore, the outlier does not change the mode of the data.

**To find the range:**

**With outlier**

$104 - 8 = 96$

The range is 96.

**Without outlier**

$16 - 8 = 8$

The range is 8.

The outlier increases the range of the data by 88.

► Statistics that indicate how data are spread out or distributed are called measures of dispersion or measures of variation. The range is one kind of description of how data are spread out. It is not a measure of central tendency because it does not represent the data values.

Sets $A$ and $B$ below have the same range but different means, medians, and modes.

- Set $A$: 4, 12, 25, 36, 40, 41, 41, 44 → Mean: 30.375  Median: 38  Mode: 41
  $44 - 4 = 40$ ← range
- Set $B$: 27, 28, 28, 29, 30, 50, 67 → Mean: 37  Median: 29  Mode: 28
  $67 - 27 = 40$ ← range

► You can use the measures of central tendency to make estimates. Below are some examples.

- DVD rental services pay close attention to customer preferences. They can estimate how many of each DVD to keep in stock by using the *mode* to order the movies people see most often.
- Statisticians use the *median* to compare how the prices of houses change from year to year or over a period of years.
- Teachers use the *mean* score on quizzes and tests to evaluate a student's performance over a period of time.

## Technology Organize Data in Spreadsheets

► You can use a spreadsheet to organize and analyze data. Spreadsheets contain numbered rows and lettered columns. A cell is where a row and a column intersect.

What is the value of cell C4? What does that value represent?

The value in cell C4 is 10.7. It represents the mean number of DVDs, in thousands, rented per month in 2006.

Mean Monthly Rentals (thousands)

| | A | B | C |
|---|---|---|---|
| 1 | Year | Videos | DVDs |
| 2 | 2004 | 12.1 | 8.9 |
| 3 | 2005 | 9.7 | 10.3 |
| 4 | 2006 | 7.8 | 10.7 |
| 5 | 2007 | 5.6 | 12.8 |
| 6 | 2008 | 2.9 | 14.4 |
| 7 | | | |

Sheet 1 / Sheet 2

column
cell C4
row

## Try These

**Identify the outlier in each data set. Then find the mean, median, and mode for the data set both with and without the outlier.**

**1.** 2, 7, 13, 5, 13, 32

**2.** 8, 9, 8, 10, 41, 10, 9, 9

**3.** 0.3, 0.8, 0.8, 0.5, 3.9, 0.8, 0.8, 0.9

**4.** Discuss and Write How does an outlier affect the mean, median, mode, and range of a data set?

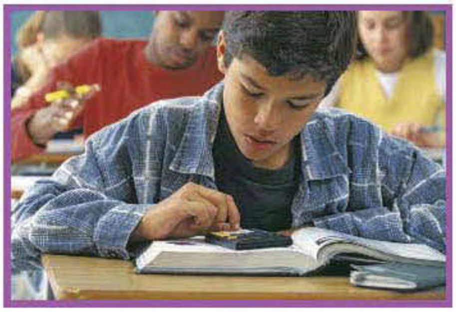
Go to PRACTICE BOOK Lesson 8-3 for exercise sets.

Update your skills. See pages 415 XIX, 416 XX.

# 8-4 Choose an Appropriate Graph

**Objective** To select and use appropriate types of graphs to display data

Manuel surveyed his math class to find the favorite sport of the 20 students in the class. The frequency table below shows the data he collected. What type of graph would be appropriate for Manuel to use in displaying the data? Create a graph to display the data in the table.

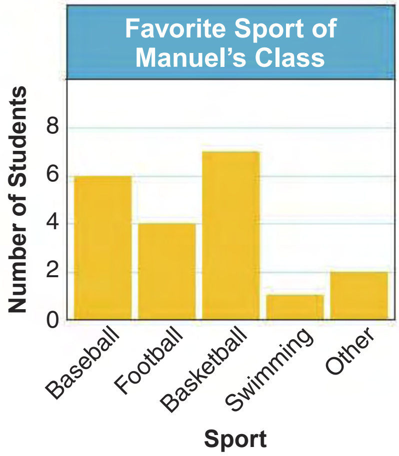


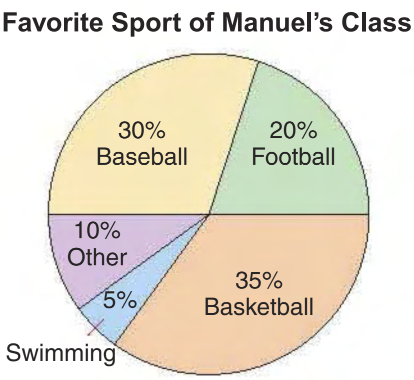
| Favorite Sport | | |
|---|---|---|
| **Sport** | **Tally** | **Frequency** |
| Baseball | ~~IIII~~ I | 6 |
| Football | IIII | 4 |
| Basketball | ~~IIII~~ II | 7 |
| Swimming | I | 1 |
| Other | II | 2 |

► To find an appropriate type of graph that Manuel could use to display the data, consider various types of graphs and their purposes.

- **Bar Graph:** Uses the length or height of bars in relation to a scale of equal intervals to *compare* two or more sets of data.
- **Line Graph:** Uses the location of points to display *changes* that occur in a data set over time. Since the data do not show change over time, a line graph would not be an appropriate way to display the data in the table.
- **Circle Graph:** Shows how a whole is divided into parts. The parts must total 100%.

The data in the table do not show change over time, so a line graph is not an appropriate way to display the data. The bar graph and the circle graph are appropriate ways to display the data.



## Example

**1** Each year for the past 5 years, Manuel's teacher has asked a student to survey the students in the math class about their favorite sport. The chart below shows the data that the teacher recorded about baseball as the favorite sport. What kind of graph would be appropriate for displaying how the percent of students choosing baseball changed over the past 5 years? Create an appropriate graph to display the data.

| 5-Year Survey Data | | |
|---|---|---|
| Year | Number of Votes | Number of Students in the Class |
| 1 | 10 | 20 |
| 2 | 7 | 28 |
| 3 | 8 | 25 |
| 4 | 12 | 25 |
| 5 | 6 | 20 |

Since the graph will show a change over time, a line graph would be the most appropriate way to display the data.

Each ratio must be expressed as a percent.

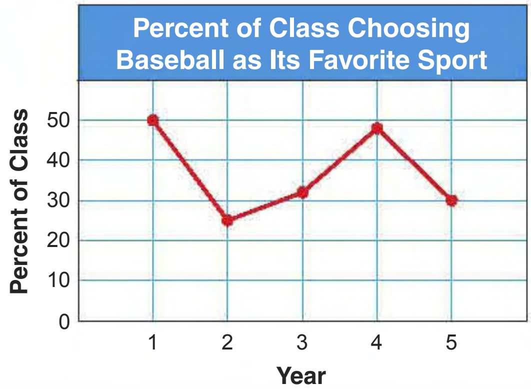
Year 1: $\frac{10}{20} = \frac{1}{2} = 50\%$

Year 2: $\frac{7}{28} = \frac{1}{4} = 25\%$

Year 3: $\frac{8}{25} = \frac{8 \bullet 4}{25 \bullet 4} = \frac{32}{100} = 32\%$

Year 4: $\frac{12}{25} = \frac{12 \bullet 4}{25 \bullet 4} = \frac{48}{100} = 48\%$

Year 5: $\frac{6}{20} = \frac{6 \bullet 5}{20 \bullet 5} = \frac{30}{100} = 30\%$




## Try These

**Which type of graph(s) would be most appropriate for displaying the data that would be collected to answer each question?**

1. Which of the five books read in English class this year is the one the students liked best?

2. Does the number of customers in a store vary at different times of the day?

3. What is the cost of an adult movie ticket at several different theaters?

4. **Discuss and Write** Consider the problem at the beginning of the lesson. Suppose that there are 25 students in Manuel's math class and Manuel surveys 20 of them. Would it still be appropriate to use a circle graph to display the results? Why or why not?

Go to PRACTICE BOOK Lesson 8-4 for exercise sets.

8-5

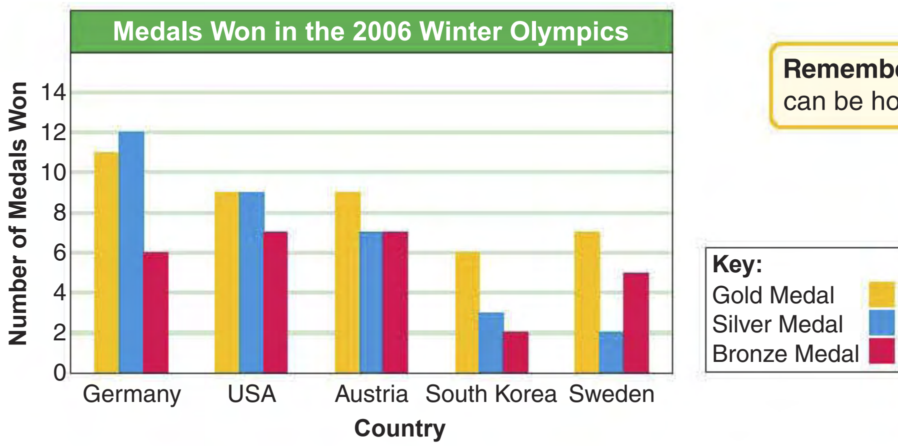
Update your skills. See page 415 XIX.

# Multiple Bar Graphs

**Objective** To read and interpret multiple bar graphs

The bar graph below shows the number of gold, silver, and bronze medals won by each of five countries in the 2006 Winter Olympics. The number of gold medals won by each country is what percent of all the medals won by the same country? For which country was that percent greatest?

► When data fall into categories such as different countries, a bar graph is a good way to display the data. Use the vertical multiple bar graph to find the ratio of the number of gold medals won by any given country to the total number of medals won by that country. Then find and compare the percents.




**Remember:** The bars of a bar graph can be horizontal or vertical.

A multiple bar graph compares related sets of data.

► To find the percent of gold medals won by each country:

1. Find the number of gold medals and the total number of medals each country won.
2. Write a ratio expressing the number of gold medals won to the total number of medals won by each country.
3. Divide. Write each ratio as a percent.

| Country | Gold Medals | Total Medals |
|---|---|---|
| Germany | 11 | 11 + 12 + 6 = 29 |
| USA | 9 | 9 + 9 + 7 = 25 |
| Austria | 9 | 9 + 7 + 7 = 23 |
| South Korea | 6 | 6 + 3 + 2 = 11 |
| Sweden | 7 | 7 + 2 + 5 = 14 |

Add the number of gold, silver, and bronze medals won by each country to find the total.

Germany: $11 : 29 \rightarrow \frac{11}{29} \approx 0.38 \rightarrow 38\%$

USA: $9 : 25 \rightarrow \frac{9}{25} = 0.36 \rightarrow 36\%$

Austria: $9 : 23 \rightarrow \frac{9}{23} \approx 0.39 \rightarrow 39\%$

South Korea: $6 : 11 \rightarrow \frac{6}{11} \approx 0.55 \rightarrow 55\%$

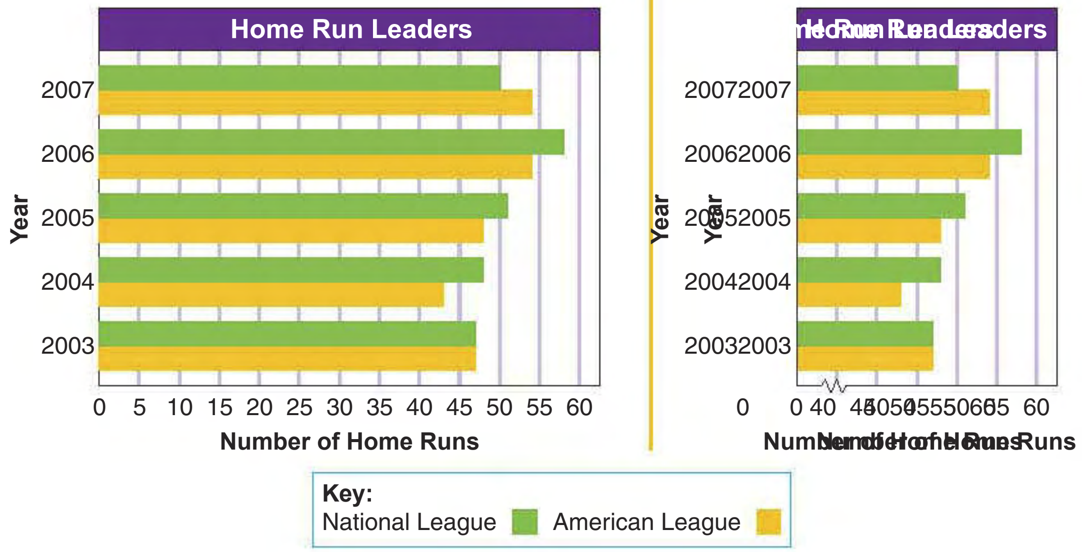
Sweden: $7 : 14 \rightarrow \frac{7}{14} = 0.5 \rightarrow 50\%$

So of the countries shown, the country for which the number of gold medals won was the greatest percent of all medals won by that country was South Korea.

► A horizontal bar graph has the scale along the horizontal axis.

This double bar graph shows the number of home runs hit by league leaders from 2003 to 2007.

The graph below shows the same data, but the break in the scale allows the graph to occupy less space.




In what year did the leaders in both leagues hit the same number of home runs? Explain how you can tell.

- 2003; the bars for both leagues are the same length for 2003.

For the years shown, in which year and in which league did the league leader hit the greatest number of home runs? Explain.

- 2006, National League; the longest bar on the graph is the National League bar for 2006.

Which league's leader had the greater number of home runs most often?

- National League; the National League leader had more home runs than the American League leader in 3 of the 5 years shown.

## Try These

**Use the graph to answer the questions.**

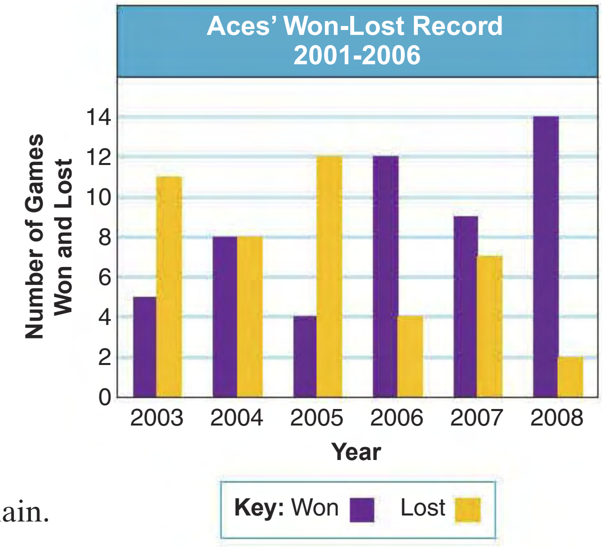
**1.** In what year did the Ashton Aces lose the same number of games as they won? In what year did they lose about twice as many games as they won?

**2.** What is the range of the data for wins and losses during this 6-year period?

**3.** What is the median number of wins? What is the median number of losses?

**4.** Summarize the data that the graph shows.

**5.** Discuss and Write Which measure(s) of central tendency are easiest to find from a bar graph? Explain.



# 8-6 Histograms

**Objective** **To make a frequency table using time intervals • To make a histogram from a frequency table • To read a histogram**

The tally chart below shows the results of a survey about the number of customers at a store during each 2-hour interval of the day from 10:00 A.M. to 8:00 P.M. How can the results be shown in a graph?

| Time Interval | Number of Customers |
|---|---|
| 10:00–11:59 A.M. | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 \|\| |
| 12:00–1:59 P.M. | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |
| 2:00–3:59 P.M. | 𝍸 𝍸 𝍸 𝍸 \|\|\|\| |
| 4:00–5:59 P.M. | 𝍸 𝍸 𝍸 \|\| |
| 6:00–7:59 P.M. | 𝍸 𝍸 𝍸 \|\|\|\| |

► To display the data shown in the tally chart visually in a graph, first make a frequency table from the data. Then make a *histogram*.

A histogram is a graph that shows frequencies of data within equal intervals. Unlike the bars on a bar graph, the bars on a histogram are next to each other without a gap, unless there is an interval that has a frequency of 0.

To make a frequency table, group the data into equal intervals. The intervals will be shown on the horizontal axis of the histogram.

The frequency table below shows the number of customers that visit the store in a 10-hour business day that is divided into 2-hour intervals.

| Interval | Frequency |
|---|---|
| 10–11:59 | 42 |
| 12–1:59 | 35 |
| 2–3:59 | 24 |
| 4–5:59 | 17 |
| 6–7:59 | 19 |

The scale along the vertical axis of the histogram will include the least data value and the greatest data value, and it will be divided into equal intervals.

Using a scale with intervals of 2 would require a very tall graph. Using a scale with intervals of 10 would not show the changes in frequency clearly enough. Using intervals of 5 results in a scale from 0 to 45.

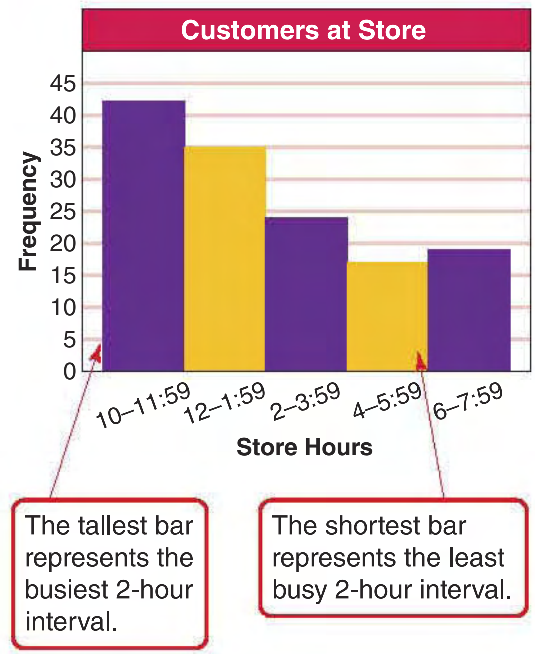
► To make a histogram:

1. Draw and label the horizontal axis and the vertical axis. The horizontal axis shows the intervals; the vertical axis shows the frequencies.
2. Use the least and greatest values in the data to choose a sensible scale for the frequencies. Use intervals of the same size throughout the scale.

3 Label equal spaces along the horizontal axis.

4 Draw bars without any gaps in between to show the frequency for each interval of data. Do not omit any interval, even if an interval has a frequency of 0.

5 Write a title for the graph.




The histogram clearly shows the change in volume of customer traffic throughout the day.

- During which time interval is the volume of customer traffic about twice as great as it is during the interval from 4:00 P.M. to 5:59 P.M.? Explain how you know.

  The volume of customer traffic is about twice as great from 12:00 to 1:59 P.M. The top of the bar for the interval from 4:00 P.M. to 5:59 P.M. is at about 17, and the top of the bar for the interval from 12:00 to 1:59 P.M. is at 35, which is about twice as great as 17.

## Try These

The frequency table shows how many people, by age group, shopped for musical instruments at a single store during one week.

**Use the data in the table to make a histogram. Then answer the questions.**

| Age Group | Frequency |
|---|---|
| 10–19 | 84 |
| 20–29 | 68 |
| 30–39 | 40 |
| 40–49 | 12 |
| 50–59 | 5 |
| 60–69 | 2 |
| 70–79 | 0 |

1. Which age group purchased the greatest number of musical instruments? How does the histogram show this?

2. What percent of shoppers who shopped at the store during that week are ages 50 to 59?

3. What is the range of the frequencies?

**Make a frequency chart and histogram for the given data.**

4. A newspaper listed the top 20 best-selling novels and indicated how many weeks each had been on the best-sellers list. The number of weeks on the list for those novels is as follows:

   7, 23, 38, 1, 2, 2, 5, 45, 14, 41, 28, 16, 17, 19, 9, 13, 6, 5, 9, 11

   To group the number of weeks, use the categories 1–10, 11–20, 21–30, 31–40, and 41–50. Which grouping contains the median number of weeks on the best-sellers list?

5. **Discuss and Write** How did you use the data in your frequency chart in question 4 to construct the histogram?

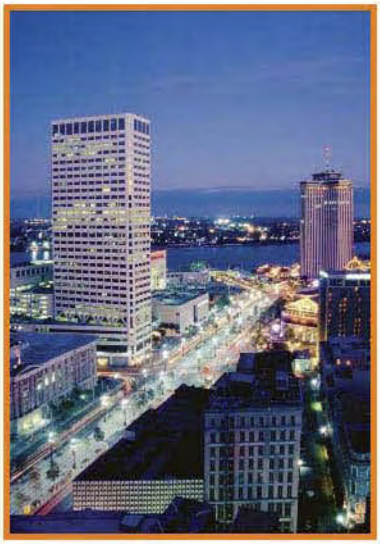
Go to PRACTICE BOOK Lesson 8-6 for exercise sets.

# 8-7 Stem-and-Leaf Plots

**Objective** To read and make stem-and-leaf plots

The table below shows the number of floors in certain tall buildings. How would you display that data in a stem-and-leaf plot?

| Building | Number of Floors |
|---|---|
| One Cherry Square | 51 |
| Newbanc Center | 53 |
| Plaza Tower | 45 |
| City Centre | 39 |
| TMP Tower | 36 |
| Deluxe Suites Hotel | 47 |
| One Founders Row | 32 |
| Peninsula Plaza | 31 |



A stem-and-leaf plot is a display that uses the digits of the numbers in a data set to show how the data are distributed. It condenses and orders the data set to let you see the values and frequencies. It is especially useful in making a large data set easier to analyze.

A stem-and-leaf plot is two sets of digits separated by a vertical line. The "stem" is written to the left of the line, and the "leaves" are written to the right of the line. Each digit in a data value that has the least place value is represented by a leaf. The remaining digit in each data value is represented by a stem. A stem-and-leaf plot shows every item in the data.

**Number of Floors in Buildings**

| | Stem | Leaves | |
|---|---|---|---|
| tens digits | 3 | 1 2 6 9 | ones digits |
| | 4 | 5 7 | |
| | 5 | 1 3 | |

**Key:** 3|1 = 31 floors

► To make a stem-and-leaf plot:

1. Order the numbers from least to greatest.
   31, 32, 36, 39, 45, 47, 51, 53
2. Since the numbers in the table range from 31 to 53, use the tens digits for the stem and the ones digits for the leaves. Draw a vertical line to separate the stems from the leaves.
3. Write the stems in ascending order on the left side of the line.
4. To the right of each stem, write the appropriate leaf digits in ascending order from left to right. Note that the same leaf digit may appear on different stems. It will also appear more than once on the same stem if a number in the data set occurs more than once.
5. Write a key to show what the stems and leaves represent.
6. Give the stem-and-leaf plot a title.

**Key Concept**

**Stem-and-Leaf Plots**

A stem-and-leaf plot has only the last digit of each number in the data set as a leaf. The front-end digit (or group of digits) is the stem.

Since the numbers in a stem-and-leaf plot are ordered, this type of display is convenient in finding the median, the mode, and the range of a set of data.

## Example

**1** Make a stem-and-leaf plot to display the mean mathematics test scores for Ms. Perry's class of 20 students.

| Mean Mathematics Test Scores for Ms. Perry's Students | | | |
|---|---|---|---|
| 85.5 | 75.5 | 75.4 | 95.0 |
| 75.5 | 83.8 | 85.0 | 92.5 |
| 87.2 | 68.3 | 89.2 | 95.2 |
| 89.5 | 78.8 | 85.3 | 83.7 |
| 92.3 | 95.2 | 92.5 | 87.6 |

- First order the scores from least to greatest.
  68.3, 75.4, 75.5, 75.5, 78.8,
  83.7, 83.8, 85.0, 85.3, 85.5, 87.2, 87.6,
  89.2, 89.5, 92.3, 92.5, 92.5, 95.0, 95.2, 95.2
- Give the stem-and-leaf plot a title.
- Use the ones and tens for the stem and the tenths for the leaves.
- Write a key to show the relation of digits.

**Mean Mathematics Test Scores for Ms. Perry's Students**

| Stem | Leaves |
|---|---|
| 68 | 3 |
| 75 | 4 5 5 |
| 78 | 8 |
| 83 | 7 8 |
| 85 | 0 3 5 |
| 87 | 2 6 |
| 89 | 2 5 |
| 92 | 3 5 5 |
| 95 | 0 2 2 |

**Key:** 68|3 = 68.3

Two sets of data can be compared using a back-to-back stem-and-leaf plot. This type of plot shows leaves on both sides of the stem. The plot below shows data about buildings in two cities.

**Number of Floors in Buildings**

| City 1 Leaves | Stem | City 2 Leaves |
|---|---|---|
| 9 6 3 2 1 | 3 | 2 2 6 7 |
| 7 5 2 | 4 | 0 2 3 |
| 3 1 | 5 | 0 2 6 |

**Key:** 51 floors ← 1|5|0 → 50 floors

Both cities have a 42-floor building.
City 1's tallest building has 53 floors.
City 2's tallest building has 56 floors.

## Try These

The ages of the first 10 American presidents at the start of their presidencies are given at the right.
**Use the data to make a stem-and-leaf plot.**
**Then answer the questions.**

| Ages of Presidents | |
|---|---|
| George Washington | 57 |
| John Adams | 60 |
| Thomas Jefferson | 57 |
| James Madison | 57 |
| James Monroe | 58 |
| John Quincy Adams | 57 |
| Andrew Jackson | 62 |
| Martin Van Buren | 54 |
| William H. Harrison | 68 |
| John Tyler | 51 |

**1.** What are the range and mode of the data?

**2.** How many of the presidents were at least 60 years old at the start of their presidencies?

**3.** What is the median age of the first 10 presidents at the start of their presidencies?

**4.** **Discuss and Write** Describe how you would make a stem-and-leaf plot using the numbers 2.6, 2.8, 1.4, 3.6, 1.9, 2.2, and 3.2.

Go to PRACTICE BOOK Lesson 8-7 for exercise sets.

# 8-8 Box-and-Whisker Plots

**Objective** **To read and make box-and-whisker plots • To determine and interpret clusters, quartiles, gaps, and outliers of data • To compare two box-and-whisker plots using the same number line**

The table at the right shows how many minutes it took each of 20 students to complete a math quiz. How could this data be displayed in a box-and-whisker plot?

| Length of Time in Minutes | | | | | |
|---|---|---|---|---|---|
| 8.8 | 7.5 | 9.5 | 8.4 | 9.4 | 6.4 |
| 10.1 | 6.7 | 8.2 | 10.5 | 7.6 | 7.3 |
| 10.5 | 9.0 | 6.5 | 7.9 | 8.9 | 8.8 |
| 6.8 | 7.1 | | | | |

A box-and-whisker plot is a graph that shows how the data in a set are distributed, but it does *not* show all the values in the data set. It summarizes the spread of data by dividing them into fourths along a number line. The median of the data separates the data into halves. The quartiles are values that divide the data into fourths. The median of the lower half is the lower quartile of the data, and the median of the upper half is the upper quartile of the data.

**Remember:** The median for an even set of numbers is the mean of the two middle numbers when the set is listed in order from least to greatest or from greatest to least.

► To make a box-and-whisker plot:

1. Draw a number line that includes the least and greatest data values.
2. Determine the median, the lower quartile, and the upper quartile of the data.

6.4, 6.5, 6.7, 6.8, 7.1, 7.3, 7.5, 7.6, 7.9, 8.2, 8.4, 8.8, 8.8, 8.9, 9.0, 9.4, 9.5, 10.1, 10.5, 10.5

Median of Lower Half: $\frac{7.1 + 7.3}{2} = 7.2$
7.2 = Lower Quartile

Median: $\frac{8.2 + 8.4}{2} = 8.3$

Median of Upper Half: $\frac{9.0 + 9.4}{2} = 9.2$
9.2 = Upper Quartile

3. Locate these three points on the number line, and draw the box. The box extends from the lower quartile to the upper quartile. Draw a vertical line segment in the box through the median. The length of the box along the number line represents the spread of the middle half of the data.
4. Draw the lines ("whiskers") on the box from the upper quartile to the greatest value, called the upper extreme, and from the lower quartile to the least value, called the lower extreme.

**Key Concept**

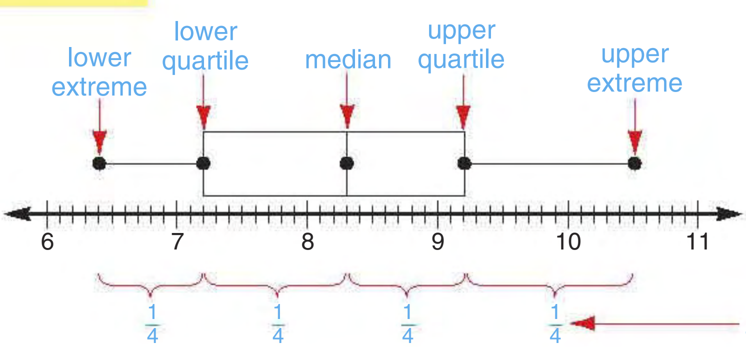
**Box-and-Whisker Plot**

upper quartile: median of the upper half of the data

lower quartile: median of the lower half of the data

upper extreme: greatest data value

lower extreme: least data value.




Each part is one fourth of the data, even though the visual may not show four parts of equal size.

The interquartile range is the difference between the upper quartile and the lower quartile. If this difference is small, then the data in the middle of the set are close or clustered. If it is large, then the data in the middle of the set are spread out and there are gaps in the data. Gaps are places on the graph with no data. One way to determine if there are any outliers, or points that are clearly separate from the set of data, is to multiply the interquartile range by 1.5.

interquartile range = upper quartile − lower quartile
= 9.2 − 7.2 = 2.0

and 1.5 • 2.0 = 3.0

So outliers are any data that are more than 3 points away from either quartile.

lower quartile − 3.0 = 7.2 − 3.0 = 4.2 ← Any data point less than 4.2 or greater than 12.2 is an outlier.
upper quartile + 3.0 = 9.2 + 3.0 = 12.2

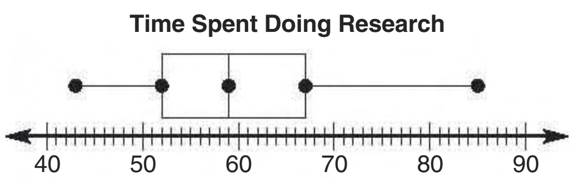
Since no points are less than 4.2 or greater than 12.2, there are no outliers.

## Example

**1** The table shows how many minutes each of 21 students spent doing research for a project. Use the data to make a box-and-whisker plot. Then interpret the data.

| Length of Time in Minutes | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 58 | 53 | 52 | 74 | 63 | 59 | 43 | 56 | 67 | 82 | 48 |
| 52 | 65 | 67 | 67 | 73 | 64 | 47 | 53 | 85 | 45 | |

43, 45, 47, 48, 52, 52, 53, 53, 56, 58, 59, 63, 64, 65, 67, 67, 67, 73, 74, 82, 85

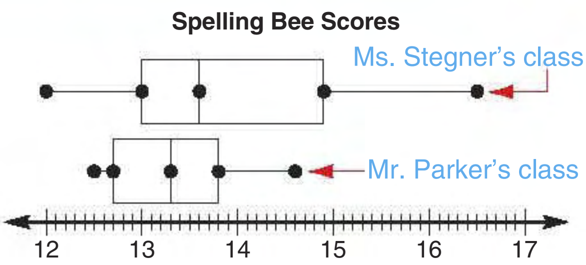



The median is 59. The upper quartile is 67. The lower quartile is 52. The upper extreme is 85. The lower extreme is 43. The interquartile range is 15.

There are no outliers: since 1.5 • 15 = 22.5; and 52 − 22.5 = 29.5 and 67 + 22.5 = 89.5. The data set does not contain any values less than 29.5 or greater than 89.5.

## Try These

The box-and-whisker plots at the right compare points scored in a spelling bee by two different groups.




**Use the plots for exercises 1–4.**

**1.** What is the median number of points scored in Ms. Stegner's class? In Mr. Parker's class?

**2.** Which class has the greater range of points scored?

**3.** For which class do the points scored cluster more closely around the median?

**4.** Discuss and Write What measure or measures of central tendency are box-and-whisker plots most useful for showing?

Go to PRACTICE BOOK Lesson 8-8 for exercise sets.

# 8-9 Venn Diagrams

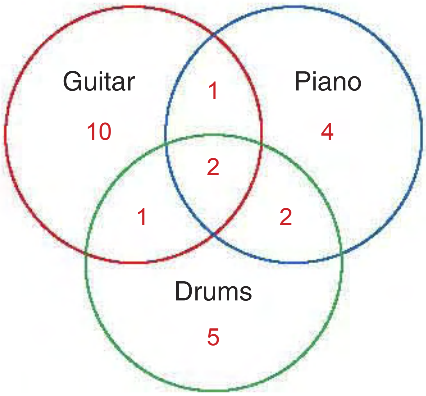
**Objective** To read and make Venn diagrams

Joy surveyed 25 music students on whether they enjoyed playing guitar, piano, or drums. The table shows the results of her survey.

How can Joy show the relationships among these data sets? How many students enjoy playing only guitar? Only piano? Only drums?

| Survey Questions | Number of Students |
|---|---|
| How many enjoy playing guitar? | 14 |
| How many enjoy playing piano? | 9 |
| How many enjoy playing drums? | 10 |
| How many enjoy playing guitar and piano? | 3 |
| How many enjoy playing guitar and drums? | 3 |
| How many enjoy playing piano and drums? | 4 |
| How many enjoy playing all three instruments? | 2 |

► To show the relationships among the data sets, you can use a *Venn diagram*. A Venn diagram is a group of two or three overlapping circles, each circle representing a single data set.




1. Draw and label three overlapping circles, one to represent each instrument.
2. Find the region where the three circles overlap. This region represents the 2 students who enjoy playing all three instruments. Write 2 in this region.
3. Four students enjoy playing both piano and drums, but 2 of them like all three. So subtract those from 4. Write 2 in the region shared by the circles labeled piano and drums.
4. Three students enjoy playing guitar and drums, but 2 of them enjoy all three instruments. So subtract those from 3. Write 1 in the region shared by the circles labeled guitar and drums.
5. Three students enjoy playing guitar and piano, but 2 of them enjoy all three instruments. So subtract those from 3. Write 1 in the region shared by the circles labeled guitar and piano.
6. Ten students enjoy drums, but 2 like all three types, 2 enjoy piano and drums, and 1 enjoys guitar and drums. So subtract $(2 + 2 + 1)$, or 5, from 10 to find the number who enjoy drums only. Write 5 in the region labeled drums.
7. Nine students enjoy piano, but 2 like all three types, 2 enjoy piano and drums, and 1 enjoys piano and guitar. So subtract $(2 + 2 + 1)$, or 5, from 9 to find the number who enjoy drums only. Write 4 in the region labeled piano.
8. Fourteen students enjoy guitar, but 2 like all three types, 1 enjoys guitar and piano, and 1 enjoys guitar and drums. So subtract $(2 + 1 + 1)$, or 4, from 14 to find the number who enjoy drums only. Write 10 in the region labeled guitar.

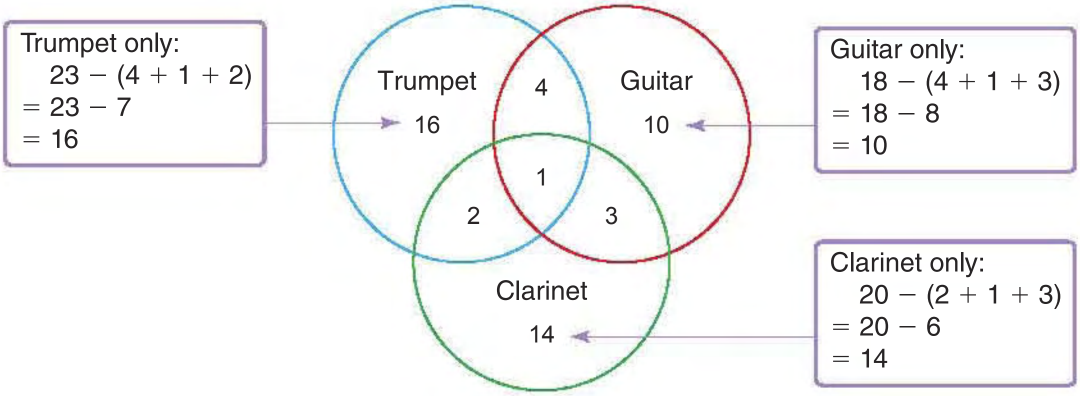
So the Venn diagram above represents the relationship among the data sets.

## Example

**1** There are 96 students at Kennedy Middle School who play instruments. Of these, 23 students in all play the trumpet; 20 students in all play the clarinet; and 18 students in all play the guitar. Of the students who play one or more of these three instruments, 4 students play the trumpet and the guitar but not the clarinet; 2 students play the clarinet and the trumpet but not the guitar; and 3 students play the guitar and the clarinet but not the trumpet. Only 1 student plays all three instruments. How many students play *only* the trumpet? How many play *only* the guitar? How many play *only* the clarinet?

To answer these questions, use a Venn diagram.

1. Draw and label three overlapping circles, one for each data set.
2. Add to find the number of trumpet players who also play one or more other instruments: $4 + 1 + 2 = 7$ students. Since the total number of trumpet players is 23, subtract 7 from 23 to find that there are 16 students who play only the trumpet. Follow this procedure for each instrument.




So 16 students play only the trumpet, 10 students play only the guitar, and 14 students play only the clarinet.

## Try These

**Draw a Venn diagram to illustrate the following.**

**1.** Al's class has 18 stamp collectors, 14 coin collectors, and 12 CD collectors. One student collects all three, 7 collect stamps and coins but not CDs, 5 collect coins and CDs but not stamps, and 5 collect stamps and CDs but not coins. How many students in Al's class collect only coins? Only stamps? Only CDs?

**2.** Show the factors of 9, 16, and 24, including their common factors.

**3.** **Discuss and Write** In Example 1 above, which numbers that are given in the problem do not appear in the Venn diagram? Explain.

Go to PRACTICE BOOK Lesson 8-9 for exercise sets.

8-10

Update your skills. See page 416 XX.

# Multiple Line Graphs

**Objective** To use multiple line graphs • To interpret trends and make predictions from line graphs

The table at the right shows the number of dinners that each of three new restaurants served on their first 6 days of business. To compare the data, represent each set of data on a line graph.

| Number of Dinners Served | | | |
|---|---|---|---|
| **Day** | **Pasta Plus** | **Gail's Grill** | **Lu's Lunch** |
| 1 | 50 | 75 | 80 |
| 2 | 60 | 90 | 85 |
| 3 | 65 | 95 | 75 |
| 4 | 75 | 120 | 60 |
| 5 | 75 | 145 | 55 |
| 6 | 80 | 155 | 55 |

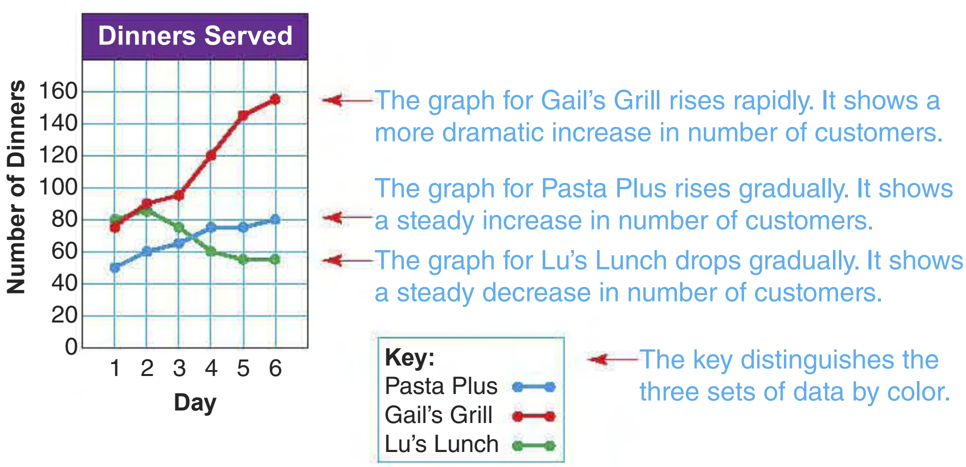
► To compare the data for the three restaurants, make a multiple line graph.

A multiple line graph compares related sets of data that change over time.

**Remember:** A line graph is used to show changes in data over a period of time. A line segment that slopes upward indicates an increase, while a line segment that slopes downward indicates a decrease. A horizontal line segment indicates no change.

► To make the multiple line graph:

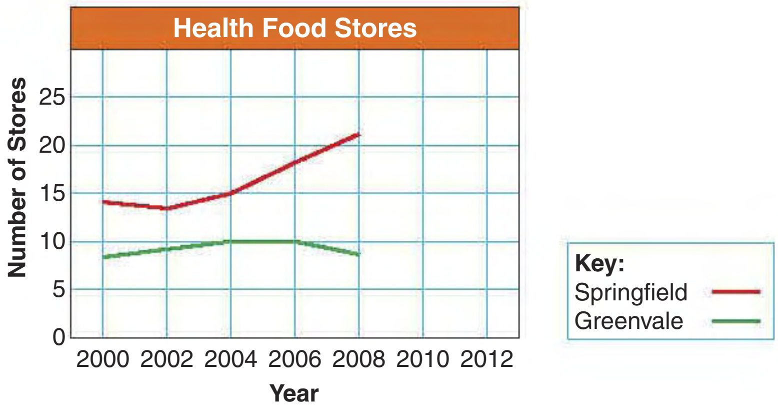
1. Write a title for the graph.
2. Draw a vertical axis and a horizontal axis.
3. List the numbers 1–6 for the days equally spaced on the horizontal axis.
4. Choose a scale with a range that will fit the data. Write the scale, starting at 0, along the vertical axis. Choose equal intervals for the scale that will show significant increases and decreases.
5. Place a point on the graph for each number of dinners served at each restaurant at the indicated time.
   - Use a line segment to connect the points for each restaurant. Use a different type of line or a different color for each restaurant.
6. Make a key to explain how to identify the restaurant represented by each line.




So the triple line graph shows that of the three new restaurants, *Gail's Grill* appears to be off to the best start.

► You can make predictions about the trends you see in a line graph.

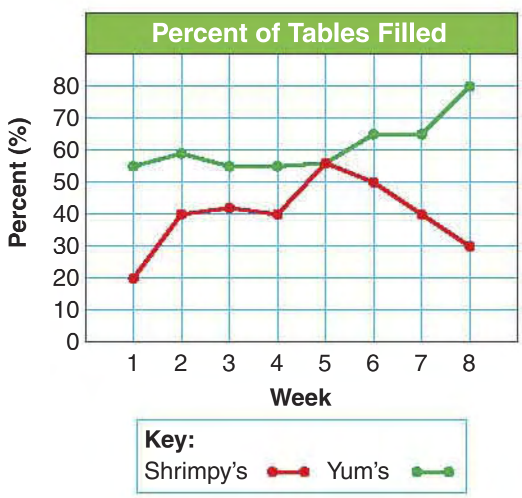
The graph below shows the number of health food stores in two cities. Predict how many stores each city will have in 2012.




In Springfield, the trend seems to be an increase in the number of stores at a rate of 2 or 3 stores every two years. In Greenvale, the trend appears to be that the number of stores does not change significantly over time. So you can predict that there will be about 25–30 health food stores in Springfield and about 6–8 health food stores in Greenvale in 2012.

## Try These

**Two restaurants opened in the same year. The graph shows the percent of tables filled at dinnertime for each restaurant during its first 8 weeks of operation. Use the graph to answer the questions.**




**1.** What percent of Yum's tables were filled in its 4th week of operation?

**2.** In which week did the two restaurants have the same percent of filled tables?

**3.** What is the approximate difference in percent of filled tables for the two restaurants in their 3rd week of operation?

**4.** Which restaurant do you predict will have a greater percent of filled tables in the 10th week of operation?

**5.** **Discuss and Write** Why is a multiple line graph an effective tool for displaying and comparing sets of data in which changes occur over time?

Go to PRACTICE BOOK Lesson 8-10 for exercise sets.

# 8-11 Scatter Plots

**Objective** To use a coordinate plane to make a scatter plot • To read scatter plots • To determine the line of best fit • To identify the type of correlation found in the data

Car dealers take many factors into account to determine how much money to give customers on the trade-in value of their vehicles. Mileage is one of the factors. How can you use the data from the table to show the correlation between the number of miles a car has been driven and the resale value of the vehicle?

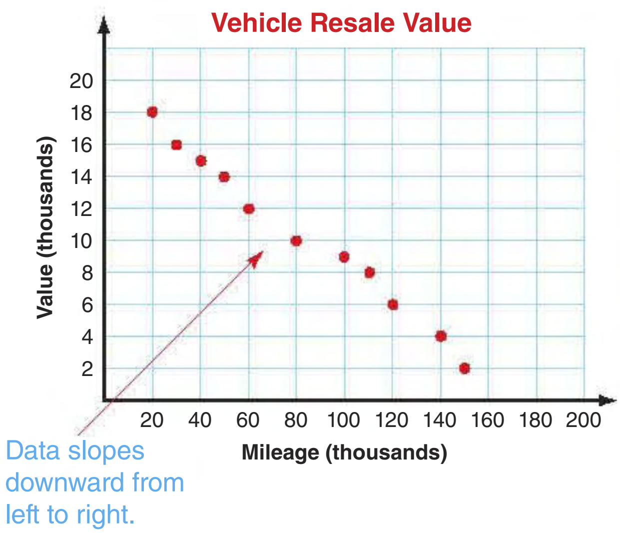
| Mileage (thousands) | Value (thousands) |
|---|---|
| 20 | 18 |
| 30 | 16 |
| 40 | 15 |
| 50 | 14 |
| 60 | 12 |
| 80 | 10 |
| 100 | 9 |
| 110 | 8 |
| 120 | 6 |
| 140 | 4 |
| 150 | 2 |

► To use the data to show the correlation (relationship) between the number of miles a car has been driven and its resale value, you can make a *scatter plot*. A scatter plot is a graph that compares two related sets of data on a coordinate plane. Each point on a scatter plot represents a pair of values.

► To make a scatter plot:

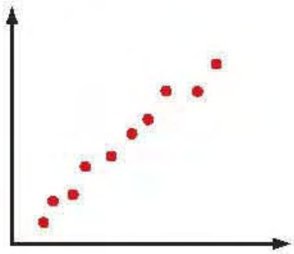
1. Decide on a title for the graph.
2. Draw a vertical and a horizontal axis.
3. Choose a scale for each axis, using a range and intervals that will fit the data.
4. Use the horizontal axis for the mileage scale and the vertical axis for the values scale.
5. Plot a point for each pair of numbers in the table.

The points show how the values of the vehicles decrease as the number of miles increases.



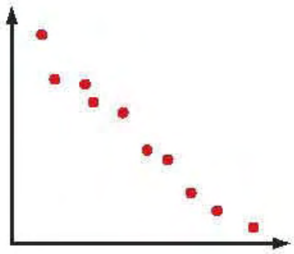

► In any scatter plot, the data displayed can be described in three ways.

**Positive Correlation**



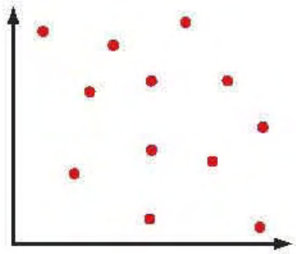
The numbers for one data set increase as the numbers for the other data set increase.

**Negative Correlation**



The numbers for one data set decrease as the numbers for the other data set increase.

**No Correlation**



There is no pattern in the way the numbers for the data sets increase or decrease.

► You can draw a line near the points of a scatter plot to clearly show the trend or correlation between two sets of data. This line is known as a line of best fit. The line of best fit is a line that is close to most of the data points.

This scatter plot shows the relationship between work experience and salaries for car salespeople at one dealership.


**Remember:** Outliers are points that are clearly separate from the set of data.

In many cases, you can approximate the line of best fit by "eyeballing" the data and using a straightedge.

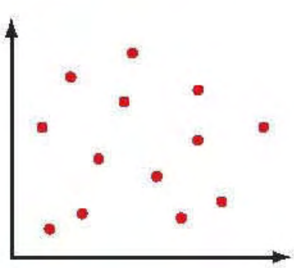
Use the straightedge to connect several points in the scatter plot so that there appear to be an equal number of data points above and below the line of best fit.

A line of best fit can be very useful in making predictions. If you know the value of one of the two variables, you can use the line to estimate a likely value for the other.

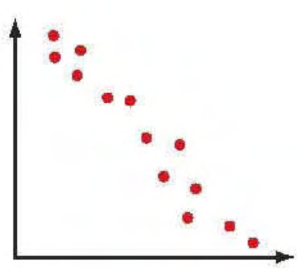
## Try These

**Tell what kind of correlation each scatter plot describes.**

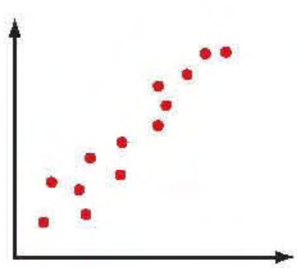
**1.**



**2.**



**3.**



**Tell whether you would expect a positive correlation, a negative correlation, or no correlation for each of the following.**

**4.** science aptitude and shirt size

**5.** the list price of a gemstone and its value

**6.** number of losses a baseball team has and the attendance at its home games

**7.** The table at the right shows data about temperature change. Use the data to make a scatter plot. Then describe the relationship between the data sets.

| Time | Noon | 2 PM | 4 PM | 6 PM | 8 PM |
|---|---|---|---|---|---|
| Temperature (°F) | 41 | 46 | 49 | 46 | 51 |

**8.** **Discuss and Write** Look at the scatter plot for Vehicle Resale Value on the preceding page. How would the correlation between the number of miles a car has been driven and its resale value change if the labels for the axes were reversed?

Go to PRACTICE BOOK Lesson 8-11 for exercise sets.

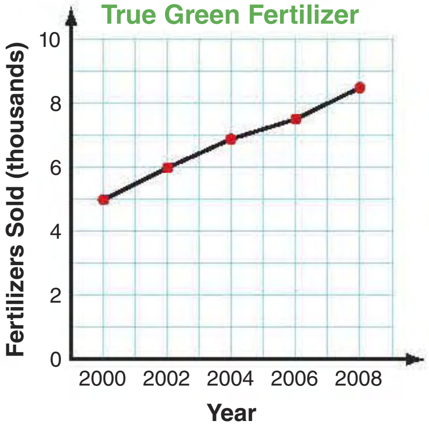
8-12

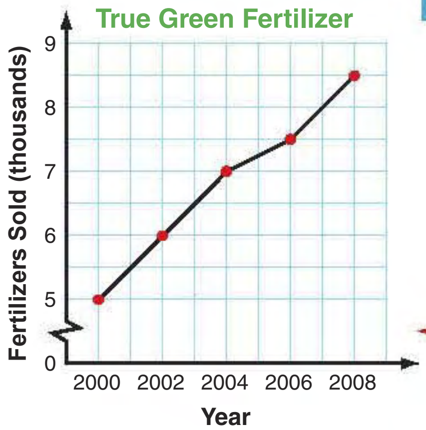
# Misleading Statistics and Graphs

**Objective** To recognize the characteristics of a misleading graph • To recognize misleading statistics

The two line graphs below show the same information about the number of units of tree fertilizer *True Green* has sold. Why do the graphs differ in appearance? Which graph could be misleading? Why?

► To find why the graphs differ and which graph could be misleading, look at the axes of each graph.




The graph at the right is misleading. Both the break in the vertical scale and the fact that two vertical units are used for each one thousand units of fertilizer makes the line appear to rise more steeply. This distorts the data, making it appear that sales are rising rapidly.

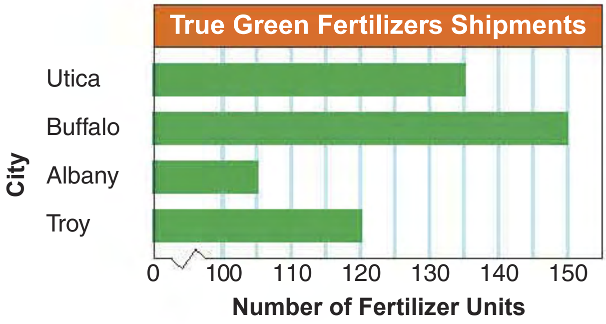
Although one graph seems to have a different value than the other, they both show the same number of units of fertilizers sold each year.

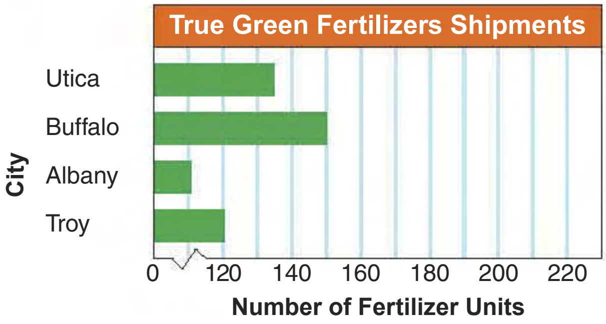
**Key Concept**

**Misleading Graphs**

- The scale is compressed or expanded, making the data appear to rise or fall more or less steeply.
- The intervals on the scales are not equal, making data appear to rise or fall more or less steeply.
- There is no title or there are no labels on the axes, or the units of measure are not indicated.
- The scale does not begin with zero, or a break in the scale distorts the appearance of the data.

The bar graphs below show the same data about *True Green* shipments for 1 month to different places. The difference of the intervals of the scales makes the graph on the right appear as if there were nearly 0 shipments to Albany when in fact there were more than 100. The graph on the right is misleading.



► Statistical measures used to describe data may also be misleading.

The results of a survey about how many kinds of tree fertilizers a group of gardeners use is shown at the right. In that summary of the data, the mean number of kinds of tree fertilizers per gardener is 3.25. Is this misleading? Which statistic would give a better measure of the typical number of kinds of tree fertilizers used?

| How many kinds of tree fertilizers do you use? |
|---|
| 1, 1, 1, 2, 2, 2, 2, 2, 3, 5, 6, 12 |

Using the mean as a measure of the typical kind of tree fertilizer used by this group of gardeners is misleading. The outlier, 12, distorts the data. The mean is not representative of the data. The mode, 2, is more representative of the data.

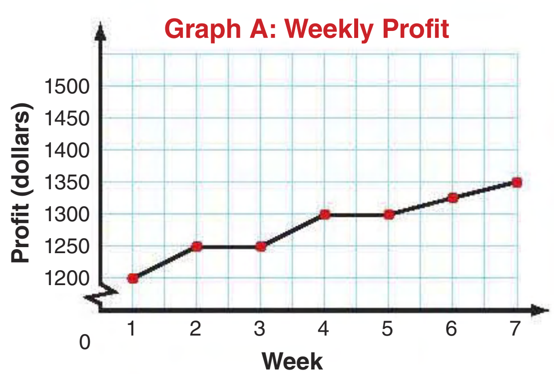
► Advertisers often present data in a way that influences how that data is interpreted. A data display or claim can distort information to help sell a product or service.

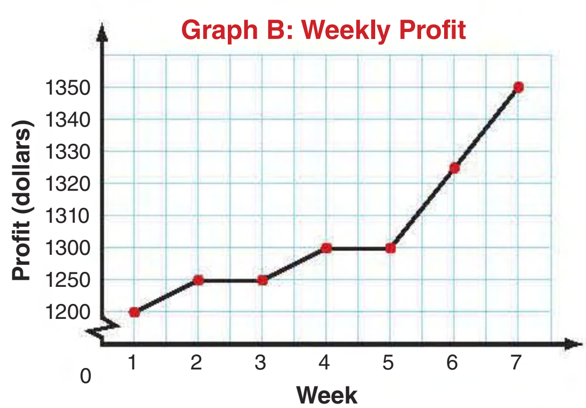
| When Advertisers Claim that: | | Think: |
|---|---|---|
| • 80% of gardeners agree. *True Green* fertilizer works best! | → | How many gardeners were asked? How many brands of fertilizer were compared? |
| • Fruity Delight contains real fruit juice. | → | What else does it contain? Sugar? Preservatives? What percent is fruit juice? |
| • The mean salary for our agency's employment opportunities is $85,000. | → | Are any of the salaries high enough to skew the mean? What is the median salary? |

## Try These

**Jane made the following graphs to show the weekly profit made by her small company. Use the graphs to answer the questions.**

**1.** Which one of Jane's two graphs is misleading? Why?




**2.** Tony got the following scores on 7 of his math tests: 68, 71, 97, 70, 67, 74, 97. When asked how well he did, he said, "Great! My statistical average was 97." How and why is his answer misleading?

**3.** Discuss and Write In what ways can graphs and statistics be misleading?

Go to PRACTICE BOOK Lesson 8-12 for exercise sets.

# 8-13 Technology: Create Graphs

**Objective** To use spreadsheet software to display data in bar graphs, line graphs, or circle graphs

The table at the right shows prices of merchandise sold at a baseball stadium. How can you use software to display and analyze the data?

| Item | Price |
|---|---|
| Batting helmet | $12 |
| Banner | $8 |
| Bat | $10 |
| Team jacket | $40 |
| Team photo | $5 |

One way to do this is to use a spreadsheet.

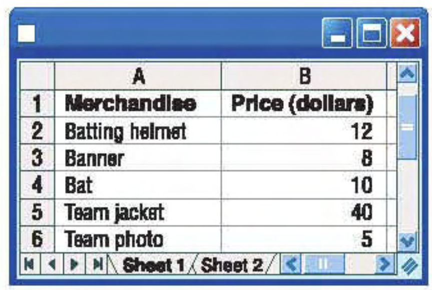
► You can use spreadsheet software to make a bar graph to represent the data.

Use the following steps.

**Step 1** Enter the data from the table into column A and column B of a spreadsheet, as shown at the right.

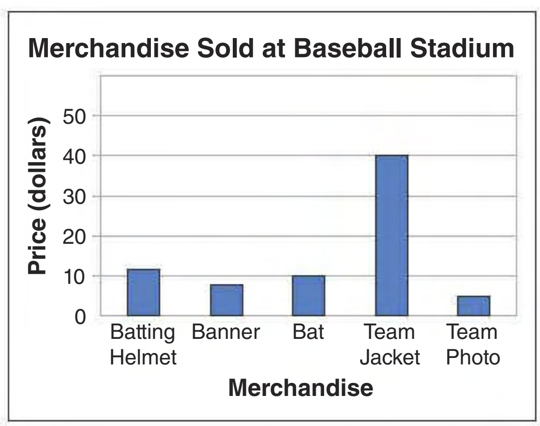
**Step 2** Highlight the data, including the column heads.

**Step 3** From the *Insert* menu, select **Chart**, or use the *Chart Wizard* icon on the toolbar. This will take you through a series of steps that enables you to create the graph. Here, select **Column** to display the data in a vertical bar graph, and input the graph title and axis labels. Click **Finish** when done.

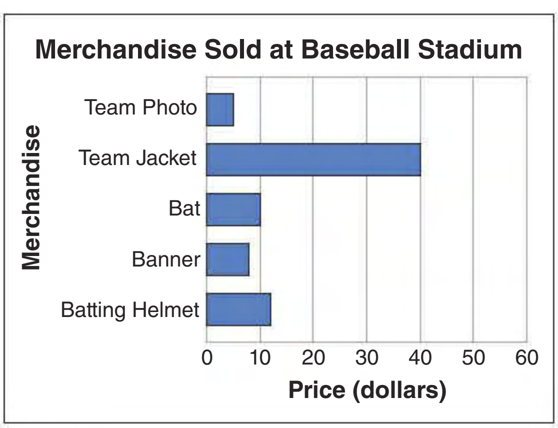


| | A | B |
|---|---|---|
| 1 | Merchandise | Price (dollars) |
| 2 | Batting helmet | 12 |
| 3 | Banner | 8 |
| 4 | Bat | 10 |
| 5 | Team jacket | 40 |
| 6 | Team photo | 5 |





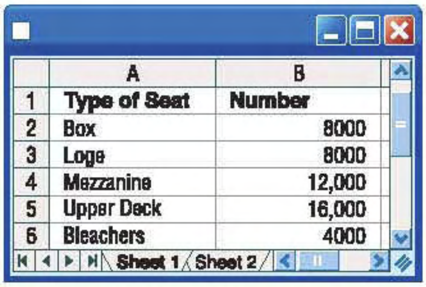

To represent the data as a horizontal bar graph, select **Bar**. Click **Finish** when done.



► You can also use spreadsheet software to make a circle graph to represent data.

A stadium seats 48,000 people. There are 8000 box seats; 8000 loge seats; 12,000 seats in the mezzanine; 16,000 seats in the upper deck; and 4000 seats in the bleachers. Use a spreadsheet program to represent this data in a circle graph.

**Step 1** Enter the data in the first two columns of the spreadsheet, as shown below.



| | A | B |
|---|---|---|
| 1 | Type of Seat | Number |
| 2 | Box | 8000 |
| 3 | Loge | 8000 |
| 4 | Mezzanine | 12,000 |
| 5 | Upper Deck | 16,000 |
| 6 | Bleachers | 4000 |

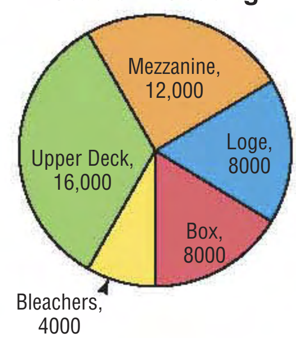


**Step 2** Highlight the data, including the column heads.

**Step 3** From the *Insert* menu, select **Chart**, or use the *Chart Wizard* icon on the toolbar. This will take you through a series of steps that enables you to create the graph. Here, select **Pie** to display the data in a circle graph, and input the graph title. Click **Finish** when done.

When you change the data in the spreadsheet, the graph is automatically updated.

**Stadium Seating**




## Try These

1. Use a spreadsheet program to represent the data in the table at the right.
2. Make a double bar graph showing the data in the table, by grade.

| Favorite Types of Books | | |
|---|---|---|
| Type of Book | 7th Graders | 8th Graders |
| Adventure | 19 | 13 |
| Biography | 8 | 12 |
| Mystery | 11 | 9 |
| Poetry | 4 | 7 |
| Other | 8 | 9 |

3. **Discuss and Write** Explain how you would use spreadsheet software to graph data about the daily mean high temperature in your city or town over the past week.

Go to PRACTICE BOOK Lesson 8-13 for exercise sets.

# 8-14 Problem Solving: Review of Strategies

Read › Plan › Solve › Check

**Objective** To solve problems using a variety of strategies

**Problem 1:** A park has two concentric square walking paths with a 10-foot distance between them. The inside path measures 240 feet per side. Sylvester walked once around the outside path in 160 seconds. At this rate, about how many seconds would it take him to walk once around the inside path?

**Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Two square paths, one inside the other, have a 10-foot gap between them. The inside (shorter) path has 240-foot sides. In 160 seconds, Sylvester walked once around the outside path.

**Question:** If Sylvester walks at the same rate around the inside path, about how many seconds will it take him to complete one lap?

**Select a strategy.**
There are many ways to approach this problem. Here are two possibilities:

- You could use the strategy *Organize Data* to try to analyze the problem situation.
- You could also try to ask a simpler question, using the strategy *Solve a Simpler Problem*.

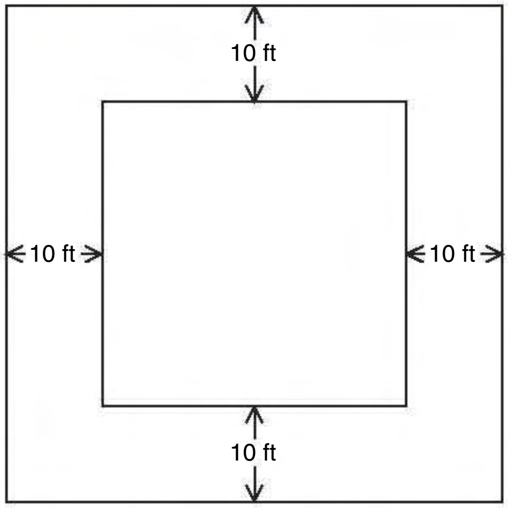
**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Apply the strategy.**

► **Method 1: Organize Data**

First, make a sketch, labeling the information you know. You do not need a scale drawing; rather, you need a rough sketch that shows the important information. The sketch at the right makes it clear that each side of the outside path is 20 feet longer than each side of the inside path. Therefore, the outside path has sides of length 260 feet, and the perimeter of the outside path is 4(260 feet), or 1040 feet. So Sylvester walked 1040 feet in 160 seconds.




The inside path has perimeter 4(240 feet), or 960 feet. To solve the problem, you must find out how much time it would take Sylvester to walk the inside path's 960 feet.

To find how much time it would take Sylvester to walk the path, organize the information to form a proportion.

$$\frac{\text{time to walk inside path}}{\text{perimeter of inside path}} = \frac{\text{time to walk outside path}}{\text{perimeter of outside path}}$$

$$\frac{t \text{ sec}}{960 \text{ ft}} = \frac{160 \text{ sec}}{1040 \text{ ft}}$$

Now solve the proportion.

$$\frac{t}{960} = \frac{160}{1040}$$

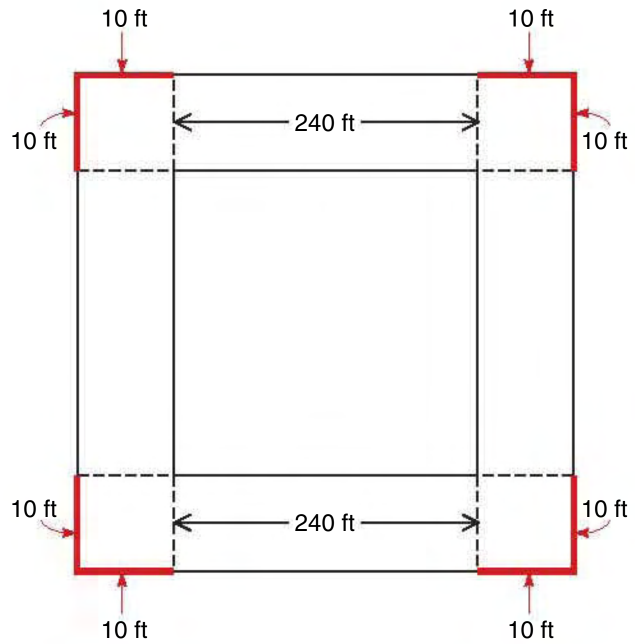
$1040t = 153{,}600$ ← Cross multiply.

$t \approx 148$ ← Simplify.

It would take Sylvester about 148 seconds to walk the inside path.

## ► Method 2: Solve a Simpler Problem

As you did for Method 1, make a rough sketch that shows the important information. Your sketch will reveal that the outside path is a total of 8(10 feet), or 80 feet, longer than the inside path.




To solve this problem, you can first answer the simpler question: "How long does it take Sylvester to walk 80 feet?"

Once you have this answer, you can subtract it from the 160 seconds it takes to walk the outside path. This will be the time it would take Sylvester to walk the inside path.

To determine that Sylvester's walking rate is 6.5 feet per second, divide 1040 feet (the length of the outside path) by 160 seconds (the length of time it took Sylvester to walk the outside path).

At a rate of 6.5 feet per second, it would take Sylvester 80 ÷ 6.5, or about 12, seconds to walk 80 feet. Taking these 12 seconds off his outside path's time of 160 seconds, you find that it would take him about 148 seconds to walk the inside path.

**Check to make sure your answer makes sense.**
The same solution was found two different ways, so you can feel confident that it is correct.

Go to PRACTICE BOOK Lesson 8-14 for exercise sets.

# Enrichment:
## Financial Spreadsheets

**Objective** **To use financial spreadsheets to organize and analyze financial data**

**Technology**

A spreadsheet is a software program that organizes data and performs calculations. Formulas are used to give the computer instructions for carrying out the calculations.

A spreadsheet is a table with columns and rows. Usually the rows are named with numbers and the columns are named with letters. The intersection of a row and a column is called a cell. Almost any kind of information can be put in a cell: words, numbers, or formulas. If you have a spreadsheet program, use it to create the spreadsheet in this lesson.

Some seventh-graders want to raise money for hurricane relief by making and selling tie-dyed T-shirts. They use a spreadsheet to plan the project. First, they enter information about the items they need to buy.

Cell D4 means the cell in column D, row 4.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Seventh-Grade Tie-Dye T-Shirt Project | | | | |
| 2 | | | | | |
| 3 | Materials Needed | Amount to Buy | Price Per Unit | Cost | |
| 4 | Can of Dye | 6 | $12.50 | | |
| 5 | Box of Rubber Gloves | 3 | $1.75 | | |
| 6 | Pound of Soda Ash | 2 | $4.85 | | |
| 7 | Bottle of Thickener | 1 | $8.90 | | |
| 8 | Box of Rubber Bands | 1 | $4.50 | | |
| 9 | Adult T-shirts, short sleeves | 12 | $3.00 | | |
| 10 | Adult T-shirts, long sleeves | 12 | $4.10 | | |
| 11 | Youth T-shirts, short sleeves | 12 | $3.25 | | |
| 12 | Youth T-shirts, long sleeves | 12 | $5.25 | | |
| 13 | | | | | |

Sheet 1 Sheet 2

Next, they use a formula to find the cost of each item. In cell D4, they enter the formula =B4*C4. This multiplies the value in cell B4 (number of cans of dye to buy) by the value in cell C4 (price per can) to get the total cost for the dye.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Seventh-Grade Tie-Dye T-Shirt Project | | | | |
| 2 | | | | | |
| 3 | Materials Needed | Amount to Buy | Price Per Unit | Cost | |
| 4 | Can of Dye | 6 | $12.50 | =B4*C4 | |
| 5 | Box of Rubber Gloves | 3 | $1.75 | | |

Sheet 1 Sheet 2

They write the formula into rows 4–12 of column D. The program automatically multiplies the number in the "Price Per Unit" column by the number in the "Amount to Buy" column.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Seventh-Grade Tie-Dye T-Shirt Project | | | | |
| 2 | | | | | |
| 3 | Materials Needed | Amount to Buy | Price Per Unit | Cost | |
| 4 | Can of Dye | 6 | $12.50 | $75.00 | |
| 5 | Box of Rubber Gloves | 3 | $1.75 | $5.25 | |
| 6 | Pound of Soda Ash | 2 | $4.85 | $9.70 | |
| 7 | Bottle of Thickener | 1 | $8.90 | $8.90 | |
| 8 | Box of Rubber Bands | 1 | $4.50 | $4.50 | |
| 9 | Adult T-shirts, short sleeves | 12 | $3.00 | $36.00 | |
| 10 | Adult T-shirts, long sleeves | 12 | $4.10 | $49.20 | |
| 11 | Youth T-shirts, short sleeves | 12 | $3.25 | $39.00 | |
| 12 | Youth T-shirts, long sleeves | 12 | $5.25 | $63.00 | |
| 13 | | | | | |

Sheet 1 Sheet 2

Next, they find the total cost by entering the formula =SUM(D4:D12) in cell D13. This computes the sum of all the values in the Cost column, from cell D4 to cell D12.

| | | | |
|---|---|---|---|
| | $63.00 | | |
| | =SUM(D4:D12) | | |
| | | | |

Finally, they use the formula =D13/48 to divide the total cost by 48. This tells them the cost of one tie-dyed T-shirt.

| | | |
|---|---|---|
| | Total Cost | $290.55 |
| | Cost for 1 T-shirt | =D13/48 |
| | | |

Their spreadsheet now looks like this:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Seventh-Grade Tie-Dye T-Shirt Project | | | | |
| 2 | | | | | |
| 3 | Materials Needed | Amount to Buy | Price Per Unit | Cost | |
| 4 | Can of Dye | 6 | $12.50 | $75.00 | |
| 5 | Box of Rubber Gloves | 3 | $1.75 | $5.25 | |
| 6 | Pound of Soda Ash | 2 | $4.85 | $9.70 | |
| 7 | Bottle of Thickener | 1 | $8.90 | $8.90 | |
| 8 | Box of Rubber Bands | 1 | $4.50 | $4.50 | |
| 9 | Adult T-shirts, short sleeves | 12 | $3.00 | $36.00 | |
| 10 | Adult T-shirts, long sleeves | 12 | $4.10 | $49.20 | |
| 11 | Youth T-shirts, short sleeves | 12 | $3.25 | $39.00 | |
| 12 | Youth T-shirts, long sleeves | 12 | $5.25 | $63.00 | |
| 13 | | | Total Cost | $290.55 | |
| 14 | | | Cost for 1 T-shirt | $6.05 | |
| 15 | | | | | |

Sheet 1 / Sheet 2

Then they learn that the company takes 50¢ off the price of each T-shirt for orders of 12 or more T-shirts of any size. They change the prices in the spreadsheet and the program automatically recalculates everything.

They decide to sell the T-shirts for $12. If they sell all of them, they will make $309.45 for the hurricane relief fund.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Seventh-Grade Tie-Dye T-Shirt Project | | | | |
| 2 | | | | | |
| 3 | Materials Needed | Amount to Buy | Price Per Unit | Cost | |
| 4 | Can of Dye | 6 | $12.50 | $75.00 | |
| 5 | Box of Rubber Gloves | 3 | $1.75 | $5.25 | |
| 6 | Pound of Soda Ash | 2 | $4.85 | $9.70 | |
| 7 | Bottle of Thickener | 1 | $8.90 | $8.90 | |
| 8 | Box of Rubber Bands | 1 | $4.50 | $4.50 | |
| 9 | Adult T-shirts, short sleeves | 12 | $2.50 | $30.00 | |
| 10 | Adult T-shirts, long sleeves | 12 | $3.60 | $43.20 | |
| 11 | Youth T-shirts, short sleeves | 12 | $2.75 | $33.00 | |
| 12 | Youth T-shirts, long sleeves | 12 | $4.75 | $57.00 | |
| 13 | | | Total Cost | $266.55 | |
| 14 | | | Cost for 1 T-shirt | $5.55 | |
| 15 | | | | | |

Sheet 1 / Sheet 2

## Try These

1. Change the number of adult and youth short-sleeve T-shirts to 36, and change the number of cans of dye to 9. What will the new total cost be? If the students sell all the T-shirts, how much money will they make?

2. **Discuss and Write** Plan a project that your class might do. Use a spreadsheet to find all the costs and the possible profit. Write a short report summarizing your results.

Go to PRACTICE BOOK pages 263–264 for exercise sets.

# Test Prep: Multiple-Choice Questions

## Strategy: Understand Distractors

Distractors in multiple-choice questions often seem correct because they are related to information given in the problem. Be sure to *answer the question asked* when choosing your answer.

**Sample Test Item**

What is the median of these six monthly salaries: \$4000, \$2800, \$3200, \$6200, \$3200, \$4600?

**A.** \$3200 **C.** \$3600
**B.** \$3400 **D.** \$4000

### Test-Taking Tips

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Read the whole test item carefully.**

- Reread the test item, including all the answer choices.
- Underline important words.
  What is the median salary?
  The *median* is the middle number in a set of ordered data.
- Restate the question in your own words.
  When the salaries are put in numerical order, what is the middle salary?

**Solve the problem.**

- Distinguish the median from the range, mode, and mean.
- Order the data. Find the median.
  \$2800, \$3200, \$3200, \$4000, \$4600, \$6200
  The median is $\frac{\$3200 + \$4000}{2} = \$3600$.

**Think**

When there is an even number of data, the median is the mean of the two middle numbers.

**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices. Watch out for distractors.

**A.** \$3200 ← This is the mode of the data. Eliminate this choice.

**B.** \$3400 ← This is the range of the data. Eliminate this choice.

**C.** \$3600 ← This is the correct choice!

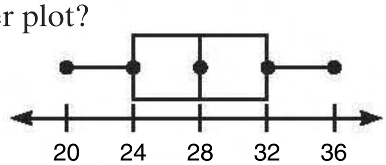
**D.** \$4000 ← This is the mean of the data. Eliminate this choice.

**Try These**

**Choose the correct answer. Explain how you used strategies.**

1. What is the range of the data shown in the box-and-whisker plot?

   **A.** 16 **C.** 24
   **B.** 20 **D.** 28




2. What is the least common multiple of 8, 16, and 24?

   **F.** 4 **H.** 48
   **G.** 8 **J.** 192

Go to PRACTICE BOOK page 265 for exercise sets.

# Two-Dimensional Geometry

CHAPTER 9

## In This Chapter You Will:

- Identify complementary, supplementary, adjacent, and vertical angles
- Identify congruent angles formed by pairs of parallel lines cut by a transversal
- Identify and classify polygons
- Identify congruent triangles
- Construct perpendicular and parallel lines and congruent triangles
- Identify central angles and inscribed angles
- Make a circle graph to display a set of data
- Apply the strategy: *Adopt a Different Point of View*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- If two figures are similar, all pairs of corresponding angles are congruent, and the ratios of the lengths of all pairs of corresponding sides are equal.
- Properties of equality and properties of rational numbers help you solve equations.
- Percents can be written as equivalent fractions or decimals.
- To find a percentage of a number, multiply the base by the percent.

**For Practice Exercises:**

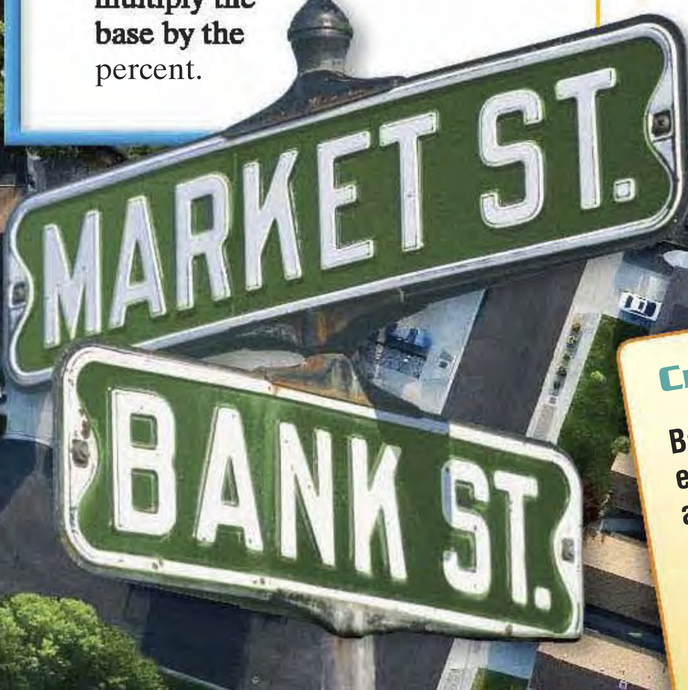
Go to PRACTICE BOOK, pp. 271–306

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

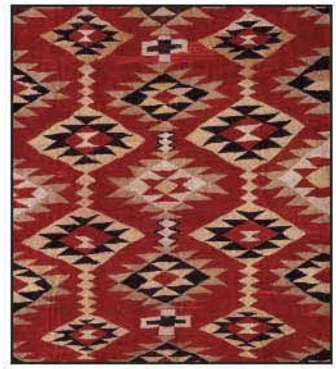



### Critical Thinking

Bank Street and Albion Road run parallel to each other. Market Street intersects Bank Street and Albion Road to form right angles at each corner. Describe how the lines that represent Market Street and Bank Street are related.

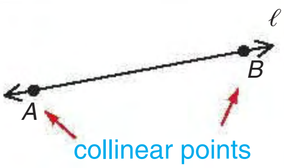
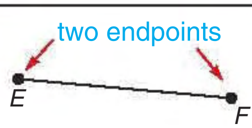
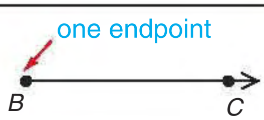
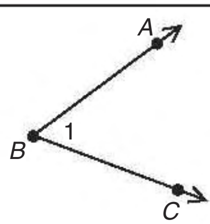
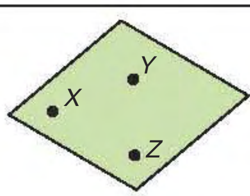
# 9-1 Points, Lines, and Planes

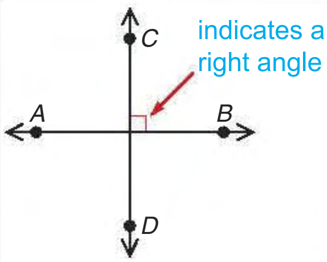
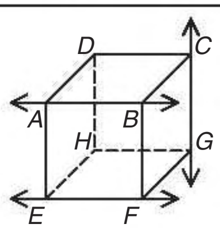
**Objective** **To identify collinear and noncollinear points, lines, line segments, rays, angles, parallel lines, intersecting lines, perpendicular lines, and skew lines • To read and write symbols for geometric figures • To identify parallel, intersecting, and perpendicular planes**



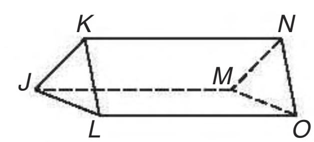
Many artists from all over the world incorporate geometric figures into their paintings, pots, textiles, and tapestries.

► Points, lines, and planes are the most basic geometric figures.

| Figure | Definition | Read/Write |
|---|---|---|
| • $A$ | A point is an exact location in space. Although represented by a dot, a point has no size. It is named by a letter. | point $A$ |
|   | A line is a continuous set of points in a straight path. It extends without end in both directions. It can be named by any two points on the line. It is sometimes named by a lower-case letter.<br>If a straight line can be drawn through a set of points, the points are collinear. If no one line can be drawn through all the points, the points are noncollinear. | line $AB$ or line $BA$<br>$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$<br>line $\ell$ |
|   | A line segment is a part of a line and has *two endpoints*. Line segment $EF$ consists of endpoints $E$ and $F$ and all the points between $E$ and $F$. A line segment is named by its endpoints. | line segment $EF$ or $FE$<br>$\overline{EF}$ or $\overline{FE}$ |
|   | A ray is part of a line that has *one endpoint*. It continues infinitely in the opposite direction. It is named by its endpoint and any other point on the ray. | ray $BC$<br>$\overrightarrow{BC}$ |
|   | An angle is a figure formed by two rays with a common endpoint, the vertex (plural: *vertices*) of the angle. The rays form the sides of the angle. An angle can be named in several different ways: You can use the letters of three points that make the angle, one point on each ray with the vertex letter as the middle letter; just the letter of the vertex; or just a number. | angle $ABC$<br>$\angle ABC$<br>angle $CBA$<br>$\angle CBA$<br>angle $B$<br>$\angle B$<br>angle 1<br>$\angle 1$ |
|   Points $X$, $Y$, and $Z$ are coplanar. | A plane is a flat, two-dimensional surface that extends infinitely in all directions. It is named by at least three points in the plane that are *not* all collinear. Points and lines that lie in the same plane are called coplanar. | plane $XYZ$ |

| Figure | Definition | Read/Write |
|---|---|---|
|  | Coplanar lines are either *intersecting* or *parallel*. Two lines that cross at exactly one point are intersecting lines. | |
|   | Lines $AB$ and $CD$ intersect at *right angles*. (A right angle has the shape of a corner of this page.) Lines $AB$ and $CD$ are perpendicular lines. | $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ "is perpendicular to" |
|   | In the figure at the left, $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ are in the same plane, but do not intersect. They are parallel lines. In the same figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$ are in *different* planes. They are *skew lines*. Skew lines lie in different planes and are neither intersecting nor parallel. | $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ "is parallel to" |

► Planes may or may not intersect. In the illustrated three-dimensional figure above, plane $ABFE$ and plane $DCGH$ are parallel planes. In the same figure, plane $ABFE$ and plane $ADHE$ are perpendicular planes. In the illustrated three-dimensional figure at the right, plane $LKNO$ and plane $JKNM$ are intersecting planes that are not perpendicular.




## Try These

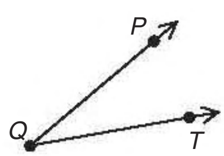
**Name and write the symbol for each figure.**

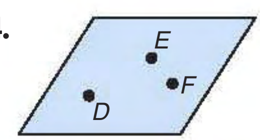
**1.**



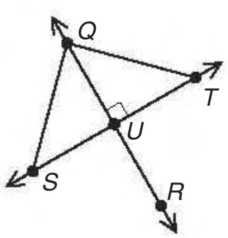
**2.**



**3.**



**4.**



**Identify the figures in the diagram.**




**5.** five points

**6.** four rays

**7.** eight line segments

**8.** two perpendicular lines

**9.** eleven angles

**10.** **Discuss and Write** Compare and contrast lines and rays; parallel and perpendicular lines; and lines and planes.

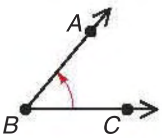
# 9-2 Classify and Measure Angles

**Objective** To classify angles by angle measure • To describe the position of points in relation to an angle • To use a right angle as a benchmark to estimate an angle measure • To use a protractor to measure angles • To use a protractor and straightedge to draw an angle of a given measure

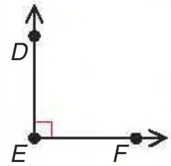
The man is boarding the bus on a wheelchair ramp. What type of angle is the ramp's angle of incline?


► Angles are measured in degrees. Angles are classified according to their degree measures.

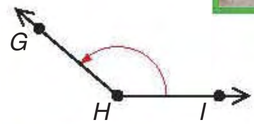
**Remember:** The measure of angle $ABC$ can be written as m$\angle ABC$.



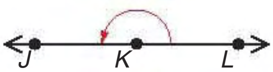

acute angle $ABC$
$0° < m\angle ABC < 90°$



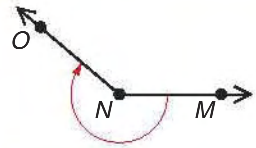

right angle $DEF$
$m\angle DEF = 90°$




obtuse angle $GHI$
$90° < m\angle GHI < 180°$



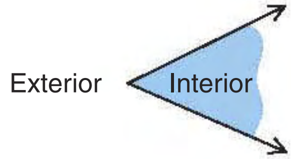

straight angle $JKL$
$m\angle JKL = 180°$




reflex angle $MNO$
$180° < m\angle MNO < 360°$

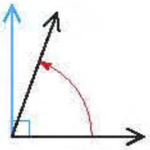
► The two rays of an angle divide a plane into three sets of points:

- points on the rays (sides or vertex of the angle)
- interior points—all points in the plane between the two rays
- exterior points—all points in the plane not part of the angle or its interior

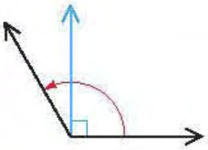



► You can use the easily recognizable shape of a right angle as a benchmark for identifying types of angles and estimating angle measures.

An angle of 70° is an acute angle.



An angle of 120° is an obtuse angle.

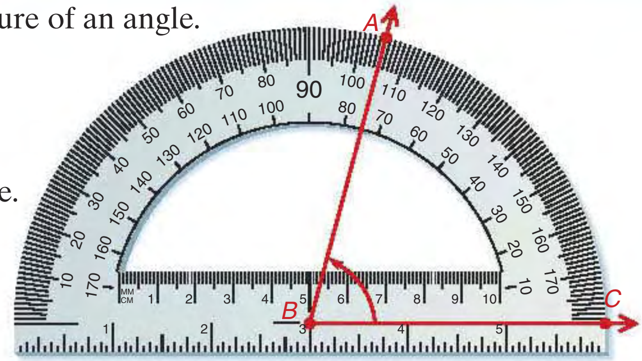


The ramp's angle of incline is less than 90°, so it is an acute angle.

► You can use a protractor to find the degree measure of an angle.

To measure an angle:

1. Place the protractor so that its zero line rests along one ray of the angle, called the *base ray*, and its center mark is at the vertex of the angle.
2. Find the 0 on the scale where the base ray, $\overrightarrow{BC}$, crosses the protractor.
3. Follow along that scale to the point where the other ray, $\overrightarrow{BA}$, crosses the protractor. The number at that point is the measure of the angle.



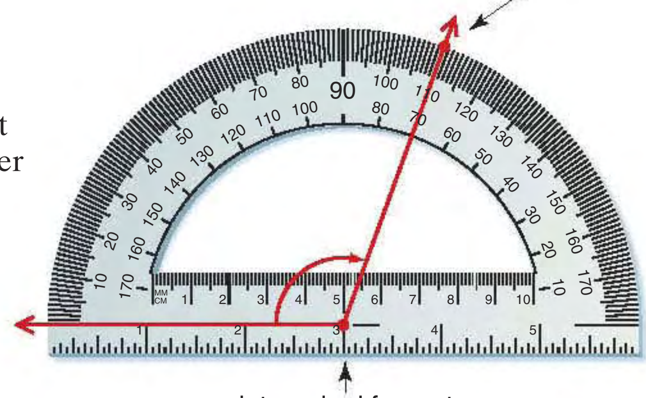

$m\angle ABC = 75°$

► You can use a protractor and a straightedge to draw an angle of a given measure.

To draw an angle:

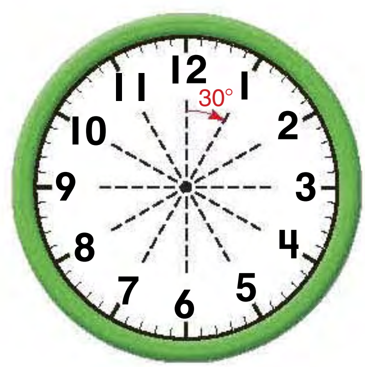
1. Mark a point for the vertex of the angle.
2. Position the center mark of the protractor at the vertex. Mark another point at 0° on either scale of the protractor. Draw a ray from the vertex through the point at 0°.
3. To draw a 110° angle, follow along the scale where the ray crosses the protractor to 110°. Mark a third point there. Draw a second ray from the vertex to that point.

mark at 110° point of protractor



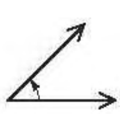
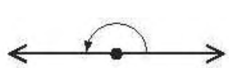
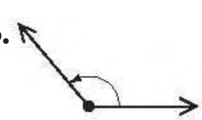
point marked for vertex

► At different times of day the hour and minute hands of a clock form angles with different measures. The minute hand of an analog clock rotates 360° every hour. It rotates one twelfth of 360°, or 30°, every five minutes.




## Try These

**Estimate the measure of each angle. Classify each as *acute*, *right*, *obtuse*, or *straight*. Then use a protractor to find the actual measure.**

**1.**  **2.**  **3.**  **4.** 

**5.** Name three times at which the hands of a clock form an acute angle.

**6.** **Discuss and Write** The angle of incline for a loading ramp is often about 20°. What might occur if the angle were greater?

Go to PRACTICE BOOK Lesson 9-2 for exercise sets.

Algebra
# 9-3 Angle Pairs

**Objective** To identify complementary, supplementary, adjacent, and vertical angles • To find missing angle measures by solving equations

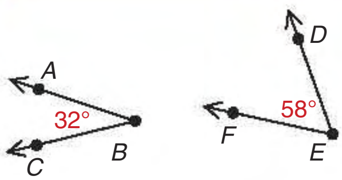
Alex places a piece of tile, with an angle measuring 36°, in one corner of a rectangular backsplash above the kitchen counter. He needs to place another piece of tile adjacent to it to complete the square corner. What should the measure of the angle on the second tile be?

► You can classify pairs of angles.

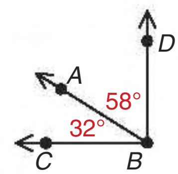
**Key Concept**

**Complementary Angles**

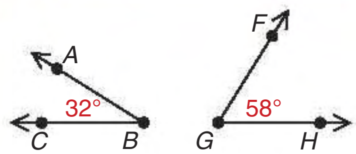
Two angles with a sum of 90° are called complementary angles. Each angle is the complement of the other.




$\angle ABC$ and $\angle DEF$ are complementary angles.
$32° + 58° = 90°$




$\angle ABC$ and $\angle ABD$ are complementary angles.
$32° + 58° = 90°$



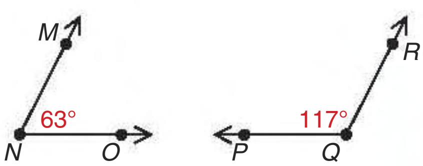

$\angle ABC$ and $\angle FGH$ are complementary angles.
$32° + 58° = 90°$

Alex needs a piece of tile with an angle that is the complement of a 36° angle. So he needs a piece of tile with an angle of $90° - 36° = 54°$.

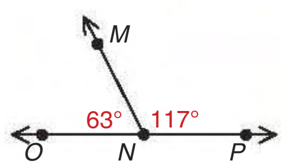
**Key Concept**

**Supplementary Angles**

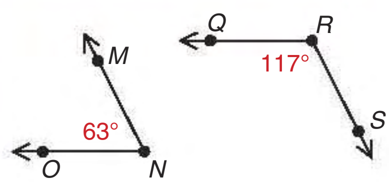
Two angles with a sum of 180° are called supplementary angles. Each angle is the supplement of the other.




$\angle MNO$ and $\angle PQR$ are supplementary angles.
$63° + 117° = 180°$



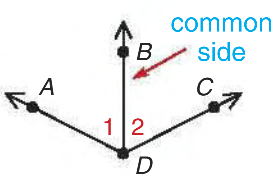

$\angle MNO$ and $\angle MNP$ are supplementary angles.
$63° + 117° = 180°$




$\angle MNO$ and $\angle QRS$ are supplementary angles.
$63° + 117° = 180°$

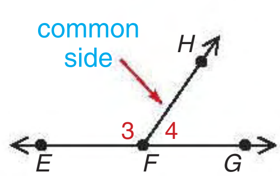
► Some pairs of angles share a common side and vertex.

Adjacent angles have a common side and vertex but have no common interior points (their interior points do not overlap).



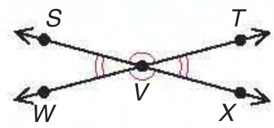

$\angle 1$ and $\angle 2$ are adjacent angles.

Adjacent angles that are supplementary form a *linear pair*. Their unshared sides are opposite rays (rays that form a straight angle).




$m\angle 3 + m\angle 4 = 180°$

► **Vertical angles** are a pair of opposite angles formed by two intersecting lines. Vertical angles are congruent.




$\angle SVW$ and $\angle TVX$ are vertical angles. So are $\angle SVT$ and $\angle WVX$.

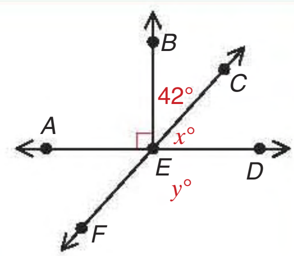
$\angle SVW \cong \angle TVX$
$\angle SVT \cong \angle WVX$

**Remember:**
$\cong$ means "is congruent to"

► You can solve equations to find missing angle measures.

## Example

**1** Use the diagram. Find the missing angle measures.




$\angle BEC$ and $\angle CED$ are complements. So $42° + m\angle CED = 90°$.

$\angle CED$ and $\angle DEF$ are supplements. So $m\angle CED + m\angle DEF = 180°$.

$x° = m\angle CED$ and $y° = m\angle DEF$

- To find the value of $x$:

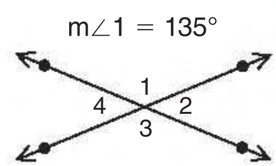
$x + 42 = 90$ ← Write an equation.
$x + 42 - 42 = 90 - 42$ ← Subtract 42 from both sides to isolate $x$.
$x = 48$
$m\angle CED = 48°$

- To find the value of $y$:

$y + x = 180$ ← Write an equation.
$y + 48 = 180$ ← Substitute 48 for $x$.
$y + 48 - 48 = 180 - 48$ ← Subtract 48 from both sides to isolate $y$.
$y = 132$
$m\angle DEF = 132°$

## Try These

**Use the figure at the right for exercises 1–4.**



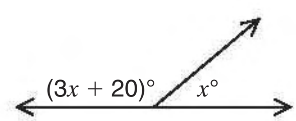

**1.** Name an angle congruent to $\angle 1$.

**2.** What is the measure of $\angle 4$?

**3.** What angle is congruent to $\angle 4$?

**4.** Which angles are supplements of $\angle 3$?

**Use the figure at the right for exercise 5.**




**5.** Write and solve an equation for $x$.

**For exercises 6–8, is the statement true? Write *always*, *sometimes*, or *never*.**

**6.** If two angles are complements, they are acute angles.

**7.** If two angles are supplements, both are obtuse angles.

**8.** If two angles are adjacent and supplementary, one is a right angle.

**9.** **Discuss and Write** Are adjacent angles always supplementary? Complementary? Could they be either supplementary or complementary? Explain.

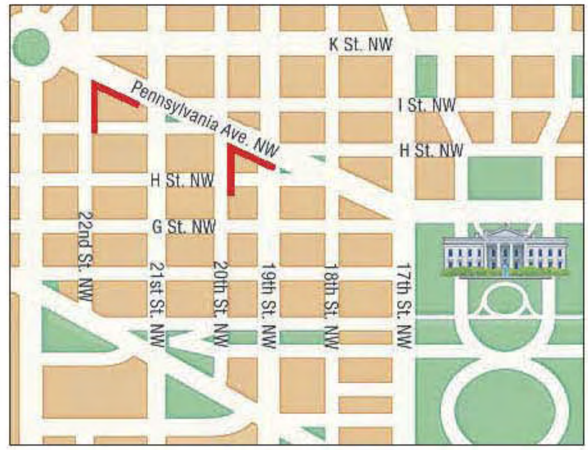
Go to PRACTICE BOOK Lesson 9-3 for exercise sets.

Algebra

# 9-4 Parallel Lines and Transversals

**Objective** **To identify the kinds of angles formed when a pair of parallel lines is intersected by a transversal • To use the properties of angle pairs and parallel lines to find missing angle measures**

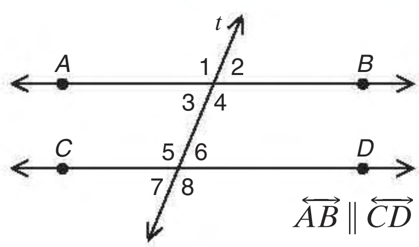
The grid of streets in many cities consists of parallel and intersecting lines. The map at the right shows some of the streets in Washington, D.C. What kinds of angles are formed whenever two parallel streets are intersected by a third street?




**Remember:** Parallel lines are in the same plane and do not intersect.

► A transversal is a line that intersects two or more lines at different points. When a pair of parallel lines is intersected by a transversal, eight angles are formed. These angles form certain special angle pairs.

In the figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines and are intersected by transversal *t*.




$\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles because they are inside the two parallel lines.

$\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ are exterior angles because they are outside the two parallel lines.

Corresponding angles are pairs of nonadjacent angles, one interior and one exterior, that are both on the *same side* of the transversal. Corresponding angles formed by parallel lines are congruent.

Corresponding angles:
$\angle 1$ and $\angle 5 \rightarrow \angle 1 \cong \angle 5$
$\angle 2$ and $\angle 6 \rightarrow \angle 2 \cong \angle 6$
$\angle 3$ and $\angle 7 \rightarrow \angle 3 \cong \angle 7$
$\angle 4$ and $\angle 8 \rightarrow \angle 4 \cong \angle 8$

Alternate interior angles are a pair of nonadjacent interior angles on *opposite sides* of the transversal. Alternate interior angles formed by parallel lines are congruent.

Alternate interior angles:
$\angle 3$ and $\angle 6 \rightarrow \angle 3 \cong \angle 6$
$\angle 4$ and $\angle 5 \rightarrow \angle 4 \cong \angle 5$

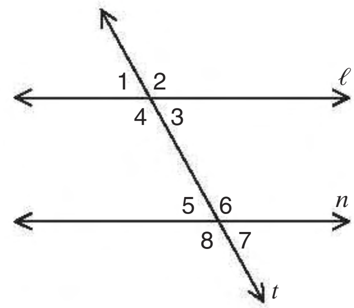
Alternate exterior angles are a pair of nonadjacent exterior angles on *opposite sides* of the transversal. Alternate exterior angles formed by parallel lines are congruent.

Alternate exterior angles:
$\angle 1$ and $\angle 8 \rightarrow \angle 1 \cong \angle 8$
$\angle 2$ and $\angle 7 \rightarrow \angle 2 \cong \angle 7$

The map of Washington, D.C. above shows that the numbered streets are parallel to one another. The numbered streets shown on this map are intersected by Pennsylvania Avenue. The angles formed by the intersection of Pennsylvania Avenue with 20th Street and with 22nd Street (shown in red on the map) are congruent because they are corresponding angles. Similar results could be found for the angles formed by the intersection of Pennsylvania Avenue with 18th Street, with 19th Street, and with 21st Street.

► You can use your understanding of parallel lines and the relationships among angle pairs to find missing angle measures.

In the figure, line $\ell \parallel$ line $n$ and $m\angle 1 = 60°$.




$m\angle 2 = 120°$ ← $\angle 1$ and $\angle 2$ are supplementary angles.

$m\angle 3 = 60°$ ← $\angle 1$ and $\angle 3$ are vertical angles.

$m\angle 4 = 120°$ ← $\angle 1$ and $\angle 4$ are supplementary angles.

$m\angle 5 = 60°$ ← $\angle 3$ and $\angle 5$ are alternate interior angles.

$m\angle 6 = 120°$ ← $\angle 2$ and $\angle 6$ are corresponding angles.

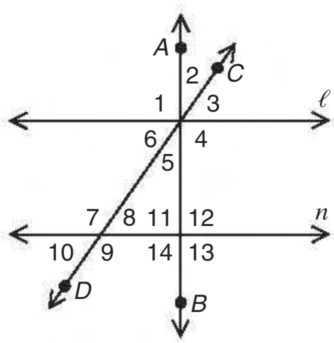
$m\angle 7 = 60°$ ← $\angle 1$ and $\angle 7$ are alternate exterior angles.

$m\angle 8 = 120°$ ← $\angle 4$ and $\angle 8$ are corresponding angles.

## Try These

**Use the figure at the right for exercises 1–8. Justify your answers.**

In the figure, line $\ell \parallel$ line $n$, $\overleftrightarrow{CD}$ is a transversal, $\overleftrightarrow{AB} \perp$ line $\ell$ and line $n$, and $m\angle 3 = 55°$.



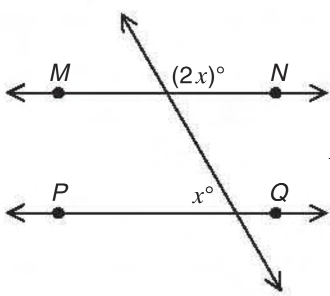

**1.** $m\angle 2 =$ ? **2.** $m\angle 4 =$ ?

**3.** $m\angle 5 =$ ? **4.** $m\angle 6 =$ ?

**5.** $m\angle 7 =$ ? **6.** $m\angle 8 =$ ?

**7.** $m\angle 9 =$ ? **8.** $m\angle 10 =$ ?

**9.** Use the figure below. Solve for $x$.




$\overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$

**For exercises 10–12, is the statement true? Write *always*, *sometimes*, or *never*.**

**10.** Alternate exterior angles are corresponding angles.

**11.** Angles that are adjacent to corresponding angles are congruent.

**12.** Alternate interior angles are supplementary.

**13. Discuss and Write** How are interior angles on the same side of the transversal related? Do exterior angles on the same side of the transversal have the same relationship? Use the figure at the top of this page to help explain your answer.

Go to PRACTICE BOOK Lesson 9-4 for exercise sets.

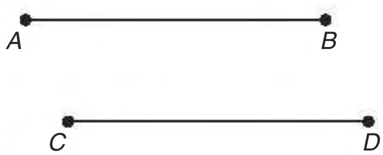
# Congruent Angles and Line Segments

**Objective** To recognize congruent angles and line segments • To identify line segment bisectors, midpoints, perpendicular bisectors, and angle bisectors • To construct the bisector of a given angle and the perpendicular bisector of a given line segment

Architects and engineers often need to construct congruent geometric figures in their daily work. How would you use a compass and straightedge to construct congruent line segments?

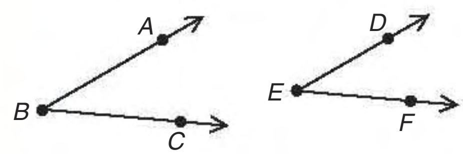
► Figures that are congruent have the same size and shape.

Congruent line segments have the *same length*.




$\overline{AB} \cong \overline{CD}$

Congruent angles have the *same degree measure*.



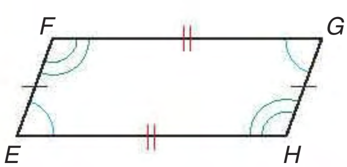

$\angle ABC \cong \angle DEF$

## Example

**1** In parallelogram $EFGH$:

$\overline{FG} \cong \overline{EH}$ and $\overline{FE} \cong \overline{GH}$

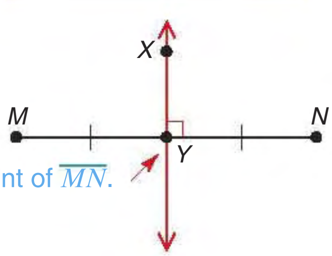
$\angle FEH \cong \angle HGF$ and $\angle EFG \cong \angle GHE$




Matching tick marks and matching angle marks indicate congruence.

► To bisect a line segment or an angle means to divide it into two congruent parts. The midpoint of a line segment is the point that divides the line segment into two congruent line segments.

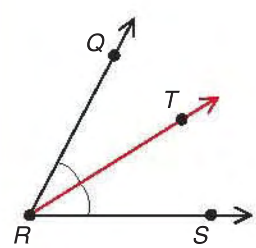
$\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{MN}$.




$Y$ is midpoint of $\overline{MN}$.

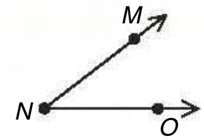
$\overline{MY} \cong \overline{YN}$

$\overrightarrow{RT}$ bisects $\angle QRS$.

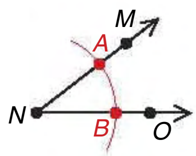



$\angle QRT \cong \angle SRT$

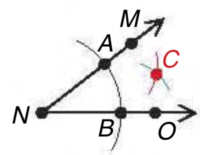
► You can use a compass and a straightedge to bisect an angle. Follow these steps to bisect angle *MNO*.




**1** With the compass tip at vertex *N*, draw an arc intersecting $\overrightarrow{NM}$ and $\overrightarrow{NO}$. Label the points of intersection as *A* and *B*.



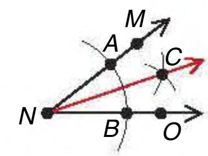

**2** Open the compass further. With the compass tip on *A*, draw an arc. With the same compass opening and the tip on *B*, draw another arc intersecting the first arc. Label the point of intersection *C*.




**3** With your straightedge, draw a ray from vertex *N* through *C*.

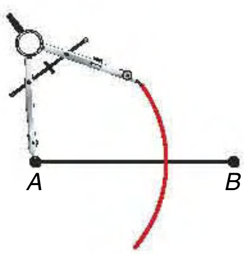
$\overrightarrow{NC}$ bisects $\angle MNO$.

$\angle MNC \cong \angle ONC$

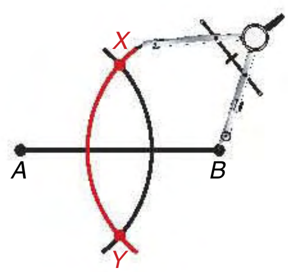



► You can use a compass and a straightedge to construct the perpendicular bisector of a line segment. Construct the perpendicular bisector of $\overline{AB}$.

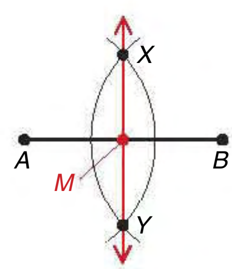

**1** With the compass tip on point *A*, construct an arc that will extend both above and below $\overline{AB}$. Be sure the opening is greater than $\frac{1}{2}\overline{AB}$.




**2** With the same setting as in Step 1, put the compass tip on *B*, and construct an arc that will extend above and below the line segment. Label the points where the two arcs intersect as *X* and *Y*.

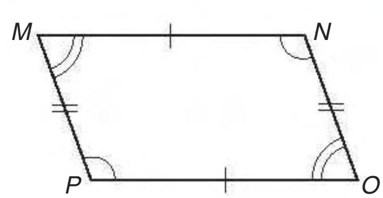



**3** Use a straightedge to draw $\overleftrightarrow{XY}$. Label the point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ as *M*. $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$. Point *M* is the midpoint of $\overline{AB}$.




## Try These

**Use the diagram at the right.**




**1.** Name 2 pairs of congruent line segments.

**2.** Name 2 pairs of congruent angles.

**Make the following constructions using a compass and straightedge.**

**3.** Draw an angle and bisect it.

**4.** Draw a line segment and its perpendicular bisector.

**5.** **Discuss and Write** Explain how to use string, chalk, and a straightedge to bisect the angle formed by two adjacent walls that meet at a corner of your classroom floor.

Go to PRACTICE BOOK Lesson 9-5 for exercise sets.

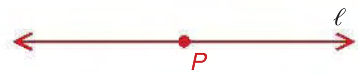
# Line Constructions

**Objective** **To construct a line perpendicular to a given line through a given point on that line • To construct a line perpendicular to a given line through a given point not on that line • To construct a line parallel to a given line through a given point**

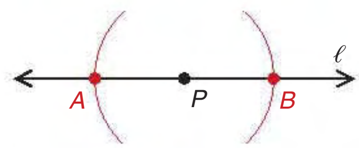
You are drawing a plan for a proposed shopping mall next to a housing development. You need to draw parallel and perpendicular lines to show streets and mall entrances and exits. The only tools you have are a pencil, a straightedge, and a compass. How can you use these tools to complete the constructions needed?

► To construct a line perpendicular to a given line through a point *on* that line:

1. Draw a line $\ell$ through a point $P$.

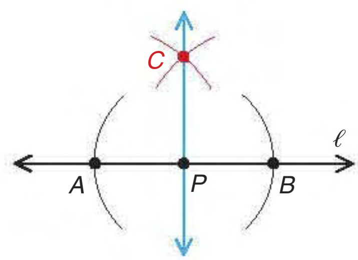



2. With your compass point on $P$, draw two arcs intersecting line $\ell$ at two points on opposite sides of point $P$. Label those points $A$ and $B$.



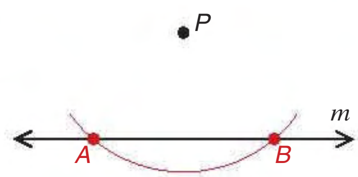

3. Widen the compass to construct two intersecting arcs above point $P$, one with the compass point at $A$, and the other with the compass point at $B$. Label the intersection of the arcs as point $C$. Draw $\overleftrightarrow{CP}$.

   $\overleftrightarrow{CP} \perp$ line $\ell$



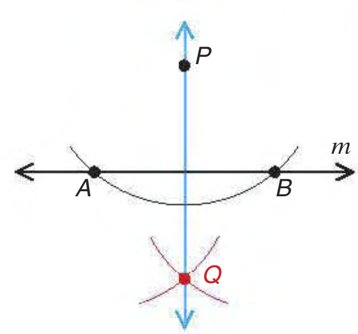

► To construct a line perpendicular to a given line through a given point *not on* the line:


1. Draw a line $m$. Draw a point $P$ at a location not on line $m$.
2. With your compass point on $P$, draw an arc intersecting line $m$ at two points. Label those points $A$ and $B$.



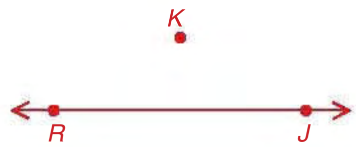

3. Using the *same* compass setting, construct two intersecting arcs below line segment $AB$, one with the compass point at $A$, and the other with the compass point at $B$. Label the intersection of the arcs as point $Q$. Draw $\overleftrightarrow{QP}$.

   $\overleftrightarrow{QP} \perp$ line $m$

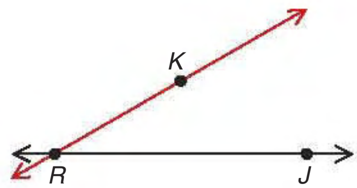


► To construct a line parallel to a given line through a given point:

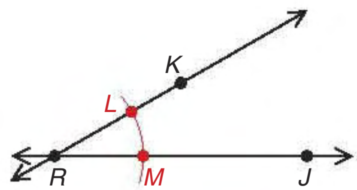
**1** Draw a line *RJ*. Draw and label a point *K* not on line *RJ*.



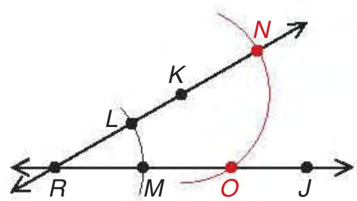

**2** Draw a line through *K* to intersect line *RJ* at point *R*.




**3** Place the compass point at point *R*, and draw an arc intersecting lines *RK* and *RJ*. Label the point where this arc crosses line *RJ* as point *M* and label where it crosses line *RK* as point *L*.

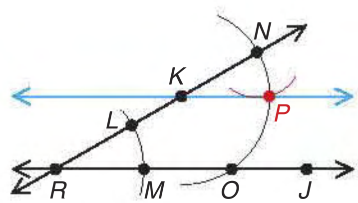



**4** Using the same compass opening, place the compass point on point *K* and draw an arc intersecting lines *RK* and *RJ*. Label the point where it crosses line *RJ* as point *O* and where it crosses line *RK* as point *N*.




**5** Open the compass to the distance between *L* and *M*. Use this new distance to draw an arc with the compass point on point *N* intersecting the arc *NO*. Label this point *P*. Draw a line through points *K* and *P*. Line *KP* is parallel to line *RJ*.

$\overleftrightarrow{KP} \parallel \overleftrightarrow{RJ}$




## Try These

**Use a compass and a straightedge to complete each construction.**

**1.** Construct a line perpendicular to a line *p* through a point *D* that is *on* line *p*.

**2.** Construct a line perpendicular to a line *q* through a point *E* that is *not* on line *q*.

**3.** Construct a line parallel to a line *r* through a point *F* that is *not* on line *r*.

**4.** **Discuss and Write** How can constructing perpendicular line segments help you construct parallel line segments? Explain.

Go to PRACTICE BOOK Lesson 9-6 for exercise sets.

Algebra

# 9-7 Polygons

**Objective** To find the exterior and interior angles of polygons • To identify regular, convex, and concave polygons • To find the sum of angle measures of polygons • To find the missing measure of angles of a polygon

PROBLEM-SOLVING STRATEGY 5

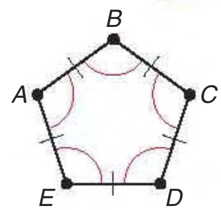
For an outdoor art project Sid is working on, he mows a large field to create several large regular triangles. What are the measures of each of the interior angles of his grass figures?

To find the measures, use the properties of polygons. A polygon is a closed plane figure that has three or more sides. Each side is a line segment. The line segments meet only at their endpoints. These are the vertices of the polygon.

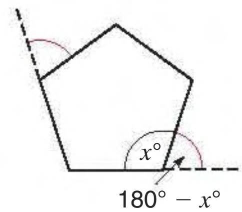
**Key Concept**

An n-gon, a polygon with $n$ sides, has $n$ vertices and $n$ angles. Polygons are named by the letters of their vertices in consecutive order.

► Polygons have both interior and exterior angles. Interior angles lie inside a polygon. The number of interior angle is the same as the number of vertices. The sides of a polygon can be extended to form exterior angles that lie outside the polygon. Each exterior angle is adjacent and supplementary to an interior angle.

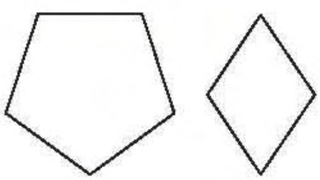


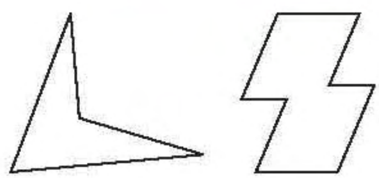
interior angles



exterior angles

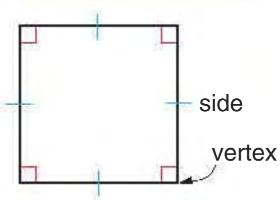
Polygons also can be either convex or concave. Each interior angle of a convex polygon measures less than 180°. A concave polygon contains one or more interior angles that each have a measure greater than 180°.


convex polygons

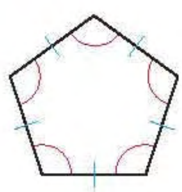

concave polygons

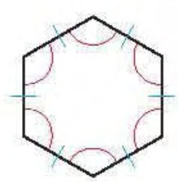
► A regular polygon has congruent sides and congruent angles.

All regular polygons are convex polygons.



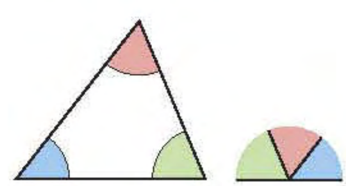
regular quadrilateral


regular pentagon


regular hexagon

A triangle is a polygon with three sides and three vertices.

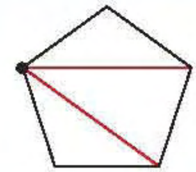
► The sum of the interior angle measures for any triangle is 180°. You can show that the sum is 180° by cutting out the angles and arranging them along a straight line. Because the measure of a straight angle is 180°, the sum of these interior angle measures is also 180°.



So each of the interior angles of Sid's grass figures has a measure of 180° ÷ 3, or 60°.

► You can find the sum of any polygon's interior angle measures by dividing the polygon into triangles.

1. Starting from one vertex, draw all possible diagonals. A diagonal is a line segment that connects two nonadjacent vertices.
2. Count the number of triangles formed by the diagonals. This number is two less than the number of sides.
3. Multiply the number of triangles by 180°.



number of sides → $5 - 2 = 3$ ← number of triangles

$3 \cdot 180° = 540°$

► You can use the sum of the interior angle measures of a polygon to find the measure of a single angle in a regular polygon.

Find the measure of one angle of a regular pentagon.

$\frac{(n-2) \cdot 180}{n}$ ← Use the formula.

$\frac{(5-2) \cdot 180}{5}$ ← Substitute 5 for $n$.

$\frac{3 \cdot \cancel{180}^{36}}{\cancel{5}_{1}}$ ← Simplify.

$3 \cdot 36 = 108$

So in a regular pentagon, each angle measures 108°.

**Key Concept**

**Measures of Interior Angles of a Regular Polygon**

- You can find the measure of each angle in a regular polygon using this formula.

  $\frac{(n-2) \cdot 180°}{n}$ ← sum of the angle measures / ← number of sides of a polygon

- You can use the numerator of this formula to find the sum of the angle measures of any polygon.

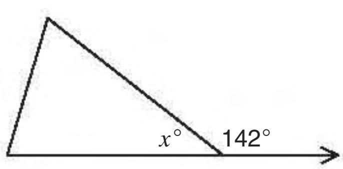
## Try These

**Find the sum of the angle measures for each polygon.**

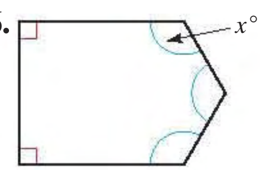
**1.** octagon (8 sides) **2.** heptagon (7 sides) **3.** 11-gon **4.** 15-gon

**Find the value of the variable in each polygon.**

**5.**




**6.**




**7.** What is the angle measure of each interior angle of a regular decagon (a polygon with 10 sides)?

**8.** What can you say about the relationship of the number of sides of a polygon and the number of diagonals that can be drawn from each vertex?

**9.** The sum of the interior angles of a regular $n$-gon is 1800°. What is the value of $n$?

**10.** **Discuss and Write** Consider any interior angle of a polygon and its adjacent exterior angle. Why is this pair of angles supplementary?

Go to PRACTICE BOOK Lesson 9-7 for exercise sets.

Algebra

9-8

# Triangles

**Objective** To classify triangles by angles, by sides, and by angles and sides
• To apply the concept of triangle inequality


Flag of Jamaica

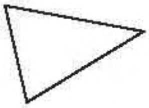
Like the flags of several other countries, the flag of Jamaica contains a few triangles. What kinds of triangles do you see in this flag?

To find out, consider ways to classify triangles.

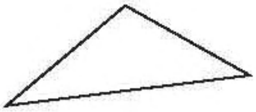
► Triangles can be classified in three ways:

- By the measures of their angles;

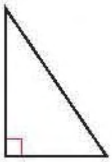
**acute triangle**
three acute angles



**obtuse triangle**
one obtuse angle

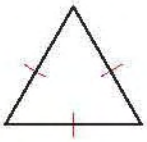


**right triangle**
one right angle



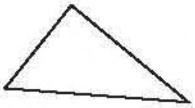
- By the lengths of their sides;

**equilateral triangle**
all sides congruent



**isosceles triangle**
two sides congruent

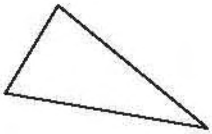
**scalene triangle**
no sides congruent



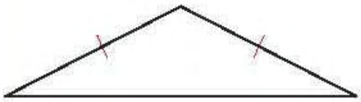
**Remember:** Matching tick marks indicate congruent sides.

- By both lengths of sides and measures of angles.

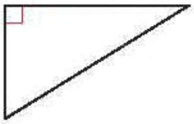
**acute scalene triangle**
three acute angles
no congruent sides



**obtuse isosceles triangle**
one obtuse angle
two congruent sides



**right scalene triangle**
one right angle
no congruent sides

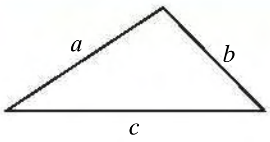


So there are two acute isosceles triangles and two obtuse isosceles triangles in the Jamaican flag.

**Key Concept**

**Classifying Triangles**
Triangles can be classified by the lengths of their sides, by the measures of their angles, and by both the lengths of their sides and the measures of their angles

► The Triangle Inequality Theorem states that the length of the third side of a triangle is always less than the sum of the lengths of the other two sides and greater than their difference.

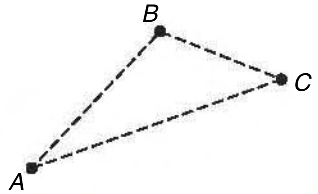



$(a + b) > c > |a - b|$

According to this theorem, if $A$, $B$, and $C$ are points in a plane:

$CA < AB + BC$

$CA > |AB - BC|$




► You can use the Triangle Inequality Theorem to find the *range* of numbers that could be the missing length of the third side of the triangle given the lengths of the other two sides.

**Key Concept**

**Triangle Inequality Theorem**

The third side of a triangle must be:

- less than the *sum* of the other sides; and
- greater than the *difference* of the other sides.

## Example

**1** For triangle $XYZ$, $XY = 3$ cm and $YZ = 4$ cm.

What are the possible lengths for $\overline{ZX}$?

Use the Triangle Inequality Theorem.

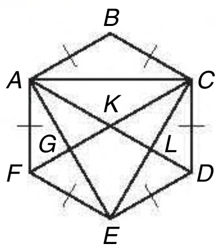
$ZX < 3 + 4$

$ZX > 4 - 3$

So the length for $\overline{ZX}$ is between 1 cm and 7 cm, or 1 cm $< ZX <$ 7 cm.

## Try These

**Use the figure to answer the questions.**




**1.** How many triangles are in the figure?

**2.** How many are equilateral triangles?

**3.** How many are obtuse isosceles triangles?

**Can the set of given measurements be used to form a triangle? Write *yes* or *no*. Justify your answers.**

**4.** 2 m, 6 m, 9 m

**5.** 4.5 ft, 7 ft, 3.2 ft

**6.** 5 in., 12 in., 13 in.

**Write *true* or *false*.**

**7.** Every acute triangle has three acute angles.

**8.** All scalene triangles are obtuse triangles.

**9.** All isosceles triangles are right triangles.

**10. Discuss and Write** Explain why there cannot be a right obtuse triangle.

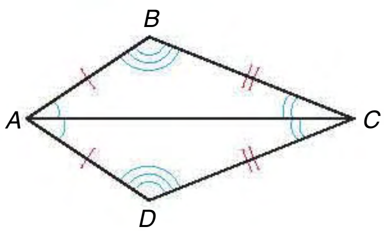
Go to PRACTICE BOOK Lesson 9-8 for exercise sets.

# Congruent Triangles

**Objective** To identify congruent triangles • To explore angle-side relations in triangles • To use congruent parts of congruent triangles to find missing angle and side measures

► **Congruent triangles** have exactly the same size and shape. Their corresponding sides and corresponding angles are congruent.

$\triangle ABC \cong \triangle ADC$ Triangle $ABC$ is congruent to triangle $ADC$.




| Corresponding sides: | Corresponding angles: |
| --- | --- |
| $\overline{AC} \cong \overline{AC}$ | $\angle ABC \cong \angle ADC$ |
| $\overline{AD} \cong \overline{AB}$ | $\angle CAB \cong \angle CAD$ |
| $\overline{DC} \cong \overline{BC}$ | $\angle ACB \cong \angle ACD$ |

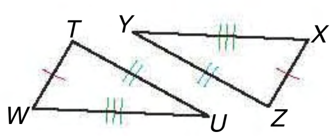
► To show that two triangles are congruent, you do not need to show that *all* corresponding parts are congruent.

Use these three statements to identify congruent triangles.

An *included angle* is the angle between two sides. An *included side* is the side between two angles.

**Side-Side-Side (SSS)**

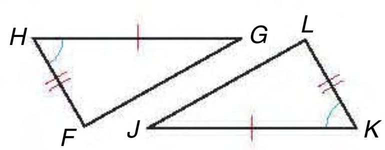
If all 3 pairs of corresponding sides of both triangles are congruent, the triangles are congruent.




$\triangle TUW \cong \triangle ZYX$

**Side-Angle-Side (SAS)**

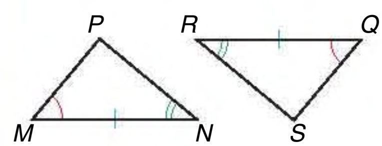
If 2 pairs of corresponding sides and the *included angles* of both triangles are congruent, the triangles are congruent.




$\triangle FGH \cong \triangle LJK$

**Angle-Side-Angle (ASA)**

If 2 pairs of corresponding angles and the *included sides* of both triangles are congruent, the triangles are congruent.



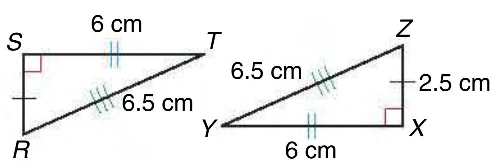

$\triangle MNP \cong \triangle QRS$

► Use the corresponding parts of congruent triangles to find missing angle measures and side lengths.

**Remember:** Matching tick marks and angle marks identify corresponding parts.

$\triangle RST \cong \triangle ZXY$

Find the measure of $\overline{RS}$.

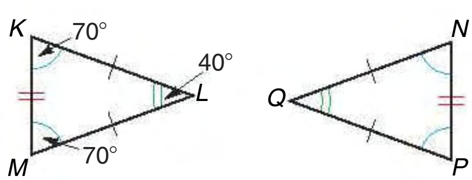



$\overline{RS}$ and $\overline{ZX}$ are corresponding sides.

$\overline{RS} \cong \overline{ZX}$, so $RS = 2.5$ cm

$\triangle KLM \cong \triangle PQN$

Find m$\angle PQN$.

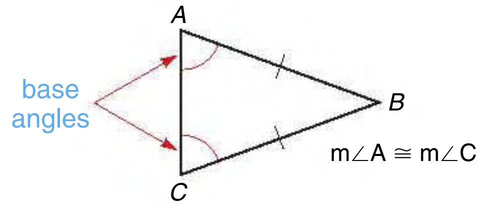


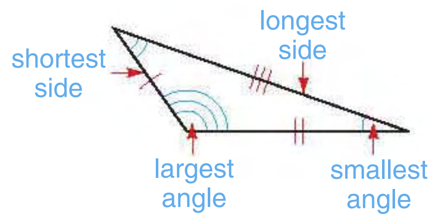

$\angle PQN$ and $\angle KLM$ are corresponding angles.

$\angle PQN \cong \angle KLM$, so m$\angle PQN = 40°$

► Here are some important relationships between the sides and angles of triangles.

- In any isosceles triangle or equilateral triangle, angles opposite congruent sides are congruent. These are called base angles.
- For all triangles, the angle with the *greatest* measure is opposite the *longest* side.
- For all triangles, the angle with the *least* measure is opposite the *shortest* side.




## Technology

You can use geometry software to explore congruent triangles.

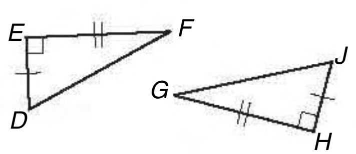
Use software to draw any three line segments that could form a triangle. Make a copy of the line segments you drew.

Use one set of the line segments. Join the line segments at their endpoints to form a triangle. Then use the other set of line segments. Can you join endpoints to make a triangle that is *not* congruent to the first triangle? Which statement discussed in this lesson is supported by this activity?

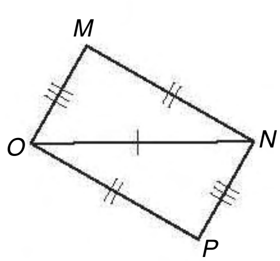
## Try These

**Which statement—SSS, SAS, or ASA—shows why the triangles are congruent? Explain.**

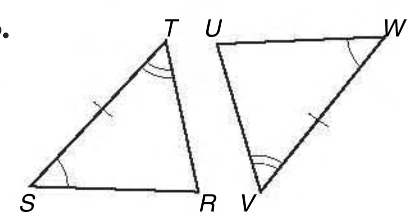
**1.**




**2.**




**3.**

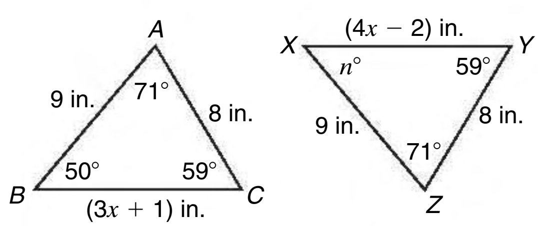



**Solve. Use the diagram.**

**4.** Find the missing side measurements.

**5.** Find m$\angle ZXY$. Explain your reasoning.

**6.** **Discuss and Write** Triangles $FGH$ and $JKL$ have three pairs of congruent corresponding angles. Why might these triangles *not* be congruent?

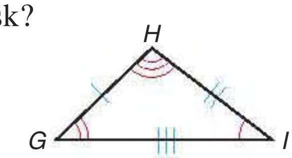



$\triangle ABC \cong \triangle ZXY$

# 9-10 Triangle Constructions

**Objective** To construct a triangle congruent to a given triangle • To construct a triangle given three line segments

Meg's task is to create a triangle congruent to triangle *GHI* without using a protractor and ruler. Can Meg use a construction to accomplish this task?




To decide if a construction can be used for this task, review what you know about *geometric constructions*.

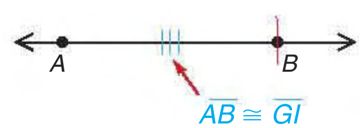
► As you have seen in earlier lessons, geometric constructions are drawings made with a compass and a straightedge.

- To create lengths, arcs, and circles, you use a compass.
- To draw line segments or sides, you use a straightedge.
- It is important to label your constructions correctly. Use tick marks and angle marks to show congruence of corresponding sides and angles.

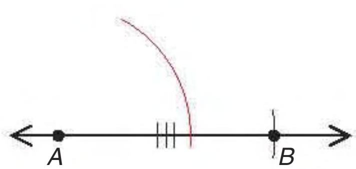
► Use compass and straightedge to construct a triangle that is congruent to $\triangle GHI$.

**1** Using your straightedge, draw a line. Draw and label a point *A* on the line.

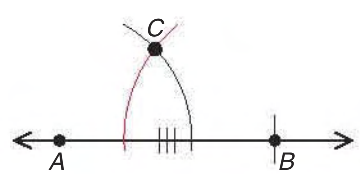

**2** Open your compass to the length of $\overline{GI}$. With the compass on point *A*, draw an arc that intersects the line. Label the point of intersection as point *B*.



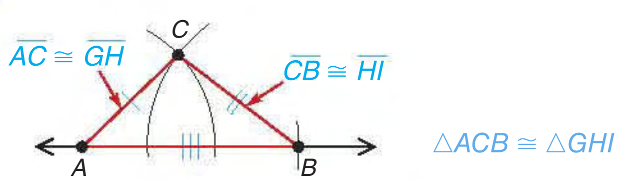

**3** Open your compass to the length of $\overline{GH}$. With your compass on point *A*, draw an arc above $\overline{AB}$.




**4** Open your compass to the length of $\overline{HI}$. With your compass on point *B*, draw an arc above $\overline{AB}$. Label the intersection of the two arcs as point *C*.



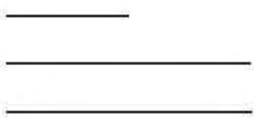

**5** Using your straightedge, draw $\overline{AC}$ and $\overline{CB}$.




Because each pair of corresponding sides are congruent, triangle *ACB* is congruent to triangle *GHI*. So a construction can be used to create congruent triangles without protractor or ruler.

► You can also use a compass and straightedge to construct a triangle from line segments.

Here are three line segments.

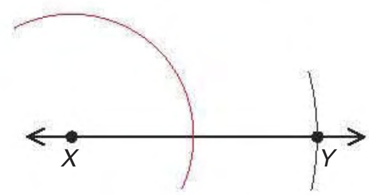


Use them to construct a triangle.

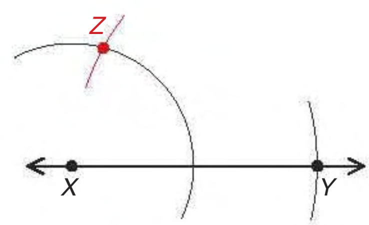
**1** Using your straightedge, draw a line and label a point $X$. This will be a vertex.


**2** Open your compass to the length of one of the longer lines given. With your compass on point $X$, draw an arc intersecting the line you drew. Label the point of intersection $Y$. This is the second vertex.

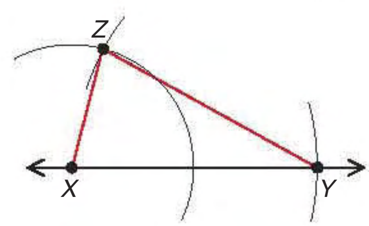

**3** Open your compass to the length of the short side. With the compass point at $X$, draw a large arc.



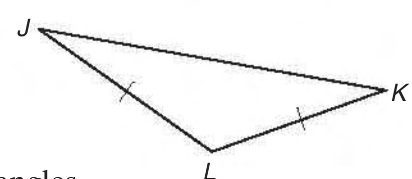

**4** Open the compass to the length of the third side. With the compass point at $Y$, make an arc intersecting the second arc. Label this point of intersection $Z$. This is the third vertex.




**5** Use the straightedge to draw the two remaining sides of your triangle.




## Try These




**1.** Use a compass and a straightedge to construct a triangle that is congruent to triangle $JKL$. Name this figure triangle $DEF$.

**2.** Tariq has a rug. Its design includes several congruent right triangles. The longest side of a right triangle is the *hypotenuse*. The other sides are the *legs*. On centimeter grid paper, draw right triangle $RST$ with legs of 5 cm and 12 cm. Then, using only a compass and straightedge, construct a triangle congruent to it.

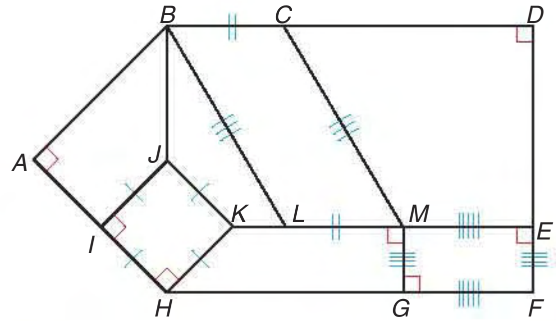
**3. Write and Discuss** Using any of the statements from Lesson 9-9 about congruent triangles, how do you know that triangles $ACB$ and $GHI$ (on the preceding page) are congruent?

Go to PRACTICE BOOK Lesson 9-10 for exercise sets.

# 9-11 Quadrilaterals

**Objective** **To identify the properties of quadrilaterals and the relationships among different types of quadrilaterals • To apply the properties of quadrilaterals in finding missing side and angle measures**

Cindy and Rod are playing a Guess the Shape game with quadrilaterals. Cindy constructed the figure shown at the right. What types of quadrilaterals are shown in Cindy's figure? Which of the quadrilaterals shown in the figure is *neither* a parallelogram *nor* a trapezoid?

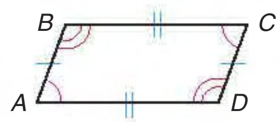



To answer the question, you need to explore the different types of quadrilaterals and their properties.

**Remember:** Quadrilaterals are polygons with four sides and four angles.

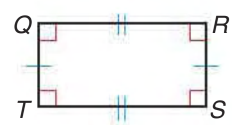
► Quadrilaterals can be classified by the special properties of their sides or angles.

A parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides and opposite angles are congruent.



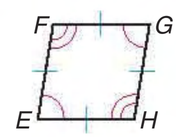

$\overline{BC} \parallel \overline{AD}$
$\overline{BA} \parallel \overline{CD}$

A rectangle is a parallelogram with four right angles.



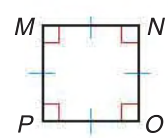

$\overline{QR} \parallel \overline{TS}$
$\overline{QT} \parallel \overline{RS}$

A rhombus is a parallelogram with four congruent sides.



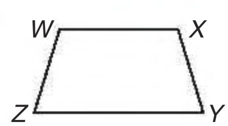

$\overline{FG} \parallel \overline{EH}$
$\overline{EF} \parallel \overline{HG}$

A square is a parallelogram with four right angles and four congruent sides.



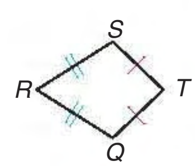

$\overline{MN} \parallel \overline{PO}$
$\overline{MP} \parallel \overline{NO}$

A trapezoid is a quadrilateral with exactly one pair of parallel sides.




$\overline{WX} \parallel \overline{ZY}$

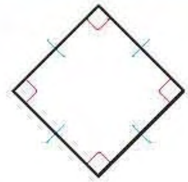
A kite is a quadrilateral with two pairs of adjacent sides that are congruent. The measures of its angles are each less than 180°.




$\overline{QR} \cong \overline{SR}$
$\overline{QT} \cong \overline{ST}$

So Cindy's construction includes three trapezoids and three parallelograms. The parallelograms include a square and another rectangle that is not a square. Since figure *JBLK* has no parallel sides, it is neither a parallelogram nor a trapezoid.

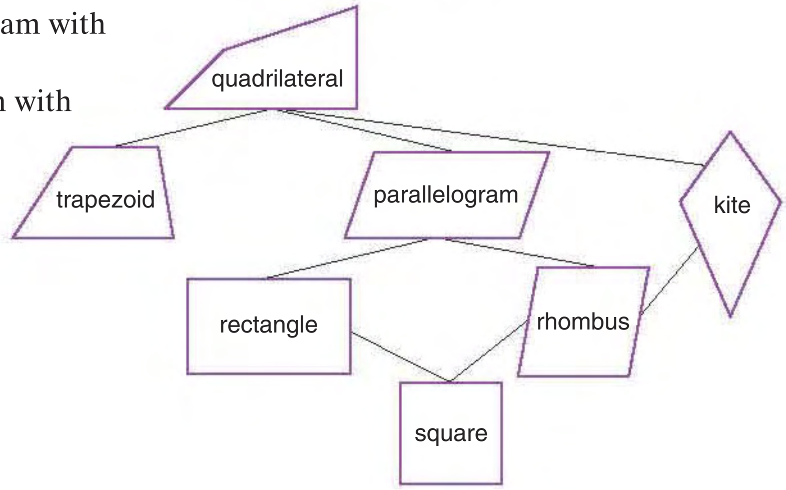
► Some quadrilaterals have several names.
The figure at the right can be classified as:



- a parallelogram because it has 2 pairs of parallel lines
- a kite because it has 2 pairs of adjacent congruent sides
- a rhombus because it is a parallelogram with 4 congruent sides
- a rectangle because it is a parallelogram with 4 right angles
- a square because it is a parallelogram with 4 right angles and 4 congruent sides

The figure is best described as a square because that name gives the most specific information.

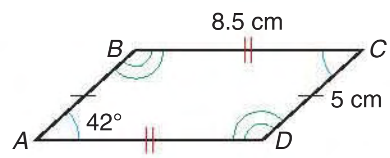
► The concept map shows the relationships among quadrilaterals.




Note that all rhombuses are kites, but not all kites are rhombuses.

► Use properties of quadrilaterals to find missing side and angle measures.

*ABCD* is a parallelogram. What is the measure of $\angle BCD$? What is the measure of side *AB*?

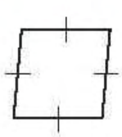


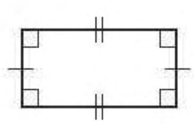

Opposite angles and opposite sides of parallelograms are congruent.
$\angle BCD$ is opposite $\angle BAD$, so $\angle BCD \cong \angle BAD$.
$\overline{AB}$ is opposite $\overline{CD}$, so $\overline{AB} \cong \overline{CD}$.

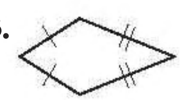
So $m\angle BCD = 42°$ and the length of $\overline{AB}$ is 5 cm.

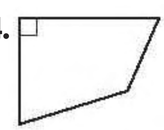
## Try These

**Classify each quadrilateral by giving the name that best describes it.**

**1.** 

**2.** 

**3.** 

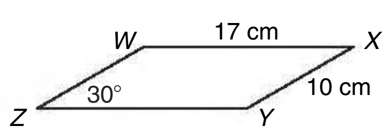
**4.** 

**For exercises 5–6, is the statement true? Write *always*, *sometimes*, or *never*.**

**5.** A square is a rhombus.

**6.** In a parallelogram, all sides are congruent and all angles are congruent.

**7.** Find the missing lengths and angle measures for parallelogram *WXYZ*.

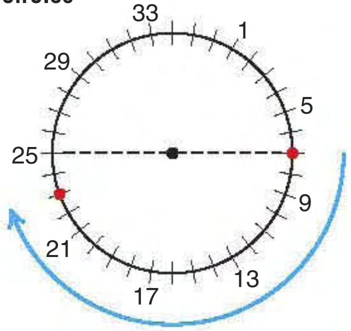



**8.** **Discuss and Write** Write a paragraph describing the relationships among quadrilaterals, trapezoids, parallelograms, rectangles, rhombuses, and squares.

Go to PRACTICE BOOK Lesson 9-11 for exercise sets.

# 9-12 Circles

**Objective** To identify parts of a circle • To name different kinds of arcs • To identify and find the measure of central angles and inscribed angles • To identify secants and tangents • To identify concentric, inscribed, and circumscribed circles



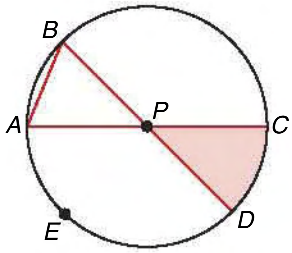

The first Ferris wheel had 36 cars. Suppose the cars of a Ferris wheel are numbered from 1 to 36. You are in car 7 and your friend is in car 23. If the wheel turns in a clockwise direction, what kind of arc will be made by your car as it moves from its current location to car 23's current location?

► A circle is a set of points in a plane, all of which are the same distance from a given point called the center. A circle is named by its center.

A chord is a line segment with its endpoints on the circle.

A radius (*plural*: radii) is a line segment from the center of a circle to a point on the circle.

A diameter is a chord that passes through the center of a circle. It is twice the length of a radius of that circle.




This is circle $P$. $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ are radii of circle $P$. $\overline{AC}$ and $\overline{BD}$ are diameters. $\overline{AB}$, $\overline{AC}$, and $\overline{BD}$ are chords.

An arc is a part of a circle with all its points on the circle. The symbol for arc $ABC$ is $\overset{\frown}{ABC}$. There are three kinds of arcs:

A semicircle connects the endpoints of a diameter. It is a half circle.

$$m\overset{\frown}{ABC} = 180°$$

A minor arc has a measure *less* than 180°.

$$m\overset{\frown}{AED} < 180°$$

A major arc has a measure *greater* than 180°.

$$m\overset{\frown}{BAC} > 180°$$

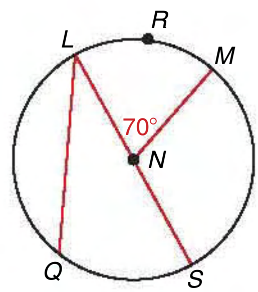
A sector of a circle is the region bounded by two radii and their intercepted arc. The shaded region is a sector of circle $P$.

The arc from car 7 to car 25's location is a semicircle. It has a measure of 180°. Car 23 comes before car 25, so the measure of the arc from car 7 to car 23 is *less* than 180°. It is a minor arc.

► The chords and radii of a circle form different kinds of angles.

A central angle has its vertex at the center of a circle.

- The sum of the measures of all the nonoverlapping central angles of a circle is 360°.
- The degree measure of a minor arc is the measure of its central angle.
- The degree measure of a major arc is 360° minus the degree measure of the minor arc that completes the circle.



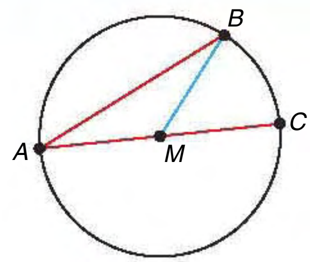

$\angle LNM$ is a central angle.

$m\overset{\frown}{LRM} = m\angle LNM = 70°$

$m\overset{\frown}{LQM} = 360° - m\overset{\frown}{LRM} = 360° - 70° = 290°$

An inscribed angle is an angle whose vertex is on the circle and whose sides intersect the circle at other points.

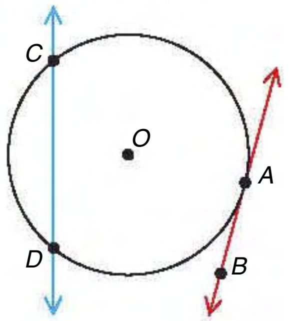
- Its measure is half of the central angle that forms the same arc. $\angle BAC$ is an inscribed angle.
- $m\angle BAC = \left(\frac{1}{2}\right)m\angle BMC$




► Tangents and secants are lines that intersect a circle.

A tangent to a circle is a line in the plane of the circle. The tangent intersects the circle at exactly one point. $\overleftrightarrow{AB}$ is tangent to circle $O$ at point $A$.

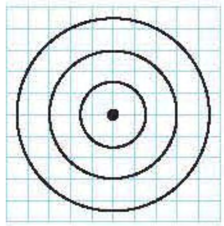
A secant is a line that intersects a circle at two points. $\overleftrightarrow{CD}$ intersects circle $O$ at points $C$ and $D$.




Secant $CD$ contains chord $CD$.

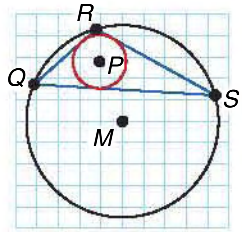
► You can identify some circle relations.

Circles are concentric if they lie in the same plane and have the same center.



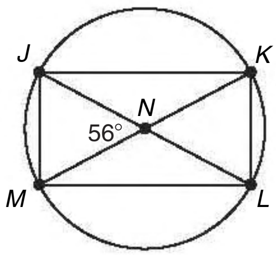
A circle is inscribed in a polygon if each side of the polygon is *tangent* to the circle. Circle $P$ is inscribed within triangle $QRS$.

A polygon is circumscribed by a circle if all the vertices of the polygon are points on the circle. Triangle $QRS$ is circumscribed by circle $M$.




## Try These

**Use the circle for exercises 1–6.**




**1.** Name two central angles and two inscribed angles.

**2.** Name a semicircle, a minor arc, and a major arc.

**3.** Name a pair of supplementary central angles.

**4.** What is the measure of $\overset{\frown}{JKL}$?

**5.** What is the measure of $\angle JLM$?

**6.** Given that $JKLM$ is a rectangle, name two pairs of parallel chords.

**7. Discuss and Write** In what ways is a circle like a polygon? How does it differ?

Go to PRACTICE BOOK Lesson 9-12 for exercise sets.

Update your skills. See pages 412 XIII, 414 XVII.

# 9-13 Make a Circle Graph

**Objective** To make a circle graph to display a set of data

The table shows the major elements that make up a typical adult's body. Why is a circle graph a good way to display this data? How can you make a circle graph to display this data?

| Element | Percent (%) |
|---|---|
| Oxygen | 65 |
| Carbon | 18 |
| Hydrogen | 10 |
| Nitrogen | 3 |
| All others | 4 |

► A circle graph is used to show parts of a whole. You can make a circle graph by constructing sectors of the circle in sizes that match the given data.

- Each labeled section of the circle will represent one of the elements in the table, and each section will be formed by a central angle.
- Use the data in the Percent column of the table to find the measure of each central angle.
- Use a decimal or fraction equivalent of each percent to find that percentage of 360°.

- There are 360° in a circle.
- To find the size of a sector, determine the measure of its central angle. You can do this by finding the percent of the circle that the sector represents.

**Remember:** To find a percentage of a number, multiply the number by the decimal or fraction equivalent to the percent.

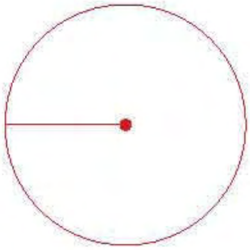
| Element | Percent (%) | Multiply | Degrees in sector |
|---|---|---|---|
| Oxygen | 65 | 0.65 • 360 | 234 |
| Carbon | 18 | 0.18 • 360 | 64.8 |
| Hydrogen | 10 | 0.10 • 360 | 36 |
| Nitrogen | 3 | 0.03 • 360 | 10.8 |
| All others | 4 | 0.04 • 360 | 14.4 |
| **Totals:** | 100% | | 360° |

### Key Concept

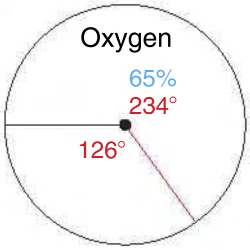
- To find the size of the central angle of a sector when you know what percent of the circle the sector represents, multiply 360° by the decimal or fractional equivalent of the percent. The result is the percentage of 360° that is the measure of the central angle of the sector.
- To find the size of the central angle of a sector given data only, first calculate the percent for each piece of data. Then multiply 360° by the decimal or fraction equivalent to that percent.

► Draw your circle graph.

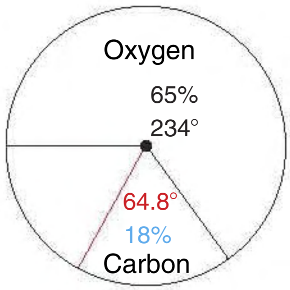
1. Use a compass to draw a circle. Make a radius using a straightedge.



2. Start by creating the section for oxygen. Since 360° − 234° = 126°, use a protractor to draw a 126° central angle. The rest of the circle will be the section representing oxygen.

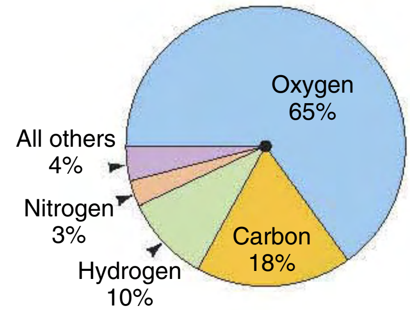


**3** Use the protractor to measure and draw the central angle for another section.




**4** Continue measuring and drawing angles until the graph is finished. Label each section with its element and percent. Give your graph a title.

**Major Elements in the Body**




## Example

**1** Iman surveys all her classmates about their favorite sport. Of students surveyed, 15 name soccer, 3 name baseball, and 12 name football. Make a circle graph. Label each section of the graph with the name of the sport and the percent of the class naming that sport as the favorite.

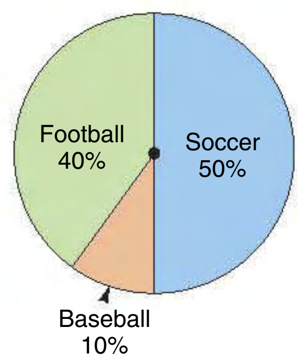
- Find the number of students surveyed.
  $15 + 3 + 12 = 30$
- Find the percent that liked each sport.
  Then find the size of each sector.

  Soccer: $\frac{15}{30} = \frac{15 \div 15}{30 \div 15} = 0.5 = 50\%$, and $0.5 \bullet 360° = 180°$

  Baseball: $\frac{3}{30} = \frac{3 \div 3}{30 \div 3} = 0.1 = 10\%$; and $0.1 \bullet 360° = 36°$

  Football: $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = 0.4 = 40\%$, and $0.4 \bullet 360° = 144°$
- Use the data to make a circle graph.

**Popularity of Sports**




## Try These

**1.** Make a graph of how you spend a typical 24-hour weekday.
- List how much time you spend each day on each activity. Use these categories for your data: school, travel, sports, eating, sleeping, and free time
- Record your data in a table.
- Use the data to make a circle graph. Label your graph and give it a title.
- Write a brief summary of what your graph shows.
- Compare your graph with those of your classmates. What similarities do you notice? How do the graphs differ?

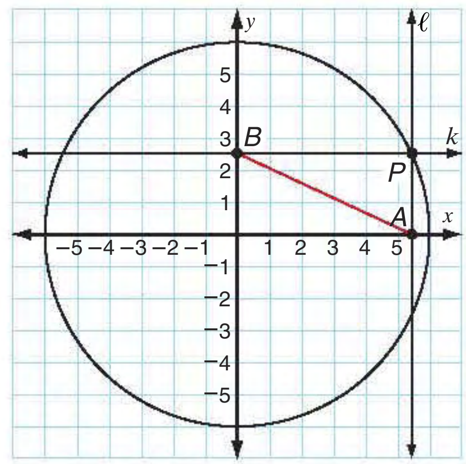
**2.** **Discuss and Write** What was the easiest part of the graph-making task? What was the most difficult part of the construction? Explain.

Go to PRACTICE BOOK Lesson 9-13 for exercise sets.

# 9-14 Problem-Solving Strategy: Adopt a Different Point of View

**Objective** To solve problems using the strategy *Adopt a Different Point of View*

**Problem 1:** The circle pictured at the right has its center at the origin and has a radius of 6 units. A point $P$ is on the circle. A vertical line $\ell$ passes through $P$ and intersects the $x$-axis at a point $A$. A horizontal line $k$ also passes through $P$ and intersects the $y$-axis at point $B$. What is the distance from point $A$ to point $B$?




**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** A circle has radius 6 units and its center at the origin.
A vertical line intersects the circle at point $P$ and the $x$-axis at $A$.
A horizontal line intersects the circle at point $P$ and the $y$-axis at $B$.

**Question:** What is the distance from $A$ to $B$?

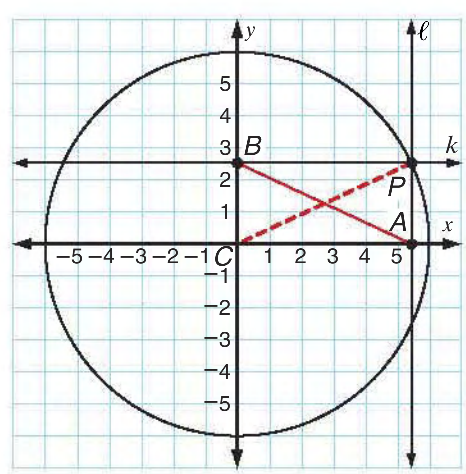
**Plan** **Select a strategy.**
If you knew the coordinates of $A$ and $B$, you could solve this problem by using the Pythagorean Theorem. Unfortunately, however, this information has not been given. The problem becomes easier if you use the strategy *Adopt a Different Point of View*.

**Solve** **Apply the strategy.**
If $C$ is the center of the circle, then polygon $CAPB$ is a rectangle, and $\overline{AB}$ is a diagonal of this rectangle.

However, $\overline{CP}$ is the other diagonal of this same rectangle. Since the diagonals of a rectangle are congruent, then $\overline{CP}$ has exactly the same length as $\overline{AB}$.

It is clear from the drawing that $\overline{CP}$ is a radius of the circle. Therefore, its length must be 6 units.

So the length of $\overline{AB}$, which is the distance from $A$ to $B$, is also 6 units.




**Check** **Check to make sure your answer makes sense.**
There is no question that the length of $\overline{AB}$ is that of the radius of this circle, or 6 units. This fact was merely hidden until the problem was looked at from a different point of view.

**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Problem 2:** A model car is moving toward a wall at the rate of 6 centimeters per second (cm/s). At the moment the car is 810 cm from the wall, a fly—which had been resting on the front of the car—flies directly toward the wall at the rate of 10 cm/s. When the fly reaches the wall, it immediately reverses direction until it again reaches the car, at which moment it reverses direction again and heads for the wall. It continues going back and forth between the car and the wall until the car hits the wall! (All the while, the fly maintains its 10 cm/s air speed.) In all, how many centimeters did this fly travel?

**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** The model car moves toward a wall at the rate of 6 cm/s.
When the car is 810 cm from the wall, the fly begins to fly back and forth between the car and the wall at a speed of 10 cm/s.
The fly continues flying until the car hits the wall.

**Question:** How far does the fly go in all?

**Plan** **Select a strategy.**

Because you know the car's speed and the fly's speed, you could determine where the car was when the fly first reached the wall. You could then record the initial 810 cm the fly flew, and then you could reset the problem from there. This could take some time, however. Perhaps a more efficient strategy would be to *Adopt a Different Point of View*.

**Solve** **Apply the strategy.**

Think about *how much time* the fly was flying rather than the distance it flew. Because it never changed its speed, if you know how long it was in the air, you can determine how far it traveled. You know the fly was flying for as long as it took the car to reach the wall. How long was that?

To find how long, use the Distance Formula.

$$\text{distance} = \text{rate} \bullet \text{time} \rightarrow \text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\text{time} = \frac{810 \text{ cm}}{6 \text{ cm/s}} = 135 \text{ s}$$

So the fly was flying for 135 seconds.

The fly's rate was 10 cm/s, so in 135 seconds, it would have flown 10 • 135, or 1350 cm.

**Check** **Check to make sure your answer makes sense.**

Because the fly moved at 10 cm/s, flying 1350 cm would have taken 135 seconds.

Because the car traveled at 6 cm/s, in 135 seconds, it would have traveled 135 • 6, or 810, cm. This is exactly the car's distance from the wall when the fly started flying.

So the answer checks.

Go to PRACTICE BOOK Lesson 9-14 for exercise sets.

# Enrichment:
## Quadrilaterals from Quadrilaterals

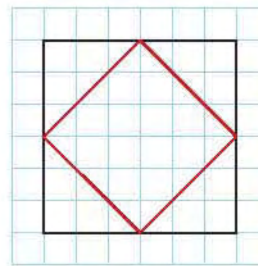
**Objective** **To investigate the nature of quadrilaterals formed by joining the midpoints of other quadrilaterals**
• **To make conjectures about the conditions needed to form a particular quadrilateral**

As you know, a square is a quadrilateral with four right angles and four congruent sides. What kind of quadrilateral would you create if you connected the midpoints of the sides of a square?

Use grid paper and a ruler.

Draw a square. Find the midpoints of its sides and join them.

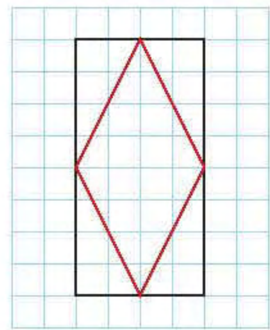
You can see that the new quadrilateral that is formed is another square. Compare drawings with other students to be sure that you all have the same results.



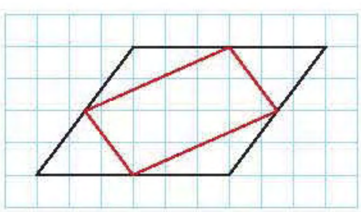
► A rectangle is a parallelogram with four right angles. What kind of quadrilateral is formed when you connect the midpoints of the sides of a rectangle?

Draw a rectangle that is not a square and join the midpoints of its sides.

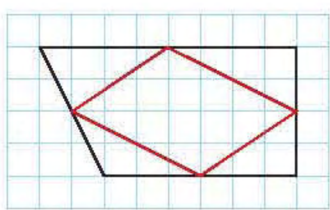
You can see that a rhombus is formed. Compare drawings with other students to check that everyone has drawn a rhombus.



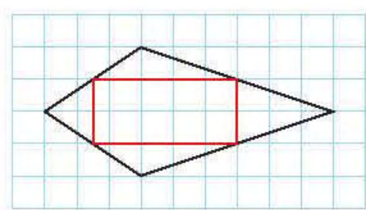
► Now, try connecting the midpoints of the sides of a non-rectangular parallelogram, a trapezoid, and a kite. Your shapes should look different than the ones below.



**Parallelogram**



**Trapezoid**

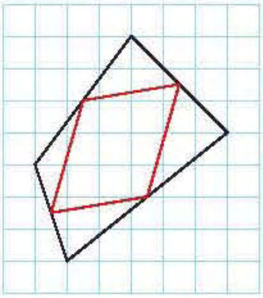


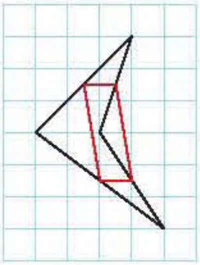
**Kite**

So far, for all the quadrilaterals you have tried, you should have found that the "midpoint quadrilateral" is a parallelogram. (Sometimes it is a square or a rectangle or a rhombus, but these are all parallelograms.)

Does this surprise you? Maybe not. After all, each of these quadrilaterals is "regular" in some way. For example, a parallelogram has two pairs of sides that are parallel and congruent, a trapezoid has one pair of parallel sides, and a kite has two pairs of sides that are the same length.

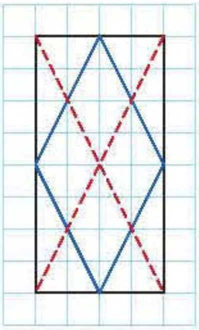
► What if you start with a quadrilateral that has no parallel or congruent sides? Try it. Make the quadrilaterals as crazy as you like. Some examples are shown at the right. Be sure to try some concave quadrilaterals (quadrilaterals with a "dent," such as the one at the far right.)

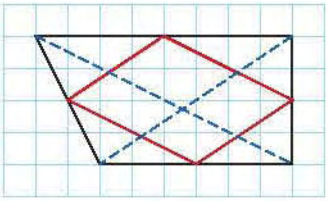


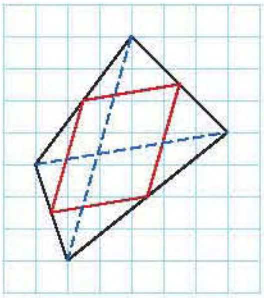


► You should have discovered that, no matter what quadrilateral you start with, the midpoint quadrilateral is a parallelogram. Why does this happen?

One way to see why is to draw the diagonals of each of the original quadrilaterals you drew. Here are some examples.







In each case, you should find that two sides of the midpoint quadrilateral are parallel to one of the diagonals and two are parallel to the other. So, in each case, the midpoint quadrilateral has two parallel sides. That is, the midpoint quadrilateral is a parallelogram.

## Try These

**Use your drawings from this lesson (or make more drawings) to answer the following questions.**

**1.** If the diagonals of the original quadrilateral are the same length, what kind of midpoint quadrilateral is formed?

**2.** If the diagonals of the original quadrilateral are perpendicular, what kind of midpoint quadrilateral is formed?

**3.** If the diagonals of the original quadrilateral are the same length *and* perpendicular, what kind of midpoint quadrilateral is formed?

**4.** **Discuss and Write** Find the midpoints of one (or more) of your midpoint quadrilaterals. Join them to form a quadrilateral. Tell how the new midpoint quadrilateral relates to the original quadrilateral.

Go to PRACTICE BOOK pages 299–300 for exercise sets.

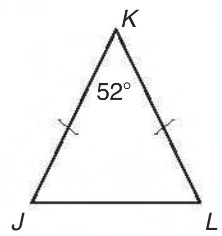
# Test Prep: Multiple-Choice Questions

## Strategy: Apply Mathematical Reasoning

When solving a multiple-choice question, *look for relationships* presented in statements, diagrams, graphs, or tables. Use mathematical reasoning to help you interpret the written and visual cues.

**Sample Test Item**

Triangle JKL is an isosceles triangle with $\overline{KJ} \cong \overline{KL}$. What is the measure of $\angle L$?




**A.** 52° **C.** 116°
**B.** 64° **D.** 128°

**Read the whole test item, including answer choices.**

- Underline important words.
  Triangle $JKL$ is an isosceles triangle.
- Restate the question in your own words.
  How many degrees is $\angle L$?

**Test-Taking Tips**

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Solve the problem.**

- Use appropriate definitions and properties.
  Triangle $JKL$ is isosceles.
  Since $\overline{KJ} \cong \overline{KL}$, $\angle J \cong \angle L$.

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

- Write and solve an equation.

$m\angle J + m\angle L + 52° = 180°$ ← The sum of the measures of angles of a triangle is 180°.

$m\angle J + m\angle L + 52° - 52° = 180° - 52°$ ← Use the Subtraction Property of Equality.

$m\angle L + m\angle L = 128°$ ← Since $m\angle J = m\angle L$, substitute $m\angle L$ for $m\angle J$.

$2m\angle L = 128°$

$\frac{\cancel{2}^{1}m\angle L}{\cancel{2}_{1}} = \frac{\cancel{128°}^{64}}{\cancel{2}_{1}}$ ← Use the Division Property of Equality.

$m\angle L = 64°$

**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices. Watch out for distractors.

**A.** 52° ← $\angle K$ and $\angle L$ are not the congruent angles. Eliminate this choice.

**B.** 64° ← $64° + 64° + 52° = 180°$. This is the correct choice!

**C.** 116° ← This is the sum of the measures of $\angle K$ and $\angle L$. Eliminate this choice.

**D.** 128° ← This is the sum of the measures of $\angle J$ and $\angle L$. Eliminate this choice.

**Try These**

**Choose the correct answer. Explain how you used strategies.**

**1.** Triangle $ABC$ is congruent to triangle $QRS$. If $BC = 15$ cm and $AC = \frac{2}{3}BC$, what is $QS$?

**A.** 30 cm **C.** 10 cm
**B.** 15 cm **D.** 5 cm

**2.** The library is 6 km north and 8 km west of Maia's house. What is the shortest distance between the library and Maia's house?

**F.** 6 km **H.** 10 km
**G.** 8 km **J.** 14 km



Go to PRACTICE BOOK page 301 for exercise sets.

# Two-Dimensional Geometry and Measurement Applications

CHAPTER 10

## In This Chapter You Will:

- Distinguish between precision and accuracy in measurements
- Use formulas to find the perimeter and area of polygons, complex figures, and circles
- Find the positive and negative square roots of perfect and nonperfect squares
- Distinguish between rational and irrational numbers
- Use the Pythagorean Theorem
- Explore the effect of changes in dimensions on the areas of polygons
- Identify line, rotational, and point symmetry
- Apply the strategy: *Account for All Possibilities*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- A point is an exact location in space.
- A line is a continuous set of points in a straight path.
- Perpendicular lines form right angles.
- A polygon is a closed plane figure composed of line segments that only intersect at their endpoints.
- A diameter of a circle is any chord that intersects the center of the circle.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 307–340

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES


## Critical Thinking

The route that the runners will follow in Saturday's race is in the form of a pentagon. The sides of that pentagon measure about $4\frac{1}{2}$ miles, 8 miles, $5\frac{3}{4}$ miles, $3\frac{3}{4}$ miles, and 4 miles. About how many miles is the perimeter of the polygon formed by the race route?

# 10-1 Precision and Accuracy in Measurement

**Objective** To explore the difference between precision and accuracy of measurements • To find the greatest possible error (GPE) and the relative error of a measurement • To measure to a given unit of precision • To apply precision and significant digits in computations

Nancy checks the weight of some vegetables. She tries two different scales in the market. The first scale weighs to the nearest pound; the second scale weighs to the nearest ounce. The vegetables weigh 4 pounds on the first scale and 67 ounces on the second scale. If the vegetables actually weigh $4\frac{1}{5}$ pounds, which measurement is more precise? Which is more accurate?

**Key Concept**

**Precision and Accuracy**
Measurements are approximate.
The *precision* of a measurement depends on the unit of measure. The smaller the unit is, the more precise the measurement is.
The *accuracy* of a measurement depends on how close the measurement is to the exact value.

► Precision refers to the smallest unit of measurement on the measuring instrument. The smaller the unit of measurement, the more precise the measurement will be. So the measurement of 67 ounces is the more precise measurement.

► Accuracy refers to how close a measurement is to the actual value. Since 4 lb = 64 oz and $4\frac{1}{5}$ lb = 67.2 oz, and since 67 ounces is closer to the actual weight of 67.2 ounces than 64 ounces is, it is a more accurate measurement than 4 pounds. Note, however, that a more precise measurement is *not always* a more accurate measurement.

How long is the key to the nearest centimeter? To the nearest millimeter?


The key is 3 centimeters or 29 millimeters depending on the precision of the measurement.

► The degree of precision required for a measurement depends on the situation. A centimeter more or less in the measurement of a baseball field makes very little difference. A centimeter more or less in a measurement made by a watchmaker is very important.

► The greatest possible error (GPE) is considered to be one half of the smallest unit that the measuring instrument can measure. So, when Nancy measures the weight of the vegetables as 4 pounds using the first scale, the GPE of her measurement is $\frac{1}{2}$ pound; and the vegetables could weigh from $3\frac{1}{2}$ to $4\frac{1}{2}$ pounds. When Nancy measures the weight of the vegetables as 67 ounces using the second scale, the GPE of her measurement is $\frac{1}{2}$ ounce; and the vegetables could weigh from $66\frac{1}{2}$ to $67\frac{1}{2}$ ounces.

► A measurement is said to be more accurate than another measurement if its *relative error* is smaller.

**Key Concept**

**Absolute Error and Relative Error**

- No measurement is exact. The uncertainty in any given measurement is referred to as *error*. The error does *not* mean "mistake."
- The absolute error of a measurement is the actual size of the error. It can be represented by the GPE.
- The relative error of a measurement is the absolute error in relation to the correct value (or to the measured value if the correct value is not known).

## Example

**1** Find the absolute error and the relative error for Nancy's measurements. Use your results to explain which measurement is more accurate.

| | actual weight − measurement = absolute error | $\frac{\text{GPE}}{\text{given measurement}}$ = relative error |
|---|---|---|
| **First Scale** | 67.2 oz − 64 oz = 3.2 oz | $\frac{0.5 \text{ lb}}{4 \text{ lb}} = 0.125 = 12.5\%$ |
| **Second Scale** | 67.2 oz − 67 oz = 0.2 oz | $\frac{0.5 \text{ oz}}{67 \text{ oz}} \approx 0.007 \approx 0.7\%$ |

So the measurement on the second scale is more accurate because it has the smaller relative error.

- Measurements obtained experimentally (by using a ruler or other measuring instrument) should be stated with a number of *significant digits*, which give a reasonable impression of the precision of the measurement. Significant digits are all nonzero digits, zeros between nonzero digits, and zeros following the last nonzero digit to the right of the decimal point. For example, 85,401 has five significant digits, and 0.050 has two significant digits.

- When adding or subtracting experimental measurements, round the final result to have the same number of decimal places to the right of the decimal point as the measurement that has the fewest decimal places to the right of the decimal point.

- When multiplying or dividing experimental measurements, round the final result to have the same number of significant digits as the measurement that has the fewest significant digits.

## Try These

**1.** Identify the unit of precision for the ruler shown on page 272.

**Give the GPE and the percent of relative error to the nearest tenth of a percent.**

**2.** 72 ft  **3.** 14 oz  **4.** 12.5 mm

**5. Discuss and Write** If you measure the width of a rectangle as 4.55 cm and you measure its length as 8.4 cm, how would you express its area? Explain.

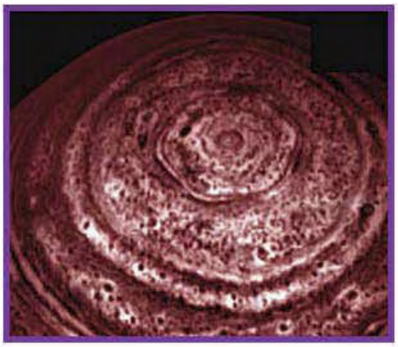
Go to PRACTICE BOOK Lesson 10-1 for exercise sets.

Algebra

# 10-2 Perimeter

**Objective** To use a formula to find the perimeter of a regular polygon • To find missing dimensions given the perimeter of a polygon • To find the perimeter of a complex figure • To explore how changing the dimensions of a polygon affects the perimeter

NASA's Cassini spacecraft photographed an enormous hexagon above Saturn's north pole. If it were a regular hexagon, with each side measuring about 12 500 km, what would its perimeter be?



► The perimeter ($P$) of a polygon is the sum of its side lengths. Let $s$ represent the length of one side of a regular hexagon.

$P = s + s + s + s + s + s = 6s$
$= 6(12{,}500)$ ← Substitute 12,500 for $s$.
$= 75{,}000$

So the perimeter of the hexagon would be 75 000 km.

► Nonsquare rectangles are *not* regular polygons because all the sides are *not* the same length. Find the perimeter of a rectangle with a length of 275.4 ft and a width of 102.3 ft.

$P = 2(\ell + w)$
$= 2(275.4 + 102.3)$ ← Substitute given values.
$= 2(377.7)$ ← Simplify.
$= 755.4$

So the perimeter of the rectangle is 755.4 ft.

**Key Concept**

**Perimeter of a Regular Polygon**
$P = ns$, where $n$ is the number of sides and $s$ is the side length

**Perimeter of a Rectangle**
$P = 2\ell + 2w$ or $P = 2(\ell + w)$, where $\ell$ is the length and $w$ is the width

► If you know the values of all but one variable in a formula, you can use algebra to find the value of the unknown variable.

The perimeter of a regular octagon is 128 inches. What is the length of each side?

$P = 8s$
$128 = 8s$ ← Substitute for $P$.
$\frac{128}{8} = \frac{8s}{8}$ ← Divide both sides by 8.
$16 = s$ ← Simplify.

**Check:** $P = 8s$
$128 \stackrel{?}{=} 8(16)$
$128 = 128$ True

So each side is 16 inches long.

A rectangle has a perimeter of 198 meters and a length of 66.4 meters. What is its width?

$P = 2\ell + 2w$
$198 = 2(66.4) + 2w$ ← Substitute for $P$ and $\ell$.
$198 = 132.8 + 2w$ ← Multiply.
$198 - 132.8 = 132.8 - 132.8 + 2w$ ← Subtract 132.8 from both sides.
$65.2 = 2w$ ← Simplify.
$65.2 \div 2 = 2w \div 2$ ← Divide both sides by 2.
$32.6 = w$

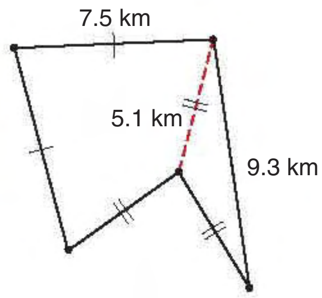
**Check:** $P = 2\ell + 2w$
$198 \stackrel{?}{=} 2(66.4) + 2(32.6) \rightarrow 132.8 + 65.2$
$198 = 198$ True

So the width is 32.6 meters.

▶ When finding the perimeter of a polygon that is not regular and has no special formula, sometimes you need to use information given in the diagram or the description to help find unlabeled dimensions.

Find the perimeter of the polygon.

**Remember:** Identical tick marks show congruency or equal measures.



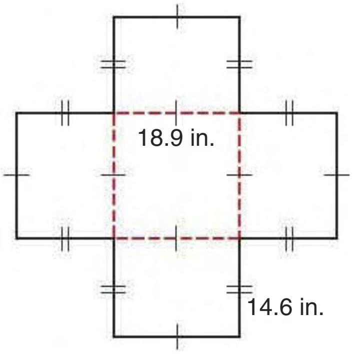

Notice that the segment labeled *5.1 km* is inside of the figure; it is not a side.

Only two sides are labeled. However, the tick marks show that two of the unlabeled sides are congruent to the segment labeled *5.1 km*, and one is congruent to the side labeled *7.5 km*.

Add the side lengths to find the perimeter.

$P = 5.1 + 5.1 + 7.5 + 7.5 + 9.3 = 34.5$ km

Find the perimeter of the polygon.




The figure has 12 sides. Only one side length is given. However, the markings show that four of the sides are 18.9 in. long and eight of the sides are 14.6 in. long.

You can use multiplication and addition to find the perimeter.

$P = 4(18.9) + 8(14.6) = 192.4$ in.

▶ If all the side lengths of a polygon are multiplied by the same number, the perimeter will also be multiplied by that number.
Find the perimeter of a square with sides of length $6x$.
Then find the perimeter if the side lengths are tripled.

Original square:

$P = 4s$
$= 4(6x)$
$= 24x$

New square:

$P = 4s$
$= 4(3 \cdot 6x)$
$= 72x$

**Think** $(24x)3 = 72x$

So tripling the side lengths of the square triples the perimeter.

## Try These

**Find the perimeter of the polygon.**

**1.** Rectangle: $\ell = 101$ m; $w = 22$ m

**2.** Regular heptagon: $s = 47.6$ cm

**Find the side length of the regular polygon with the given perimeter.**

**3.** Nonagon: $P = 48.6$ in.

**4.** Square: $P = 1024$ mm

**5.** Hexagon: $P = 3913.2$ ft

**6.** **Discuss and Write** If you know the perimeter of a rectangle, can you find its dimensions? Explain your answer.

Go to PRACTICE BOOK Lesson 10-2 for exercise sets.

Algebra 10-3

Update your skills. See page 411 XI.

# Squares and Square Roots

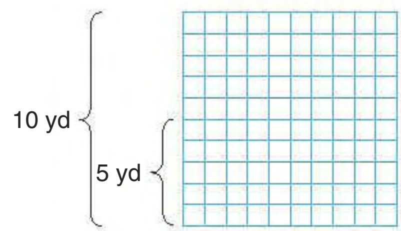
**Objective** **To find the square root of a perfect square • To understand negative square roots of perfect squares • To simplify expressions involving perfect squares and square roots • To use a calculator to find square roots**

An archaeologist uses a grid map to divide a dig site into four square regions of the same size, one for each of his four students to explore and dig. The dimensions of the dig site are 10 yards by 10 yards. What are the area and dimensions of each student's square region?

► To find the area of a square with side length $s$, multiply $s$ by itself. Multiplying a number by itself, or raising it to the second power, is finding the square of the number.

Use the formula for the area of a square, $A = s^2$, to find the area of the total dig site.

$A = s^2$ ← area formula for a square
$= (10 \text{ yd})^2$ ← Substitute the side length for $s$.
$= (10 \text{ yd})(10 \text{ yd})$ ← Evaluate the exponent.
$= 100 \text{ yd}^2$ ← Simplify




There are 4 students, so each student has a square region that is $100 \text{ yd}^2 \div 4$, or $25 \text{ yd}^2$ ("25 square yd") to dig.

► The numbers 100 and 25 are examples of square numbers, or *perfect squares*. Perfect squares are squares of the counting (or *natural*) numbers, 1, 2, 3, 4, 5, . . . . So the perfect squares are 1, 4, 9, 16, 25, 36, 49, and so on. The third row in the table below shows the first ten perfect squares.

| Counting Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Square of the Number | $1^2$ | $2^2$ | $3^2$ | $4^2$ | $5^2$ | $6^2$ | $7^2$ | $8^2$ | $9^2$ | $10^2$ |
| Perfect Square | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |

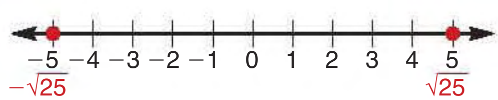
A square root of a number is a number that equals the original number when multiplied by itself. Every positive number has both a *positive square root* and a *negative square root*.

The symbol $\sqrt{\phantom{x}}$, the radical sign, stands for the positive square root of a number: $\sqrt{36} = 6$ and $\sqrt{144} = 12$.
A positive square root is also called the principal square root.

The positive square root of 100 is 10 because $10 \cdot 10 = 10^2 = 100$. The positive square root of a number $n$ is the side length of a square with area $n$. So a square with an area of $100 \text{ yd}^2$ has a side length of 10 yd. The positive square root of 25 is 5 because $5 \cdot 5 = 5^2 = 25$.

The negative square root of a number is the opposite of the number's positive square root.

Since $(-5)(-5) = 25$, $-5$ is the negative square root of 25. In symbols, $-\sqrt{25} = -5$. The negative square root cannot represent the side length of a square.




So each archaeology student has a 5 yd by 5 yd square region to explore and dig.

► You can use prime factorization to find the square root of a perfect square. Find the square root of 225.

$x = \sqrt{225}$ ← Square both sides. $(\sqrt{225})^2 = 225$
$x^2 = 225$
$= 3 \bullet 3 \bullet 5 \bullet 5$ ← Find the prime factorization of 225.
$= (3 \bullet 5)(3 \bullet 5)$ ← Rewrite as a product of two identical factors.
$= (15)(15)$ ← Simplify.
$= 15^2$

So $x = 3 \bullet 5 = 15$, and $\sqrt{225} = 15$.

**Remember:**
The prime factorization of a number shows the number as the product of prime factors.

► You can use what you know about perfect squares to simplify radical expressions.

## Examples

**1** Find the square root of 324.

$n = \sqrt{324}$
$n^2 = 324$
$= 2 \bullet 2 \bullet 3 \bullet 3 \bullet 3 \bullet 3$
$= (2 \bullet 3 \bullet 3)(2 \bullet 3 \bullet 3)$
$= (2 \bullet 3 \bullet 3)^2$
$= (2 \bullet 9)^2$
$= 18^2$

So $\sqrt{324} = 18$.

**2** Simplify the expression $\sqrt{8^2 - (-5)^2 + 10}$.

$\sqrt{8^2 - (-5)^2 + 10}$
$= \sqrt{64 - 25 + 10}$ ← Evaluate powers.
$= \sqrt{49}$ ← Add or subtract under radical sign.
$= 7$ ← Simplify radical.

**3** Simplify the expression $\sqrt{400 + 225} + 3$.

$\sqrt{400 + 225} + 3$ ← Add under radical sign.
$= \sqrt{625} + 3$ ← Find the square root.
$= 25 + 3$ ← Simplify.
$= 28$

## Technology

To use a calculator to find $\sqrt{2809}$, press the following keys:

2ND $x^2$ 2 8 0 9 ENTER → 53

So $\sqrt{2809} = 53$.

## Try These

**Find both square roots of each number.**

**1.** 169 **2.** 196 **3.** 10,000 **4.** 20,736

**Simplify each expression.**

**5.** $\sqrt{13^2 - 12^2} + 15$ **6.** $\sqrt{64} - \sqrt{9}$ **7.** $\sqrt{1} - 1$ **8.** $\sqrt{196} + \sqrt{25 - 16}$

**9.** **Discuss and Write** Use graph paper and try to construct a square with whole number side lengths and an area of 12 square units. Why is it not possible to construct this square?

Go to PRACTICE BOOK Lesson 10-3 for exercise sets.

Algebra 10-4

# Irrational Numbers

**Objective** To distinguish rational and irrational numbers • To approximate the square root of a number that is not a perfect square • To locate irrational numbers on a number line

A square region of farmland has an area of 44 square kilometers. What is the length of each side of the region of land? Round the length to the nearest tenth of a kilometer.

The length of a side of a square with an area of 44 $km^2$ is $\sqrt{44}$. Use this idea to solve the problem.

► You can use estimation to approximate numbers such as $\sqrt{44}$. To estimate the square root of a number that is not a perfect square, consider the perfect squares just before and just after the number under the radical sign.

1. To estimate $\sqrt{44}$, identify the perfect squares that 44 is between. — 44 is between 36 and 49.
2. Compare these as inequalities. — $36 < 44 < 49$
3. Take the square root of the numbers, and evaluate the perfect squares. — $\sqrt{36} < \sqrt{44} < \sqrt{49}$

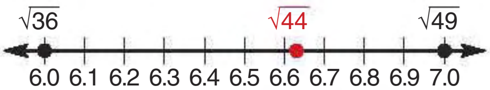
So $\sqrt{44}$ is between 6 and 7. It is closer to 7 than to 6. — $6 < \sqrt{44} < 7$

► You can use the square root key on your calculator to find a better estimate.

Press 2ND $x^2$ 4 4 ENTER 6.633249581

This tells you that $\sqrt{44}$ is a little more than 6.6.

► You can locate numbers such as $\sqrt{44}$ on a number line.




So the length of each side of the square region of farmland is about 6.6 kilometers.

## Examples

**1** Approximate $\sqrt{23}$ to the nearest whole number.

Identify the perfect squares closest to $\sqrt{23}$. — $\sqrt{16} < \sqrt{23} < \sqrt{25}$

Find the positive square roots of the perfect squares. — $4 < \sqrt{23} < 5$

Since 23 is closer to 25 than to 16, $\sqrt{23}$ to the nearest whole number is about 5.

**2** Approximate $\sqrt{31}$ to the nearest tenth.

Press 2ND $x^2$ 3 1 ENTER 5.567764363

Round the answer to the nearest tenth.

$5.567764363 \approx 5.6$

The square root of 31 rounded to the nearest tenth is 5.6.

► Irrational numbers are numbers such as $\sqrt{44}$ that cannot be expressed as quotients of two integers, $\frac{a}{b}$, where $b \neq 0$. When an irrational number is expressed in decimal form, the digits do not terminate and do not repeat.

- The numbers 0.01011011101111 . . . and −12.34567819 . . . are examples of irrational numbers.
- The square root of any whole number that is not a perfect square is an irrational number.
- The number pi, which is symbolized by the Greek letter $\pi$, is an example of an irrational number. Pi is the relationship between the circumference ($C$) of any circle and its diameter ($d$).

So $\sqrt{44}$ is an irrational number because it is the square root of a whole number that is not a perfect square.

## Example

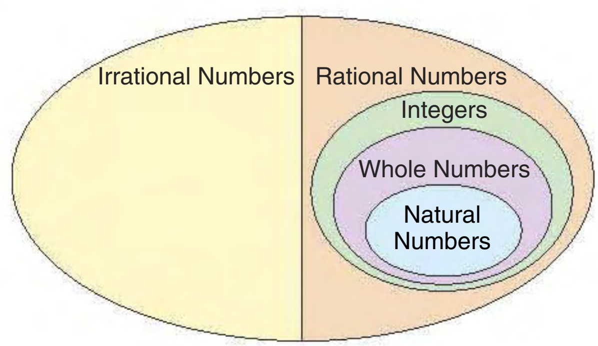
**1** Which of the following numbers is irrational? Explain why.

| Number | Type and Explanation |
|---|---|
| $\sqrt{35}$ | Irrational; 35 is not a perfect square. |
| $\frac{17}{41}$ | Rational; $\frac{17}{41}$ is a quotient of two integers. |
| $-\sqrt{100}$ | Rational; 100 is a perfect square and $-\sqrt{100} = -10$. |
| 1.65 | Rational; 1.65 is a terminating decimal. |

**Remember:** The set of rational numbers includes natural (counting) numbers, whole numbers, integers, proper fractions, improper fractions, mixed numbers, terminating decimals, repeating decimals, and percents.

► The set of rational numbers and the set of irrational numbers together make up the set of real numbers.

**Real Numbers**




## Try These

**Classify each number as *rational* or *irrational*.**

**1.** $\sqrt{99}$ **2.** $\frac{\pi}{2}$ **3.** $3.\overline{33}$ **4.** 0.875

**Identify the closest perfect squares.**

**5.** $\sqrt{67}$ **6.** $\sqrt{80}$

**Find the square root to the nearest whole number.**

**7.** $\sqrt{6}$ **8.** $\sqrt{19}$

**9. Discuss and Write** Find three irrational numbers between 7 and 8. Explain how you found your answers. How many irrational numbers are between 7 and 8?

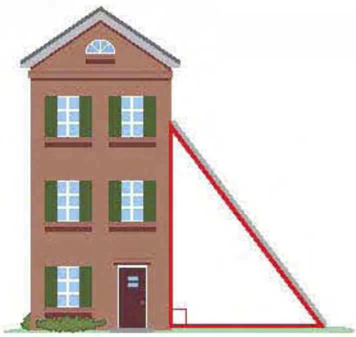
Go to PRACTICE BOOK Lesson 10-4 for exercise sets.

Algebra

# 10-5 Pythagorean Theorem

Update your skills. See page 408 III.

**Objective** To use the Pythagorean Theorem to find a missing side of a right triangle • To determine whether a given triangle is a right triangle



The third-story window of a building is 12 meters above the ground. A firefighter places the base of a ladder 9 meters from the building. What length is necessary for the ladder to reach the window?

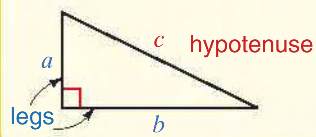
► To solve this problem, notice that the ladder forms the hypotenuse of a right triangle, so you can apply the *Pythagorean Theorem*.

**Key Concept**

**Pythagorean Theorem**

In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. That is, if $a$ and $b$ are the lengths of the legs and $c$ is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

**Remember:** In a right triangle, the legs are the sides that form the right angle. The hypotenuse is the side *opposite* the right angle. It is the longest side of the triangle.




You can use the Pythagorean Theorem to find the length of the ladder.

$a^2 + b^2 = c^2$ ← Pythagorean Theorem

$12^2 + 9^2 = c^2$ ← Substitute the given values 12 and 9 for $a$ and $b$.

$144 + 81 = c^2$ ← Evaluate the squares.

$225 = c^2$ ← Find the sum.

$\sqrt{225} = \sqrt{c^2}$ ← Find the positive square root of both sides.

$15 = c$ ← Simplify.

So the ladder must be 15 meters long in order to reach the window.

► If you know the length of any two sides of a right triangle, you can use the Pythagorean Theorem to find the length of the third side.

A television screen measures 20 inches along its diagonal and 15 inches along its base. What is the height of the television screen to the nearest tenth? Let $a = 15$ inches and $c = 20$ inches.

$a^2 + b^2 = c^2$ ← Pythagorean Theorem

$15^2 + b^2 = 20^2$ ← Substitute the known values.

$225 + b^2 = 400$ ← Evaluate the squares.

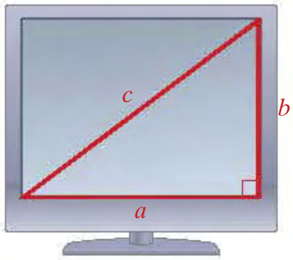
$225 - 225 + b^2 = 400 - 225$ ← Subtract 225 from both sides.

$b^2 = 175$ ← Simplify.

$\sqrt{b^2} = \sqrt{175}$ ← Find the positive square root of both sides.

$b \approx 13.22875656\ldots$ ← Simplify. Use your calculator.

$b \approx 13.2$ ← Round to the nearest tenth.

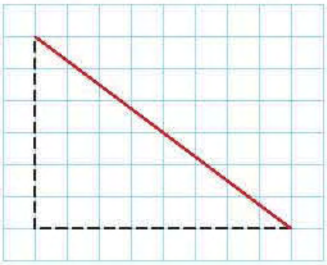



So the height of the television screen is about 13.2 inches.

► You can use the Pythagorean Theorem equation to decide whether or not a given triangle is a right triangle. You can use a calculator to verify whether or not the equation $a^2 + b^2 = c^2$ is true for the side lengths $a$, $b$, and $c$ of a given triangle.

► You can apply the Pythagorean Theorem to find the length of a line shown on a grid.

Find the length of the red line segment on the grid at the right. Assume that the side of each small square on the grid is 1 unit.



1. Draw a vertical line segment from the upper endpoint of the red line segment straight down to the horizontal grid line that aligns with the other endpoint of the red line segment.
2. Draw a horizontal line segment that connects the vertical line segment to the red line segment. The corner of the grid box will help you form a right angle.
3. Since you have constructed a right triangle, you can now use the Pythagorean Theorem to find the length of the hypotenuse (the red line segment). Count units to determine the lengths of the legs as 6 units and 8 units.

$a^2 + b^2 = c^2$ ← Pythagorean Theorem

$6^2 + 8^2 = c^2$ ← Substitute 6 and 8 for $a$ and $b$.

$36 + 64 = c^2$ ← Evaluate the squares.

$100 = c^2$ ← Find the sum.

$\sqrt{100} = \sqrt{c^2}$ ← Find the positive square root of both sides.

$10 = c$ ← Simplify.

So the length of the red line segment is 10 units.

► Each group of three numbers that can form the side lengths of a right triangle is called a Pythagorean triple. Common Pythagorean triples are 3-4-5, 5-12-13, and 8-15-17. Multiples of these triples are also Pythagorean triples. For example, 3, 4, and 5 can each be multiplied by 3 to produce another Pythagorean triple, 9-12-15.

## Try These

**Determine whether a triangle with the given side lengths is a right triangle.**

**1.** 14 cm, 15 cm, 16 cm **2.** 30 in., 40 in., 50 in. **3.** 3.5m, 12 m, 12.5 m

Suppose that $a$ and $b$ are the lengths of the legs of a right triangle and that $c$ is the length of the hypotenuse. **Find the length of the missing side to the nearest tenth. You may use a calculator.**

**4.** $a = 16$ km, $b = 63$ km **5.** $b = 100$ in., $c = 150$ in. **6.** $a = 13$ miles, $c = 17$ miles

**7. Discuss and Write** When each leg of a right triangle is doubled in length, what happens to the length of the hypotenuse? Is this always true? Explain.

Go to PRACTICE BOOK Lesson 10-5 for exercise sets.

Algebra

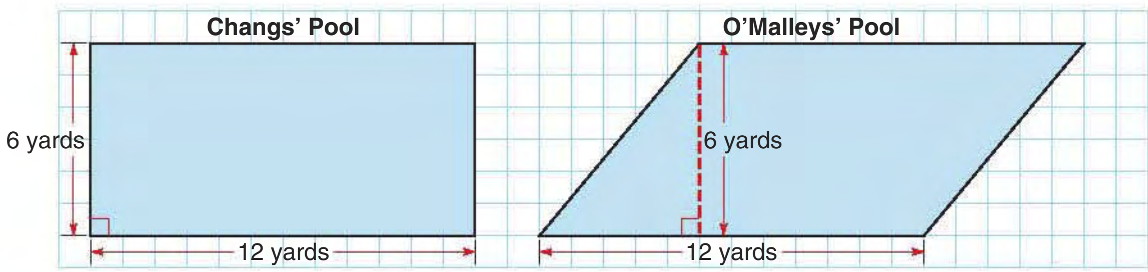
10-6

*Update your skills. See page 410.*

# Area of Parallelograms

**Objective** **To use a formula to find the area of a parallelogram • To rename area units in equivalent forms • To explore the effect of a change in the base or the height on a parallelogram's area • To find an unknown base or height given a parallelogram's area**

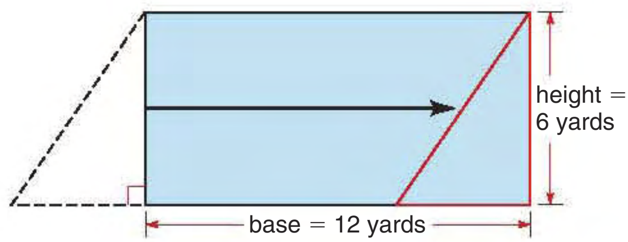
The Changs and the O'Malleys have swimming pools in their backyards. Diagrams of the pools are shown below. What are the areas of the two pools?



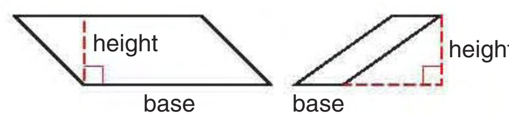

► Area is the number of square units that cover a figure. The Changs' pool is shaped like a rectangle. To find its area, use the formula for the area of a rectangle: $A = \ell w$, where $\ell$ = length and $w$ = width.

$$A = \ell w$$
$$= 12 \text{ yd} \bullet 6 \text{ yd}$$
$$= 72 \text{ yd}^2$$

The O'Malleys' pool is a parallelogram. To find its area, imagine removing the triangular region on the left side of the pool and shifting it right. You can then see that the two pools have areas that are *exactly* the same. The base and the height remain the same in the imaginary rectangle as in the parallelogram.




► The base of a parallelogram is the length of any of its sides. The altitude is a perpendicular line segment from a base to the opposite side. The height of a parallelogram is the length of the altitude. To find the height of some parallelograms, it is necessary to drop a perpendicular line segment to an extension of the base.




You can use the formula for the area of a parallelogram to find the area of the O'Malleys' pool.

$$A = bh$$
$$= 12 \text{ yd} \bullet 6 \text{ yd}$$
$$= 72 \text{ yd}^2$$

**Key Concept**

**Area of a Parallelogram**

$A = bh$, where $b$ is the base and $h$ is the height

The formula for the area of a parallelogram is related to the formula for the area of a rectangle that has the same length and width as the base and height of the parallelogram.

So the two pools have the same area, 72 $\text{yd}^2$.

► You can rename the area of a figure by finding its equivalent in different units. For example, since 1 yd = 3 ft, then 1 $\text{yd}^2$ = 9 $\text{ft}^2$. So to find the area of the Changs' pool in square feet, multiply: 72 • 9 = 648. The area of the Changs' pool is 648 $\text{ft}^2$.

**Think**

$$\begin{array}{ccccc} 1 \text{ yd} & \bullet & 1 \text{ yd} & = & 1 \text{ yd}^2 \\ \downarrow & & \downarrow & & \downarrow \\ 3 \text{ ft} & \bullet & 3 \text{ ft} & = & 9 \text{ ft}^2 \end{array}$$

So the number of square feet is 9 times the number of square yards.

► A parallelogram has a base of 2.5 cm and a height of 6 cm.
What would happen to the area of the parallelogram if its height were tripled?

- First, find the area of the original figure.

$A = bh$
$= 2.5 \text{ cm} \bullet 6 \text{ cm}$
$= 15 \text{ cm}^2$

- Then find the area of the new figure.

New height: $6 \text{ cm} \bullet 3 = 18 \text{ cm}$
$A = bh$
$= 2.5 \text{ cm} \bullet 18 \text{ cm}$
$= 45 \text{ cm}^2$

So the area of the new parallelogram would be three times the area of the original parallelogram.

**Key Concept**

**Changing Dimensions**

If you multiply either the base or the height of a parallelogram by a number $n$, the new area will be $n$ times the area of the original.

If you multiply *both* the base and the height of the parallelogram by $n$, the new area will be $n^2$ times the area of the original.

► What would happen to the area of the original parallelogram if *both* its height and its base were tripled?

New base: $2.5 \text{ cm} \bullet 3 = 7.5 \text{ cm}$
New height: $6 \text{ cm} \bullet 3 = 18 \text{ cm}$
$A = bh$
$= 7.5 \text{ cm} \bullet 18 \text{ cm}$
$= 135 \text{ cm}^2$

The new area would be 9 times the original area ($15 \text{ cm}^2 \bullet 9 = 135 \text{ cm}^2$).

► If you know the area of a parallelogram and either the base or the height, you can use the area formula to find the unknown dimension.

A wall hanging in the shape of a parallelogram has an area of 206.25 in.$^2$ and a base that measures 8.25 inches. The space for the hanging is 2 feet high. Will the wall hanging fit in the space?

$A = bh$ ← Use the formula for the area of a parallelogram.
$206.25 = 8.25h$ ← Substitute the known values.
$\frac{206.25}{8.25} = \frac{8.25h}{8.25}$ ← Divide both sides by 8.25 to isolate $h$.
$25 = h$ ← Simplify.

The hanging has a height of 25 inches. The space is 2 feet, or 24 inches, tall. So the hanging will not fit because 2 ft < 25 in.

## Try These

**Find the area of the parallelogram for the given dimensions ($b$ = base and $h$ = height).**

1. $b = 400$ in.; $h = 330$ in.
2. $b = 17\frac{1}{4}$ ft; $h = 100$ ft
3. $b = 16.65$ m; $h = 15.30$ m

**Find the unknown measurement in the parallelogram ($A$ = area; $b$ = base; and $h$ = height).**

4. $A = 841 \text{ mm}^2$; $b = 29$ mm
5. $A = 132 \text{ ft}^2$; $h = 12$ ft
6. $A = 195{,}325 \text{ mi}^2$; $h = 300\frac{1}{2}$ mi
7. $A = 54.02 \text{ m}^2$; $b = 14.6$ m
8. **Discuss and Write** If the base and height of a parallelogram were halved, how would its new area compare with its original area? Use an example to explain.

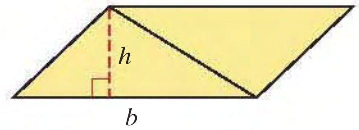
Go to PRACTICE BOOK Lesson 10-6 for exercise sets.

Algebra

# 10-7 Area of Triangles and Trapezoids

**Objective** **To use a formula to find the area of a triangle • To use a formula to find the area of atrapezoid • To rename area units in equivalent forms • To find an unknown base or height given the area of a triangle or a trapezoid**

The area formula for a parallelogram can help you understand the area formulas for a triangle and for a trapezoid.



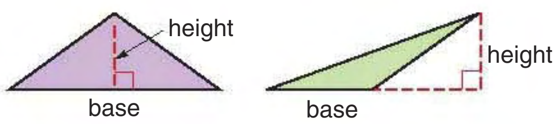

► You can put any two congruent triangles together to form a parallelogram, as shown in the figure. The base and the height of each triangle are the same as the base and the height of the parallelogram. You can see that the area of each triangle is half the area of the parallelogram. Since the area of a parallelogram is $A = bh$, the area of a triangle is $A = \frac{bh}{2}$.

**Key Concept**

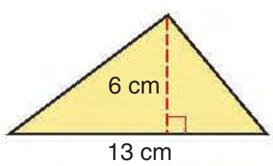
**Area of a Triangle**

$A = \frac{1}{2}bh$, where $b$ is the base and $h$ is the height

The base of a triangle can be any one of its sides. The altitude is the perpendicular line segment from a vertex to the base opposite that vertex. The height is the length of the altitude. As with parallelograms, the height may be determined by dropping a perpendicular line segment from a vertex to an extension of the base that is opposite that vertex.

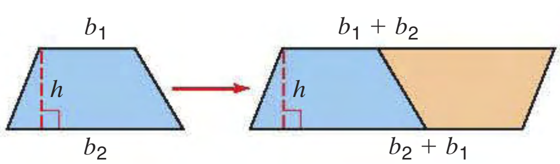



• Find the area of the triangle at the right.

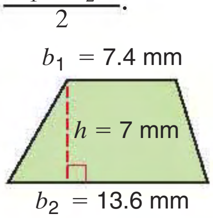



$$A = \frac{1}{2}bh \rightarrow A = \frac{1}{2}(13)(6)$$
$$= \frac{1}{2}(78) = 39 \text{ cm}^2$$

► You can put two congruent trapezoids together to form a parallelogram. The height of the parallelogram is the same as the height of either trapezoid. The base of the parallelogram equals the sum of the bases of either trapezoid. The bases of a trapezoid are the parallel sides. The height is the length of a perpendicular line segment from one base to the other. The area of each trapezoid equals half the area of the parallelogram. Since the area of that parallelogram is $(b_1 + b_2)h$, the area of the trapezoid is $\frac{(b_1 + b_2)h}{2}$.




• Find the area of the trapezoid.




$$A = \frac{1}{2}(b_1 + b_2)h \rightarrow A = \frac{1}{2}(7.4 + 13.6)7$$
$$= \frac{1}{2}(21)7$$
$$= \frac{1}{2}(147) = 73.5 \text{ mm}^2$$

**Key Concept**

**Area of a Trapezoid**

$A = \frac{1}{2}(b_1 + b_2)h$, where $b_1$ and $b_2$ are the bases and $h$ is the height

The bases of a trapezoid are not always horizontal lines, but they are always parallel lines.

► Remember that you can rename the area of a figure by finding its equivalent in different units. Since 1 cm = 10 mm, then 1 $\text{cm}^2$ = 100 $\text{mm}^2$. So to find the area of the trapezoid on the preceding page in square centimeters, divide:

$73.5 \div 100 = 0.735$

The area of the trapezoid is 0.735 $\text{cm}^2$.

**Think**

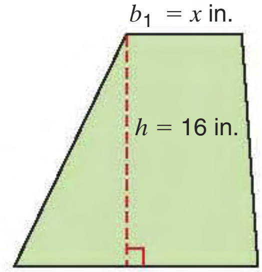
1 cm • 1 cm = 1 $\text{cm}^2$
↓ ↓ ↓
10 mm • 10 mm = 100 $\text{mm}^2$

So the number of square centimeters is the number of square millimeters divided by 100.

► As with parallelograms, if you know the area of a triangle or of a trapezoid and either the base or the height is unknown, you can use an area formula to find the unknown dimension.

## Example

**1** The area of the trapezoid at the right is 184 $\text{in}^2$. Find the missing base length.




$A = \frac{1}{2}(b_1 + b_2)h$ ← Use the formula for area of a trapezoid.

$184 = \frac{1}{2}(x + 17)16$ ← Substitute the known values.

$184 = \frac{1}{2} \bullet 16(x + 17)$ ← Apply the Commutative Property.

$184 = 8(x + 17)$ ← Simplify.

$184 = 8x + (8 \bullet 17)$ ← Apply the Distributive Property.

$184 = 8x + 136$ ← Simplify.

$184 - 136 = 8x + 136 - 136$ ← Subtract 136 from both sides.

$48 = 8x$ ← Simplify.

$\frac{48}{8} = \frac{8x}{8}$ ← Divide both sides by 8.

$6 = x$ ← Simplify.

The missing base length is 6 inches.

## Try These

**Find the area of the polygon.**

**1.** Trapezoid: $b_1 = 30.3$; $b_2 = 41.7$; $h = 4$.

**2.** Triangle: $b = 15.6$; $h = 24$.

**3.** Trapezoid: $b_1 = 52$; $b_2 = 63$; $h = 10$.

**Find the unknown dimension of the polygon.**

**4.** Triangle: $b$ = 60.25 mm; $h$ = ?; $A$ = 964 $\text{mm}^2$.

**5.** Triangle: $b$ = ?; $h = 101\frac{1}{2}$ in.; $A$ = 1116.5 $\text{in.}^2$

**6.** Trapezoid: $b_1$ = 225 ft; $b_2$ = 375 ft; $h$ = ?; $A$ = 3300 $\text{ft}^2$.

**7. Discuss and Write** A trapezoid has bases that measure 16 cm and 20 cm; it has a height of 4 cm. If you were to double the length of each base, how would the area of the trapezoid be affected? What if you were to double only the height? What if you were to double both the bases and the height?

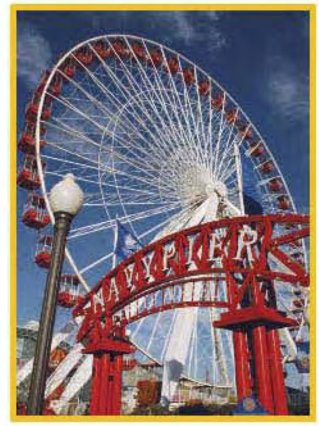
Go to PRACTICE BOOK Lesson 10-7 for exercise sets.

Algebra

# 10-8 Circumference and Area of a Circle

**Objective** To use a formula to find the circumference of a circle • To use a formula to find the area of a circle • To find the radius or diameter of a circle given its circumference or area

The Navy Pier Ferris Wheel in Chicago, Illinois, has a diameter of 140 feet. The circumference of the wheel is about 440 feet. What is the ratio of the circumference of the wheel to its diameter?



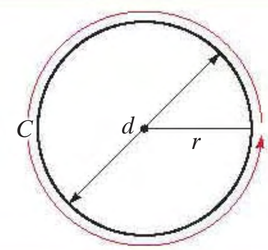

► The distance around a circle is called its circumference. The circumference of the Ferris wheel is approximately 440 feet. Because $440 \div 140 \approx 3.142857$, the circumference of the wheel is a little more than three times the diameter.

For any circle, the ratio of the circumference to the diameter is a constant value, $\frac{C}{d} = \pi$, where $\pi$ is the Greek letter *pi*.

You can multiply both sides of $\frac{C}{d} = \pi$ by $d$ to get $C = \pi d$.

The number $\pi$ is irrational, with a value of 3.141526. . . , but 3.14 or $\frac{22}{7}$ is sometimes used as an approximation for $\pi$.

**Remember:** The diameter of a circle is twice its radius: $d = 2r$.




**Key Concept**

**Circumference of a Circle**
$C = \pi d$, or $C = 2\pi r$, where $d$ is the diameter of the circle and $r$ is the radius of the circle

Find the circumference of a circle with a diameter of 4.5 miles. Express the solution in terms of $\pi$. Then estimate $C$ to the nearest 0.1 mile.

$C = \pi d \longrightarrow C = 4.5\pi$ miles

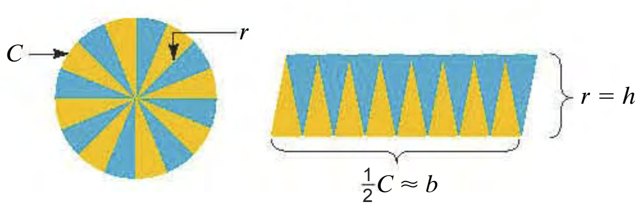
Find the estimated circumference using 3.14 for the approximate value of $\pi$.

$C = \pi d \longrightarrow C \approx 4.5(3.14)$ miles $\longrightarrow C \approx 14.13$ miles

The circle's circumference is $4.5\pi$ miles or about 14.1 miles.

Whenever any value—even the value supplied by your calculator's $\pi$ key—is substituted for $\pi$ in a formula, the solution will be an approximation. Use the $\approx$ symbol to express the approximate solution.

► You can divide a circle into equal sectors and then assemble the sectors so that they create a shape that looks like a parallelogram.




Notice that the base of the "parallelogram" above is approximately equal to $\frac{1}{2}C$, and its height is equal to $r$. So the area formula for a parallelogram helps you develop an area formula for a circle.

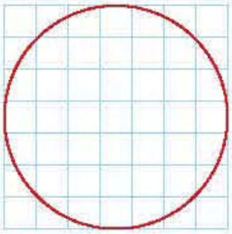
$A = bh \longrightarrow A = \left(\frac{1}{2}C\right)r \longrightarrow A = \frac{1}{2}(2\pi r)r \longrightarrow A = \pi r^2$

In the formulas for circumference and for area, the *constant* is $\pi$. The variables in these formulas are $d$ and $r$.

**Key Concept**

**Area of a Circle**
$A = \pi r^2$, where $r$ is the radius of the circle

Find the area of a circle with a radius of 3.5 inches. Use the $\pi$ key on your calculator, and round to the nearest tenth. The circle on the grid at the right can help you approximate the area before you do the calculations. (Assume each square in the grid represents 1 in.$^2$)



$A = \pi r^2 \longrightarrow A = \pi(3.5)^2 \longrightarrow A = 12.25\pi \longrightarrow A \approx 38.5 \text{ in.}^2$

The circle's area is approximately 38.5 in.$^2$

► If you know the circumference or area of a circle, you can use algebra to find its radius or diameter.

The circumference of the circular face of a wristwatch is 70 mm. Use the circumference formula and your calculator's $\pi$ key to find the maximum length of the minute hand (the radius of the circle). Round your answer to the nearest hundredth.

$C = 2\pi r$

$70 = 2\pi r$ ← Substitute 70 for $C$.

$\frac{70}{2\pi} = \frac{2\pi r}{2\pi}$ ← Divide both sides by $2\pi$ to isolate $r$.

$11.14 \approx r$ ← Simplify.

The maximum length of the watch's minute hand is approximately 11.14 mm.

## Example

**1** A round garden has an area of 154 ft$^2$. Find its diameter. Use the $\pi$ key on your calculator. Round to the nearest whole number.

$A = \pi r^2$ ← Use the formula for area of a circle.

$154 = \pi r^2$ ← Substitute 154 for the area of the circle.

$154 \div \pi = \pi r^2 \div \pi$ ← Divide both sides by $\pi$.

$\sqrt{154 \div \pi} = \sqrt{r^2}$ ← Use your calculator to take the square root of both sides.

$7.0014\ldots \approx r$ ← Round to the nearest whole number.

$7 \approx r$

The radius is about 7 ft, so the diameter is 2(7), or about 14 ft.

## Try These

**Find the area or the circumference of the circle in terms of $\pi$.**

**1.** $d = 28$ mm; $A = ?$ **2.** $r = 82.3$ km; $C = ?$ **3.** $r = 100$ yd; $A = ?$

**For exercises 4–9, use your calculator's $\pi$ key. Round to the nearest whole unit. Find the area and the circumference of the circle.**

**4.** $r = 16$ mm **5.** $d = 55$ km **6.** $r = 100$ yards

**Find the unknown measurement.**

**7.** $C = 20$ in.; $d = ?$ **8.** $A = 200.96$ cm$^2$; $r = ?$ **9.** $C = 62.8$ ft.; $d = ?$

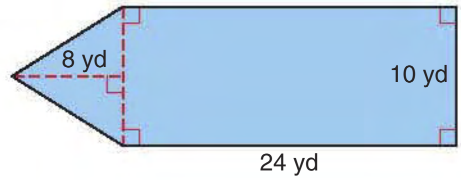
**10. Discuss and Write** If you know the circumference of a circle, can you find its area? Explain.

Go to PRACTICE BOOK Lesson 10-8 for exercise sets.

# Area of Complex Figures

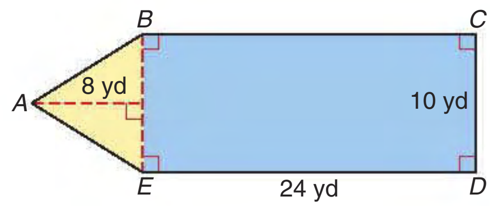
**Objective** **To identify polygons and circles within a complex figure • To find or estimate the area of complex figures involving polygons and circles • To find missing dimensions in a complex figure given its area**

An auditorium has the floor plan shown at the right. What is the area of the auditorium's floor?




► A complex figure is a figure made up of two or more shapes. You can find the area of a complex figure. Separate the figure into shapes whose areas you know how to find.

There are two shapes in the complex figure $ABCDE$. They are triangle $ABE$ and rectangle $BCDE$.




► Once you've identified the different shapes, you can determine the area of each shape. The area of the complex figure will be the sum of the areas of the shapes.

**1** Find the area of triangle $ABE$:

$A = \frac{1}{2}bh = \frac{1}{2}(10)(8) = 40 \text{ yd}^2$

**2** Find the area of rectangle $BCDE$:

$A = \ell w = (24)(10) = 240 \text{ yd}^2$

**3** Find the sum of the two areas:

Area of the floor = Area of triangle + Area of rectangle

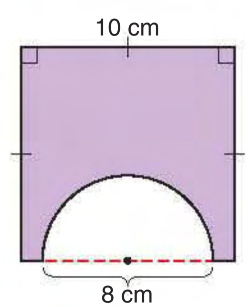
$= 40 \text{ yd}^2 + 240 \text{ yd}^2 = 280 \text{ yd}^2$

So the area of the auditorium's floor is 280 square yards.

**Remember:**
**Area formulas**
rectangle: $A = \ell w$
parallelogram: $A = bh$
triangle: $A = \frac{1}{2}bh$
trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$
circle: $A = \pi r^2$

► Sometimes the area of a complex figure is calculated by finding the *difference* of the areas of two or more shapes.

What is the area of the shaded figure at the right?




To find the area of the shaded figure, first find the area of the square and the area of the semicircle. Then subtract the area of the semicircle from that of the square.

**1** Find the area of the square:

$A = s^2 = 10^2 = 100 \text{ cm}^2$

**2** Find the area of the semicircle:

$A = \frac{1}{2}(\pi r^2)$ ← The area of a semicircle is half the area of a circle.

$\approx \frac{1}{2}(3.14 \bullet 4^2)$ ← Substitute the known value. Use 3.14 for $\pi$.

$\approx 25.12 \text{ cm}^2$ ← Simplify.

**Think**
The diameter of the semicircle is 8 cm, so its radius is 4 cm.

**3** Subtract the areas.

Area of the shaded figure = Area of square − Area of semicircle

$\approx 100 \text{ cm}^2 - 25.12 \text{ cm}^2$

$\approx 74.88 \text{ cm}^2$

So the area of the shaded figure is about 74.88 square centimeters.

► You can also find an unknown measurement, given the area of a complex figure.

**5** Problem-Solving Strategy

Figure *MNPQRSTU* has an area of 873 square feet. What is the length of side *NP*?

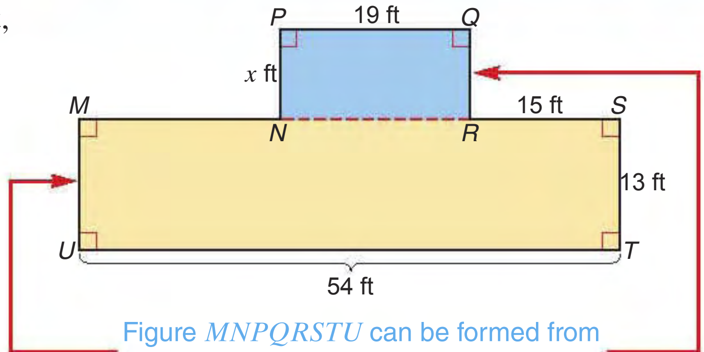



Figure *MNPQRSTU* can be formed from rectangle *MSTU* and rectangle *NPQR*.

As with finding the area of a complex figure, identify the different shapes in the figure.

Then find the area of each shape.

Area of rectangle *MSTU* ⟶ $54 \bullet 13 = 702$
Area of rectangle *NPQR* ⟶ $19 \bullet x = 19x$

The sum of these areas is 873 $ft^2$.

Use this information to write an equation and solve for $x$.

$$702 + 19x = 873$$

$$702 - 702 + 19x = 873 - 702 \longleftarrow \text{Subtract 702 from both sides.}$$

$$19x = 171 \longleftarrow \text{Simplify.}$$

$$\frac{19x}{19} = \frac{171}{19} \longleftarrow \text{Divide both sides by 19 to isolate } x.$$

$$x = 9 \longleftarrow \text{Simplify.}$$

So the length of side *NP* is 9 feet.

**Key Concept**

**Area of Complex Figures**

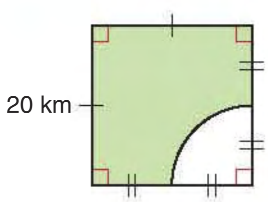
When solving area problems with complex figures:

- Identify shapes that create the figure.
- Use formulas for area to find each shape's area.
- Find the sum (or difference) of the areas, as necessary.
- If given a total area, use it in an equation, and solve for the unknown.

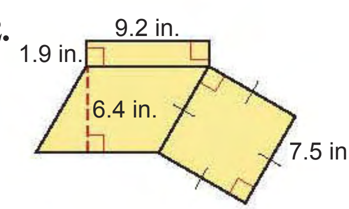
## Try These

**Find the area of the shaded region. If necessary, round to the nearest tenth.**

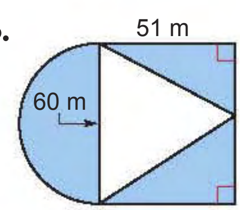
**1.**



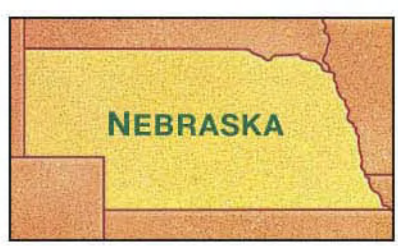

**2.**




**3.**




**4. Discuss and Write** Describe two different ways you can use polygons to estimate the area of the state of Nebraska. Illustrate each of your answers.



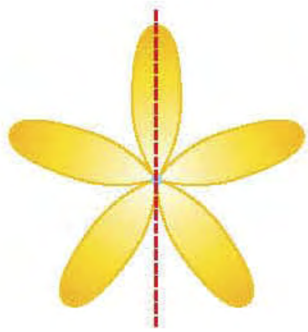

Go to PRACTICE BOOK Lesson 10-9 for exercise sets.

10-10

# Symmetry

**Objective** **To identify line symmetry and lines of symmetry, rotational symmetry, and point symmetry • To draw figures that have line symmetry, rotational symmetry, or point symmetry**

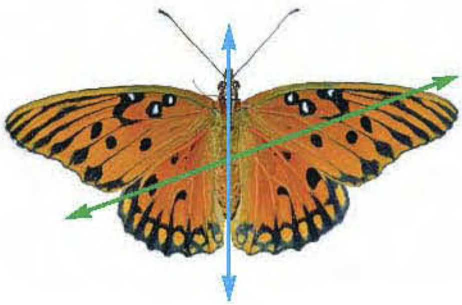
In the flower shown at the right, the dashed line divides the flower into mirror-image halves. If you were to fold the flower along the dashed line, the two halves would match exactly. If you turn the flower one fifth of a full turn, the rotated flower would look the same as the original.



Are there other imaginary lines that divide the flower into mirror-image halves? Are there other turns that would create a flower that looks just like the original?

► The quality that makes the flower above appear "balanced" is called symmetry.

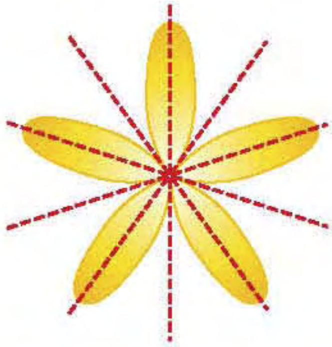
► A figure has line symmetry if a real or imaginary line divides the figure into mirror-image halves. This line is called the line of symmetry. When you fold a figure in half along a line of symmetry, the two sides match each other exactly.



In the picture of the butterfly at the right, the blue line is a line of symmetry. If you were to fold the picture on the line, the halves would match.

The green line is *not* a line of symmetry. If you were to fold on the green line, the halves would not match.

The flower shown at the top of the page has five lines of symmetry.



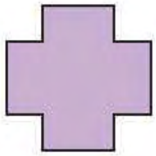
► A figure has rotational symmetry if it matches the original figure after rotating *less than a full turn* around a central point. This point is called the *center of rotation*. The center of rotation may be a point on the figure, or it may be some other point.

► The smallest turn that creates a match is an angle of rotation. To find its measure, count the number of times a figure can be rotated within a full turn. Then divide 360° by this number. A multiple of the angle of rotation also creates a match.

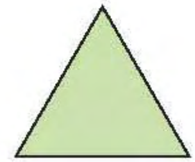
A complete rotation is 360°.
A half rotation is 180°.
A quarter rotation is 90°.

## Examples

Each of the following figures has rotational symmetry.

**1** 
This figure matches for turns of 90°, 180°, 270°, and 360°.

**2** 
This figure matches for turns of 180° and 360°.

**3** 
This figure matches for turns of 120°, 240°, and 360°.

The flower at the top of the preceding page has 5 possible rotations. So the angle of rotation is $\frac{360°}{5}$, or 72°. The angle measures of the other turns are multiples of 72°. They are 144°, 216°, 288° and 360°.

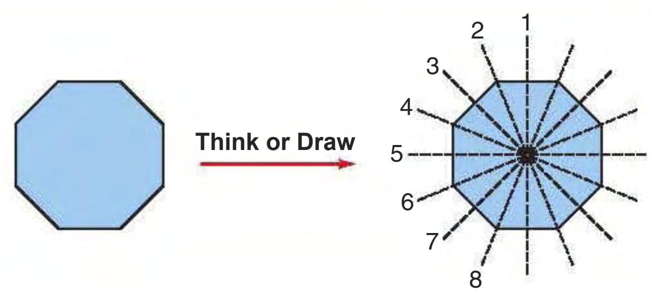
► Point symmetry is a special kind of rotational symmetry. A figure has point symmetry if it matches the original figure after rotating the figure *half a turn* around its center point. Figures with point symmetry always have an angle of rotation of 180°.

**Think**
Figures with an angle of rotation of 180° look the same upside down as right-side up!

## Example

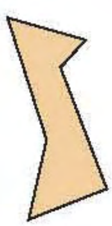
**1** Describe the symmetries of a regular octagon.

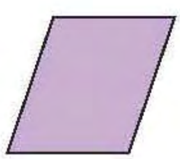
A regular octagon has line symmetry.
It has 8 lines of symmetry.
A regular octagon also has rotational symmetry. It can be rotated 8 times.
Its angle of rotation is 45°.
A regular octagon also has point symmetry.
It can be rotated 180°.




## Try These

**Identify the types of symmetry that exist for each figure. Then describe them.**

**1.** 

**2.** 

**3.** 

**4.** 

**5. Discuss and Write** Draw a figure that has both line symmetry and rotational symmetry. Draw the line(s) of reflection and the center of rotation, and list the angles of rotation.

Go to PRACTICE BOOK Lesson 10-10 for exercise sets.

# 10-11 Tessellations

**Objective** **To identify polygons that tessellate • To identify translation tessellation and rotation tessellation • To make figures for translation tessellations and rotation tessellations**

Sadiq has been commissioned to create a large mural for the entrance to a local art museum. He decides to use tessellations to create this mural.

Before creating the mural, Sadiq knows that he must first choose a polygon that tessellates. What are some of the possibilities from which he could choose?

► Tessellations are the covering of a plane with congruent copies of the same figure or figures with no overlaps or gaps. This kind of design has been used in art for centuries.

► A translation tessellation is a pattern made by translating or sliding a figure. Any parallelogram can be used to create a translation tessellation.

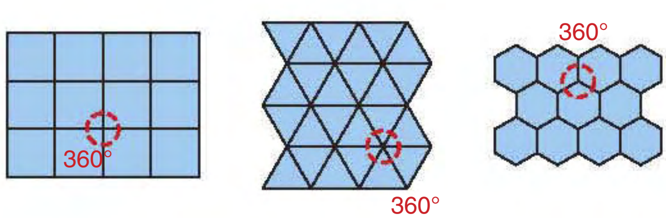
**Remember:** Rhombuses, rectangles, and squares are all different types of parallelograms.

You can also create translation tessellations from more interesting shapes by following these steps:

1. Start with any parallelogram, and cut a shape from one side.
2. Translate (or slide) the cut-out shape to the opposite side of the parallelogram.
3. If desired, repeat Steps 1 and 2 with the other pairs of sides.
4. Fit copies of the figure together.


► A rotation tessellation uses rotated figures to create a pattern. A *regular* rotation tessellation uses regular polygons. Only figures with angles that are factors of 360° can be used to create rotation tessellations. At the point where vertices meet, the total angle measure of all the angles must equal 360° to form a tessellation. So squares, regular hexagons, and equilateral triangles all tessellate: $90° \bullet 4 = 360°$; $60° \bullet 6 = 360°$; $120° \bullet 3 = 360°$.

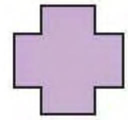
**Remember:**
A regular polygon has all congruent sides and all congruent angles.
Each angle of a square is 90°.
Each angle of an equilateral triangle is 60°.
Each angle of a regular hexagon is 120°.



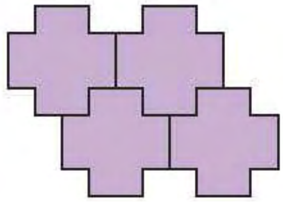

So Sadiq could use parallelograms, equilateral triangles, or regular hexagons for the mural.

## Examples

**1** Does the nonregular dodecagon (a 12-sided figure) at the right tessellate?

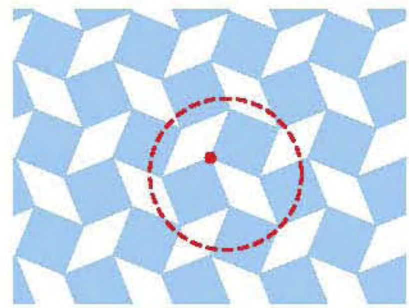


Yes, the dodecagon can make a tessellation, as shown below.



**2** Identify the polygon(s) used to create the tessellation.

Choose a point in the tessellation where four polygons meet. Identify the figures around that point.



The red dashed circle and the red dot inside should help you see that around each point there are two squares and two rhombuses.

So this tessellation was created not with one type of polygon, but with two different types of polygons. You do not have to use copies of just one type of polygon to make a tessellation.

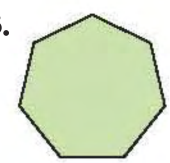
## Try These

**Draw the polygon on another piece of paper. Try to make a tessellation using each polygon. Tell whether the polygon tessellates.**

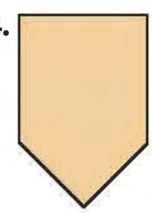
**1.** Right triangle

**2.** Regular pentagon

**3.**

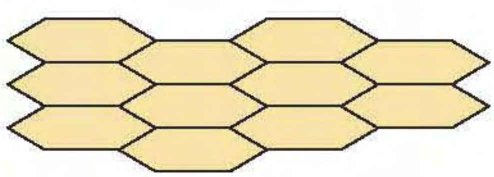


**4.**

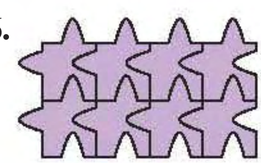


**Identify each type of tessellation.**

**5.**



**6.**



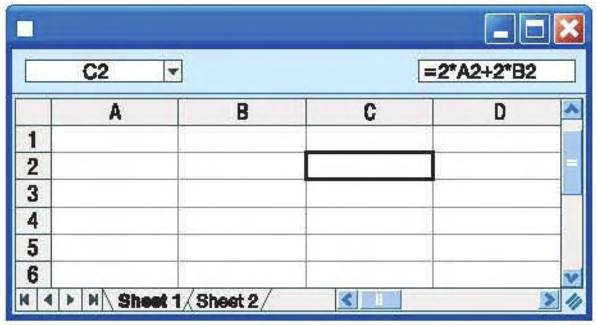
**7. Discuss and Write** On the preceding page is an example of a tessellation with equilateral triangles. Can you use scalene or isosceles triangles to make a tessellation? Explain. Use drawings to justify your reasoning.

Go to PRACTICE BOOK Lesson 10-11 for exercise sets.

# 10-12 Technology:
## Relate Perimeter and Area

**Objective** To use spreadsheets to calculate perimeter and area of rectangles • To explore the effect of changing dimensions on perimeter and area of rectangles

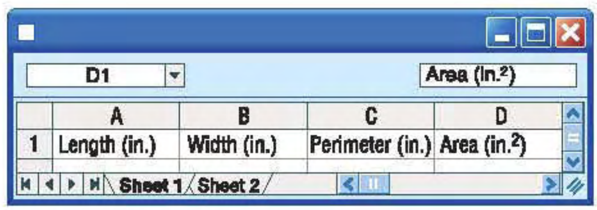
You can use a spreadsheet program to find the perimeter and area of a rectangle. A spreadsheet program can also help you explore how changing one or more dimensions affects the area or perimeter of a rectangle.




► Use a spreadsheet program to find the area and perimeter of a rectangle with a length of 13 inches and a width of 6 inches.

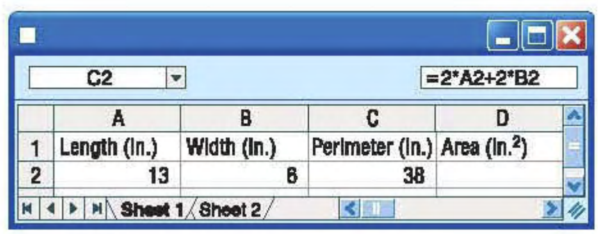
To enter text into a cell, place the cursor on the cell, type the information, and press **ENTER**. Use the arrow keys (←, ↑, →, ↓) to move from one cell to the next. The name of the cell your cursor is in appears at the top left corner of the spreadsheet, above the letter *A*.

**Step 1** Enter these headings in row 1.
Cell A1: *Length (in.)*
Cell B1: *Width (in.)*
Cell C1: *Perimeter (in.)*
Cell D1: *Area (in.$^2$)*



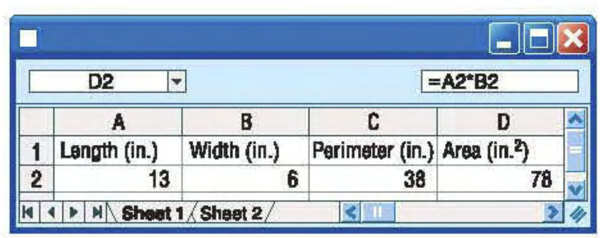

**Step 2** Enter the length in inches in cell A2 and the width in inches in cell B2.

**Step 3** Enter the formula for perimeter in cell C2. To enter the formula for perimeter, type **=2*A2+2*B2**. The symbol * is used for multiplication. This instructs the program to multiply the value in cell A2 by 2, to multiply the value in cell B2 by 2, and to add the results. The spreadsheet calculates the perimeter as 38 inches.




**Step 4** Enter the formula for area in cell D2. To enter the formula for area, type **=A2 * B2**. This instructs the program to multiply the value in cell A2 by the value in cell B2.

After you type in the formula for area, the spreadsheet automatically calculates the value for cell D2, the area of the rectangle, as 78 square inches.



► You can use a spreadsheet to see how the perimeter and area of a figure are affected when one or both dimensions change.

Explore what happens when one or both dimensions of a rectangle are *doubled*. Enter the new dimensions described below into the appropriate cells.

**Step 1** Double the original length by typing 26 into cell A2, which is twice the original length of the rectangle.

**Step 2** Type 6 into cell B2 to represent the original width. The program automatically calculates the new perimeter and area.

D2 =A2*B2

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Length (in.) | Width (in.) | Perimeter (in.) | Area (in.$^2$) |
| 2 | 26 | 6 | 64 | 156 |

Sheet 1 / Sheet 2

The new perimeter is 64, and the new area is 156. The change in the perimeter is an increase of 26, or twice the original length. The new area is twice the original area.

► Use a spreadsheet to divide one or both dimensions in half.

Explore what happens when one or both dimensions of a rectangle are *halved*. Enter the new dimensions described below into the appropriate cells.

First divide the length in half, and use the given width of 6 inches.

- Divide the original length of 13 inches by 2. Type 6.5, which is half of the original length of the rectangle, into cell A2. The width remains the same, 6 inches.

  The new perimeter changes to 25 inches, and the new area changes to 39 square inches. The change in the perimeter is a decrease of 13, the original length. The new area is half of the original area.

D2 =A2*B2

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Length (in.) | Width (in.) | Perimeter (in.) | Area (in.$^2$) |
| 2 | 6.5 | 6 | 25 | 39 |

Sheet 1 / Sheet 2

Now divide the width in half, and use the original length of 13 inches.

- Divide the original width by 2. Type 3, which is half of the original width of the rectangle, into cell B2.

The new perimeter is 32, and the new area is 39. The change in the perimeter is a decrease of 6, which is the original width. The new area is half of the original area.

D2 =A2*B2

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Length (in.) | Width (in.) | Perimeter (in.) | Area (in.$^2$) |
| 2 | 13 | 3 | 32 | 39 |

Sheet 1 / Sheet 2

## Try These

1. **Make a conjecture, and verify it with a spreadsheet.** What would happen to the perimeter and area if you were to double the original length and the original width of the rectangle? Use a spreadsheet to verify your conjecture. Describe the results.

2. **Discuss and Write** Explain how you would use spreadsheet software to find the area and perimeter of a square that has sides measuring 4.7 cm.

Go to PRACTICE BOOK Lesson 10-12 for exercise sets.

# 10-13 Problem-Solving Strategy: Account for all Possibilities

**Objective** To solve problems using the strategy *Account for All Possibilities*

**Problem 1:** Eight rods are on a table. Their lengths are 1, 2, 3, 4, 5, 6, 7, and 8 inches. How many different triangles can be formed using exactly three rods at a time? (You may make different triangles using the same rod again, but you may not use the same rod more than once in the same triangle.)

**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** You are given eight rods of lengths 1, 2, 3, 4, 5, 6, 7, and 8 inches.

You form triangles using the rods as sides.

You may use the same rod in different triangles but not within the same triangle.

**Question:** How many different triangles can you make?

**Problem-Solving Strategies**

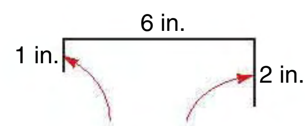
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. **Account for All Possibilities**
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**

You could attempt to account for all such possible triangles.

**Solve** **Apply the strategy.**

The key to this problem is the Triangle Inequality Theorem, which says that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. So to be able to form a triangle with three rods, the sum of the two shortest lengths must be greater than the longest length. For example, you cannot form a triangle from the 1-in., 2-in., and 6-in. rods because 1 + 2 is not greater than 6.




Swing these around. They will never meet. So no triangle is possible.

To find all the possible triangles, take each rod in turn, beginning with the shortest, and write all possible groups of rods that can form triangles, as shown in the following table.

| Shortest Length in Inches | Possible Sides | Total |
|---|---|---|
| 1 | none | 0 |
| 2 | 2-3-4 2-4-5 2-5-6 2-6-7 2-7-8 | 5 |
| 3 | 3-4-5 3-4-6 3-5-6 3-5-7 3-6-7 3-6-8 3-7-8 | 7 |
| 4 | 4-5-6 4-5-7 4-5-8 4-6-7 4-6-8 4-7-8 | 6 |
| 5 | 5-6-7 5-6-8 5-7-8 | 3 |
| 6 | 6-7-8 | 1 |
| 7 | none | 0 |
| 8 | none | 0 |
| | | 22 |

In all, 22 possible triangles can be formed.

**Check** **Check to make sure your answer makes sense.**
You could cut thin paper strips of these eight lengths and then form the triangles listed in the table. In this process, it would also be clear why certain combinations (such as 1-2-3 or 3-4-8) do *not* form triangles.

**Problem 2:** The four digits 1, 2, 3, and 4 can be used to form exactly 24 different four-digit numbers. Of these, how many are evenly divisible by 4?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** 24 four-digit numbers can be formed from 1, 2, 3, and 4.

**Question:** How many of these 24 numbers are divisible by 4?

**Plan** **Select a strategy.**
Because there are only 24 such numbers, you could attempt to consider all the possibilities.

**Solve** **Apply the strategy.**
Recall the divisibility criteria for 4: A number is divisible by 4 if and only if the number formed by the last two digits of the number is divisible by 4. For example, 520 is divisible by 4 because 20 is divisible by 4; 526 is not divisible by 4 because 26 is not divisible by 4. Also, note that for any fixed last two digits, there are two possible four-digit numbers. For example, if the last two digits are 12, then the first two digits must be 34 or 43. So the two possible numbers are 3412 and 4312.

Using these ideas, you can sort through the possibilities pretty quickly.

- There are six numbers ending in 1: two each with last two digits 21, 31, and 41. None of these is divisible by 4.
- There are six numbers ending in 2: two each with last digits 12, 32, 42. The two with endings 12 (3412 and 4312) and 32 (1432 and 4132) are divisible by 4. So far, this makes 4 numbers that are divisible by 4.
- There are six numbers ending in 3: two each with last digits 13, 23, 43. None of these is divisible by 4.
- There are six numbers ending in 4: two each with last digits 14, 24, 34. Only the two with ending 24 (1324 and 3124) are divisible by 4. This adds 2 more to the list of numbers divisible by 4.

In all, then, there are only 6 of these numbers that are divisible by 4.

**Check** **Check to make sure your answer makes sense.**
A complete list of the 24 different numbers shows that the solution is correct.
1234, 1243, <u>1324</u>, 1342, 1423, <u>1432</u>,
2134, 2143, 2314, 2341, 2413, 2431,
<u>3124</u>, 3142, 3214, 3241, <u>3412</u>, 3421,
4123, <u>4132</u>, 4213, 4231, <u>4312</u>, 4321

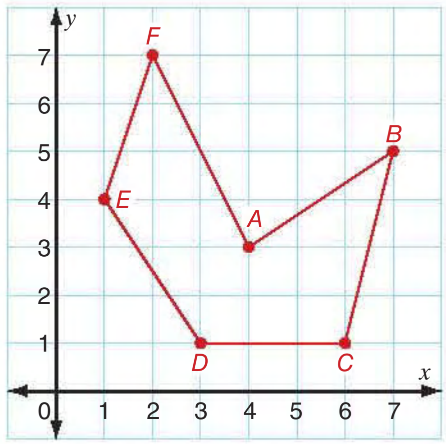
Only the six underlined numbers are divisible by 4.

Go to PRACTICE BOOK Lesson 10-13 for exercise sets.

# Enrichment:
## Area of Irregular Polygons

**Objective** To explore various methods for finding the area of irregular polygons

There are several ways to find the area of an irregular polygon if you can locate its vertices on a coordinate grid. You will use three of these methods to find the area of hexagon $ABCDEF$ at the right.




Vertices of Hexagon $ABCDEF$

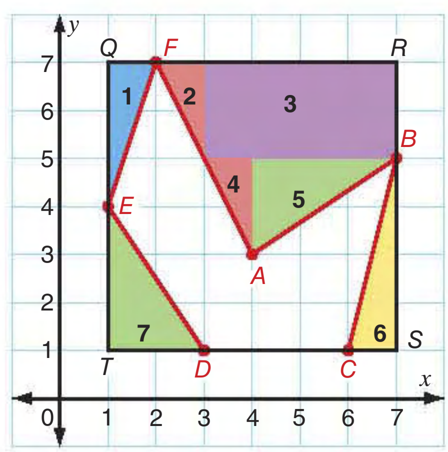
$A$ (4, 3)
$B$ (7, 5)
$C$ (6, 1)
$D$ (3, 1)
$E$ (1, 4)
$F$ (2, 7)

### Method 1
### Subtract From a Rectangle

The steps below describe how to find the area of an irregular figure by forming a rectangle, finding its area, and then subtracting the area of the shapes surrounding the irregular figure from the area of the rectangle.

1. Enclose the polygon in a rectangle so that as many polygon vertices as possible are on the rectangle.
2. Make and label right triangles and rectangles as needed to fill the area outside the polygon.
3. Use the grid to find the area of the large rectangle and other figures outside the polygon and inside the rectangle.
4. Subtract the sum of the areas of the figures outside the polygon from the area of the large rectangle.

Here is how you would use this method for finding the area of hexagon $ABCDEF$.

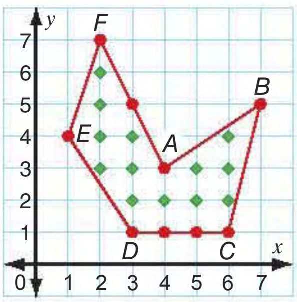



1. Draw a rectangle $QRST$ to enclose the hexagon $ABCDEF$.
2. Form $\triangle 1$, $\triangle 2$, $\square 3$, $\triangle 4$, $\triangle 5$, $\triangle 6$, and $\triangle 7$ within the rectangle $QRST$ that are outside the hexagon $ABCDEF$.
3. • Area of rectangle $QRST$:
   $A_{\square QRST} = 6 \text{ units} \bullet 6 \text{ units} = 36 \text{ units}^2$
   • Areas of polygons within the rectangle $QRST$ that are outside the hexagon $ABCDEF$:
   $A_{\triangle 1} = 1.5 \text{ units}^2$ $\quad A_{\triangle 2} = 1 \text{ unit}^2$ $\quad A_{\square 3} = 8 \text{ units}^2$ $\quad A_{\triangle 4} = 1 \text{ unit}^2$
   $A_{\triangle 5} = 3 \text{ units}^2$ $\quad A_{\triangle 6} = 2 \text{ units}^2$ $\quad A_{\triangle 7} = 3 \text{ units}^2$
   Total area of polygons: $19.5 \text{ units}^2$
4. Area of hexagon $ABCDEF$:
   area of $QRST$ − total area of polygons $= 36 \text{ unit}^2 - 19.5 \text{ units}^2$
   $= 16.5 \text{ units}^2$

### Method 2
### Use Pick's Formula

For this method, each vertex of the polygon must be on a *lattice point* (the intersection of two grid lines).

1. Mark and count all lattice points on the boundary (sides) of the polygon. Be sure to count the vertices.
2. Mark and count all the interior lattice points (those that are inside the polygon).
3. Use Pick's formula: $A = I + \frac{B}{2} - 1$, where $I$ is the number of interior points and $B$ is the number of boundary points.




**Using Pick's Formula for Hexagon $ABCDEF$**

Number of Boundary Points: 9

Number of Interior Points: 13

$$A = 13 + \frac{9}{2} - 1$$
$$= 16.5 \text{ units}^2$$

### Method 3
### Use the Cross Products of the Coordinates

1. In a table, record the coordinates of the vertices in clockwise order, starting and ending with the same point.
2. Find the cross products of the coordinates from top left to bottom right, and from top right to bottom left. The example shows how to find the first two cross products.
3. Add each set of cross products.
4. Find the difference of the sums of Cross Products. The area of the polygon is equal to half of the difference.

| Cross Products (right to left) | Coordinates $x$ | $y$ | Cross Products (left to right) |
|---|---|---|---|
| | 4 | 3 | |
| 3 • 7 = 21 ← | 7 | 5 | → 4 • 5 = 20 |
| 30 | 6 | 1 | 7 |
| 3 | 3 | 1 | 6 |
| 1 | 1 | 4 | 12 |
| 8 | 2 | 7 | 7 |
| 28 | 4 | 3 | 6 |
| **91** | ← Sums of Cross Products → | | **58** |

Difference of the sums = 91 − 58 = 33

Area of $ABCDEF = \frac{1}{2}$ difference = 16.5

## Try These

**Graph each polygon. Then find its area. Use each method at least once.**

1. $A$ (2, 2), $B$ (4, 7), $C$ (7, 4)
2. $A$ (1, 3), $B$ (5, 7), $C$ (8, 5), $D$ (7, 1)
3. $A$ (2, 2), $B$ (4, 7), $C$ (9, 2), $D$ (5, 3)
4. $A$ (2, 4), $B$ (4, 6), $C$ (6, 4), $D$ (4, 2)
5. $A$ (2, 3), $B$ (3, 9), $C$ (8, 10), $D$ (12, 6), $E$ (8, 5), $F$ (6, 1)
6. **Discuss and Write** Tell which method you prefer to use and why.

Go to PRACTICE BOOK pages 333–334 for exercise sets.

# Test Prep: Multiple-Choice Questions

## Strategy: Apply Mathematical Reasoning

When solving multiple-choice questions, you can sometimes estimate the answer and then use your estimate to determine which answer choices are reasonable and which can be eliminated. Even if only one answer choice seems reasonable, you should check that answer to be sure it is correct.

**Read the whole test item, including the answer choices.**

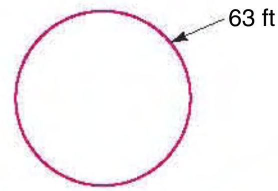
- Underline important words.
  Cleo uses about 63 feet of rope.
  What is the radius to the nearest foot?
- Restate the question in your own words.
  About how long is the radius if the circumference is 63 feet?

**Solve the problem.**

- Use an estimate of $\pi$ to simplify the formula for the circumference.
  $\pi \approx 3$ so $C \approx 2 \cdot 3 \cdot r$, or $6r$.
- Apply mathematical reasoning and use mental math to estimate the radius.
  $C \approx 6r$ so $63 \approx 6r$.
  Using mental math, the value of $r$ is about 10.

**Sample Test Item**

Cleo uses about 63 feet of rope to put a border around a circular garden.




circumference $= 2\pi r$

What is the radius, to the nearest foot, of the garden?

**A.** 3 ft  **C.** 10 ft
**B.** 5 ft  **D.** 20 ft

**Test-Taking Tips**

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices.
  Watch out for distractors.

**A.** 3 ft ← Three is an estimate of $\pi$. Eliminate this choice.

**B.** 5 ft ← This length is too short. Eliminate this choice.

**C.** 10 ft ← This is the correct choice!

**D.** 20 ft ← This is the approximate length of the diameter. Eliminate this choice.

**Try These**

**Choose the correct answer. Explain how you used strategies.**

**1.** Which whole number is closest to $\sqrt{70}$?

**A.** 7
**B.** 8
**C.** 10
**D.** 70

**2.** Randy is buying a DVD that is regularly \$17.89 and is on sale for $\frac{1}{3}$ off. About how much should Randy expect to pay for the DVD?

**F.** \$6  **H.** \$12
**G.** \$9  **J.** \$15

Go to PRACTICE BOOK page 335 for exercise sets.

CHAPTER 11

# Three-Dimensional Geometry

## In This Chapter You Will:

- Classify three-dimensional figures
- Make isometric and orthographic drawings
- Draw and use nets to find surface area
- Estimate the surface area of prisms and cylinders
- Use formulas to find the surface area and volume of prisms, pyramids, cylinders, and cones
- Find unknown dimensions given the volumes of three-dimensional figures
- Find the surface area and volume of complex three-dimensional figures
- Relate changes in scale and dimension to changes in volume and surface area
- Apply the strategy: *Work Backward*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- Pythagorean Theorem: For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.
- Area of a rectangle: $A = lw$
- Area of a triangle: $A = \frac{1}{2}bh$
- Area of a circle: $A = \pi r^2$
- Circumference of a circle: $C = \pi d$ or $2\pi r$

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 341–372

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

## Critical Thinking

The Dehejias are remodeling their house and changing the landscaping of their yard. The drawing at the right is a diagram of the deck at the back of their house. In order to put more trees in their yard, the Dehejias plan to cut the shaded areas off the deck. What will the new area of the Dehejias' deck be?

# 11-1 Three-Dimensional Figures

**Objective** To define, identify, and classify polyhedrons by their characteristics • To distinguish between regular and not regular polyhedrons • To define, identify, and classify solid figures that have curved surfaces

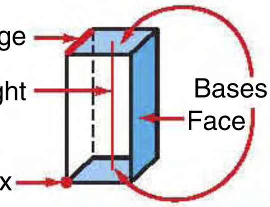
What three-dimensional figures are represented by these buildings?

**Building A**

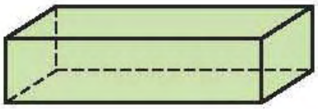
**Building B**

**Building C**

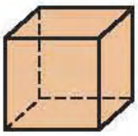
► Three-dimensional figures have length, width, and height. A polyhedron is a three-dimensional figure with faces that are all polygons. Each line segment where two faces meet is called an edge. The point of intersection of three or more edges of a polyhedron is called a vertex (plural: *vertices*). A polyhedron with faces that are all congruent is a regular polyhedron.



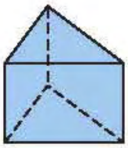

► A prism is a polyhedron with two congruent and parallel faces called bases. A prism's lateral faces are rectangles. Any polygon can form the bases of a prism. The shape of the base determines the name of the prism.



Rectangular Prism



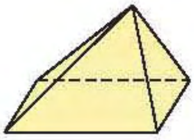
Square Prism (Cube)



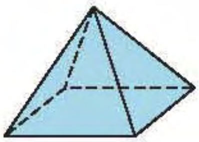
Triangular Prism

Building A is shaped like a prism.

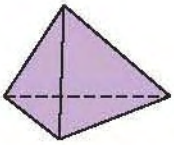
► A pyramid is a polyhedron with only one base. The base can be any polygon. The other faces of a pyramid are triangles. The shape of the base determines the name of the pyramid.



Rectangular Pyramid



Square Pyramid

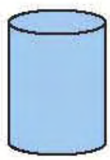


Triangular Pyramid

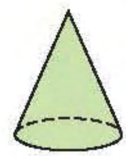
Building B is shaped like a pyramid.

► There are also three-dimensional figures that have curved surfaces. These include cylinders, cones, spheres, and hemispheres.

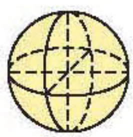
A **cylinder** has two circular congruent bases that are parallel.



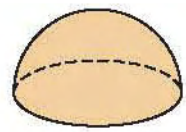
A **cone** has one circular base and one curved surface that comes to a point called the vertex.



A **sphere** is a figure with all points the same distance from the center.



A **hemisphere** is half of a sphere.



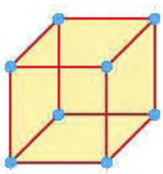
Building C is shaped like a cylinder.

► You can use a formula known as *Euler's formula* to determine how many faces, edges, or vertices a three-dimensional figure has.

**Key Concept**

**Euler's Formula**
For any polyhedron, if $F$ is the number of faces, $E$ is the number of edges, and $V$ is the number of vertices, then $F + V = E + 2$.

Use this square prism to check Euler's formula.



6 faces
12 edges
8 vertices

By counting, you can see that a square prism has 6 faces, 8 vertices, and 12 edges.

Use Euler's formula to check.

$F + V = E + 2$ ← Euler's formula

$6 + 8 \stackrel{?}{=} 12 + 2$ ← Substitute the known values.

$14 = 14$ True

## Try These

**Write the name of the three-dimensional figure each object is most like.**

**1.**

**2.**

**3.**

**4. Discuss and Write** Imagine what a hexagonal prism looks like, then describe it with words and a sketch. Count edges, vertices, and faces. Use Euler's formula to verify your counting.

Go to PRACTICE BOOK Lesson 11-1 for exercise sets.

# 11-2 Draw Three-Dimensional Figures

**Objective** To interpret and create isometric drawings • To interpret and create orthographic drawings

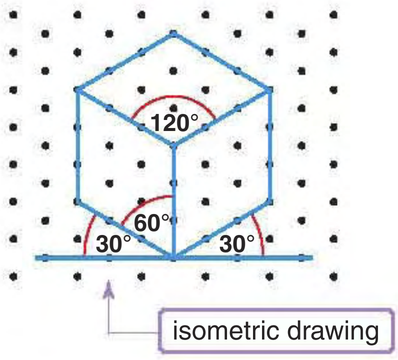
Computer game designers often use drawings to help them design game figures. How can a three-dimensional figure be represented by a two-dimensional drawing?

To represent a three-dimensional figure by a two-dimensional drawing, you can use *isometric* and *orthographic drawings*.

► An isometric drawing is a pictorial view created on an isometric dot grid of a three-dimensional figure. It is made by three types of lines:

- vertical lines
- 30° lines going to the right
- 30° lines going to the left

The drawing at the right shows an isometric drawing of a cube. The 90º angles in the faces of the cube are drawn as 120º and 60º angles.

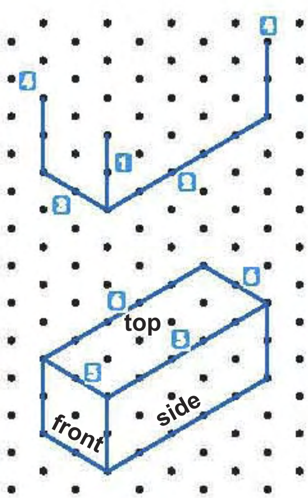



Do you see a regular hexagon in the drawing? An isometric drawing of a cube is a regular hexagon, but you imagine a cube when you look at it!

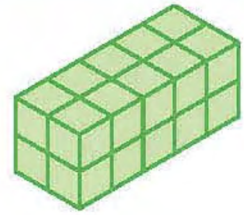
When making an isometric drawing, it is helpful to refer to a model. To draw a prism, first use cubes to build a $5 \times 2 \times 2$ rectangular prism. Have the prism rest on a $5 \times 2$ face. Turn the prism so you are looking at one of the vertical edges.

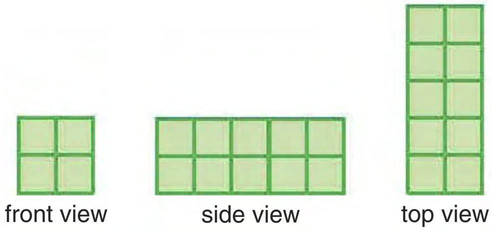
Now follow the steps below. Each step is associated with a number in the drawing.

1. Draw a 2-unit vertical line segment for the height.
2. From the bottom of the vertical line segment, draw a 30º line segment to the right that is 5 units long.
3. From the bottom of the vertical line segment, draw a 30º line segment to the left that is 2 units wide.
4. From the endpoints of the connected line segments, draw two vertical line segments, each 2 units tall.
5. Connect the endpoints of the vertical line segments.
6. Draw the length and width parallel to the length and width you drew in step 5.



► You could also build that same rectangular prism using cubes, as shown at the right. If you were then to look at each of the faces straight on, you would see the views below.

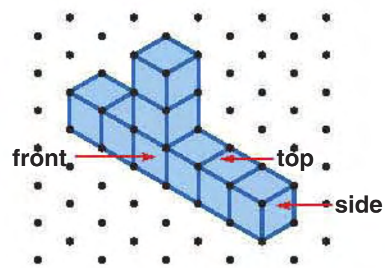


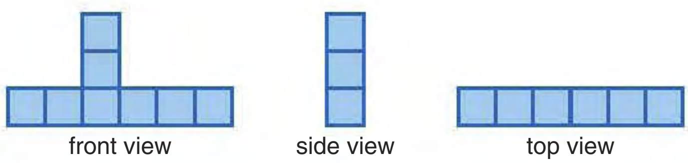



These two-dimensional views of the prism are called orthographic drawings. In this type of drawing, each view shows only one face, and the angles in the drawing are congruent to the angles in the *actual figure*—unlike the angles in isometric drawings. So a rectangular face is drawn with four 90º angles.

The drawing at the right is an isometric drawing of a figure made from 8 cubes.

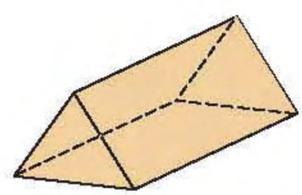
The drawings below show its orthographic views.




## Example

**1** What kind of figure would have the front, side, and top views shown? Draw the figure.

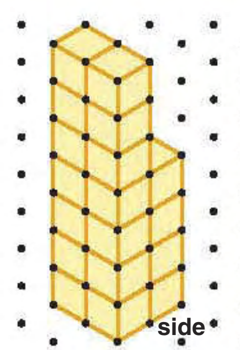
The figure would be a triangular prism.

## Try These

**For each set of dimensions, make an isometric drawing of a rectangular prism. (*h* = height; *w* = width; *ℓ* = length).**

**1.** $h = 4, w = 4, \ell = 4$ **2.** $h = 7, w = 1, \ell = 4$ **3.** $h = 2, w = 5, \ell = 3$

**4.** Draw orthographic top, side, and front views of the figure at the right.




**5.** **Discuss and Write** Make a rough sketch of your school using isometric and orthographic drawings. Use centimeter cubes if you need help. Is it easier to start with isometric or orthographic drawings? Why?

Go to PRACTICE BOOK Lesson 11-2 for exercise sets.

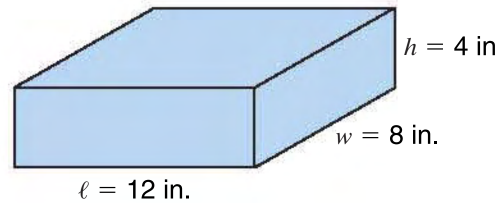
Algebra
11-3

Update your skills. See page 410.

# Surface Area of Prisms

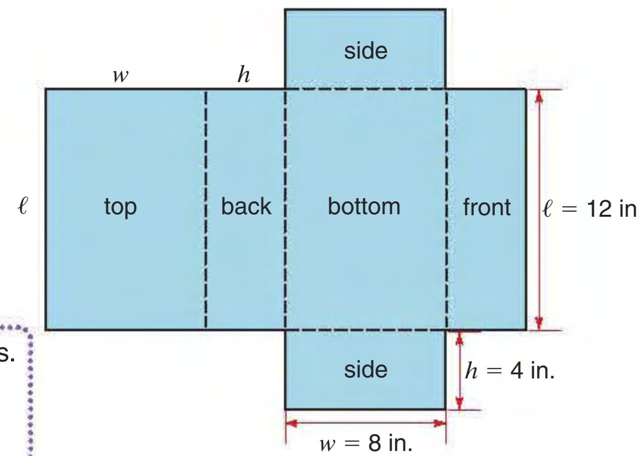
**Objective** To draw and use nets to find the surface area of prisms
• To use formulas to find the surface area of prisms

Samuel is using construction paper to cover a grab-bag box for a class party. The box has the same dimensions as the box shown at the right. How many square inches of construction paper does he need to cover the box completely?




► To find how much construction paper Samuel needs, find the surface area of the prism.

The surface area of a prism is the sum of the areas of its faces. A net can help you visualize the faces. You can find the surface area of a prism by finding the area of its *net*.




**Remember:** A net is a two-dimensional shape that can be folded to form a three-dimensional object.

**Think**
A rectangular prism has three pairs of congruent faces.
Area of top = Area of bottom
Area of front = Area of back
Area of left side = Area of right side

**Method 1** Find the area of each face. Then add the areas.

Area of top → $\ell w = 12 \bullet 8 = 96$
Area of bottom → $\ell w = 12 \bullet 8 = 96$
Area of front → $\ell h = 12 \bullet 4 = 48$
Area of back → $\ell h = 12 \bullet 4 = 48$
Area of left side → $wh = 8 \bullet 4 = 32$
Area of right side → $wh = 8 \bullet 4 = 32$

$96 + 96 + 48 + 48 + 32 + 32$
$2(96) + 2(48) + 2(32)$
$192 + 96 + 64$
$352 \text{ in.}^2$

**Method 2** Use a formula.

$S = 2\ell w + 2\ell h + 2wh$,
where $S$ = surface area, $\ell$ = length, $w$ = width, and $h$ = height.

$\ell = 12$ in. $\quad w = 8$ in. $\quad h = 4$ in.

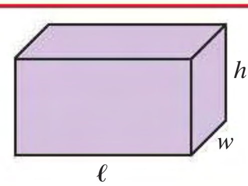
$$\begin{aligned} S &= 2(12)(8) + 2(12)(4) + 2(8)(4) \\ &= 2(96) + 2(48) + 2(32) \\ &= 192 + 96 + 64 \\ &= 352 \text{ in.}^2 \end{aligned}$$

So Samuel needs at least 352 square inches of construction paper to cover the box completely.

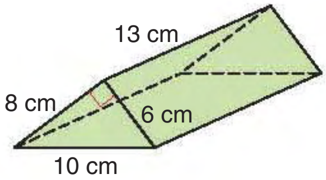
**Key Concept**

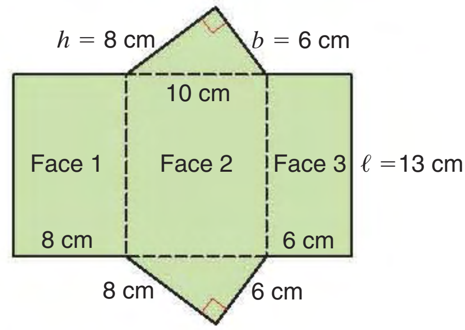
**Surface Area ($S$) of a Rectangular Prism**

$S = 2\ell w + 2\ell h + 2wh$, where $S$ = surface area, $\ell$ = length, $w$ = width, and $h$ = height.



► You can also use a net to find the surface area of a triangular prism.



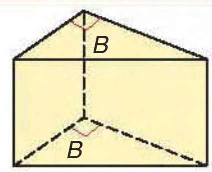

| | |
|---|---|
| Find the area of a base. | $A = \frac{1}{2}bh \rightarrow \frac{1}{2}(6)(8) = 24\text{ cm}^2$ |
| Find the area of the other faces. | Area of Face 1 $\rightarrow h\ell = 8 \bullet 13 = 104\text{ cm}^2$<br>Area of Face 2 $\rightarrow \ell w = 13 \bullet 10 = 130\text{ cm}^2$<br>Area of Face 3 $\rightarrow \ell b = 13 \bullet 6 = 78\text{ cm}^2$ |
| Find the sum of the areas. | $2(24) + 104 + 130 + 78 = 48 + 312 = 360\text{ cm}^2$ |

So the surface area of the triangular prism is 360 square centimeters

## Key Concept

**Surface Area ($S$) of a Triangular Prism**

$S = 2B + A_1 + A_2 + A_3$, where $S$ = surface area,
$B$ = area of the triangular base, $A_1$ = area of rectangular face 1,
$A_2$ = area of rectangular face 2, $A_3$ = area of rectangular face 3.




## Example

**1** Use the formula to find the surface area of a triangular prism in which the area of the triangular base is 108 yd$^2$ and each of the three rectangular faces is 24 yards long and 15 yards wide.

$S = 2B + A_1 + A_2 + A_3$ ← Formula for surface area of a rectangular prism
$= 2(108) + (24 \bullet 15) + (24 \bullet 15) + (24 \bullet 15)$ ← Substitute known values.
$= 2(108) + 3(24 \bullet 15)$ ← Simplify.
$= 216 + 1080$
$= 1296$

So the surface area of the triangular prism is 1296 square yards.

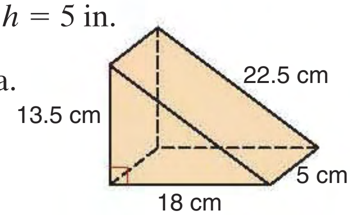
## Try These

**Find the surface area of a rectangular prism with the given dimensions.**

**1.** $\ell = 10$ m; $w = 2.5$ m; $h = 14$ m

**2.** $\ell = 3\frac{1}{2}$ in.; $w = 2$ in.; $h = 5$ in.

**3.** Draw a net of the triangular prism at the right and find its surface area.




**4. Discuss and Write** Explain how you would find the surface area of a cube and then write a formula.

Go to PRACTICE BOOK Lesson 11-3 for exercise sets.

Algebra

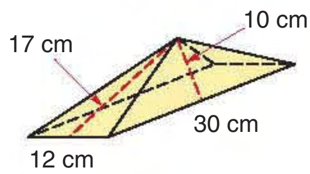
# 11-4

Update your skills. See page 408 III.

# Surface Area of Pyramids

**Objective** **To draw and use nets to find the surface area of a rectangular pyramid or a triangular pyramid • To use formulas to find the surface area of a rectangular pyramid or a triangular pyramid • To rename surface area units in equivalent forms**

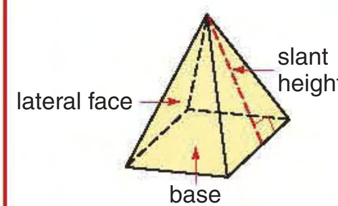
Veronica is designing a three-dimensional wooden model of a rectangular pyramid. She will cut out each face from a thin sheet of wood and glue the edges of the faces together. The dimensions of the model are shown at the right. How many square centimeters of wood will she use?




To find the amount of wood Veronica will use, find the surface area of the model rectangular pyramid.

► To find the surface area, you need to know the parts of a triangular pyramid. The triangular sides of a pyramid are called lateral faces. The height of each lateral face is called the slant height. The sum of the areas of the lateral faces is the lateral area of a pyramid. The surface area of a pyramid is the sum of the lateral area and the area of the base.

**Parts of a Pyramid**




The *slant height* is the height of the triangular face *not* the height of the pyramid.

To find the surface area of the model rectangular pyramid, find the area of the base and each lateral face. Then add the areas to find the total surface area. Use a net to help.

Area of the rectangular base:

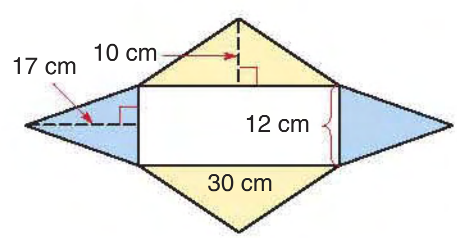
$A = \ell w \longrightarrow 30 \bullet 12 = 360 \text{ cm}^2$

Area of each blue triangular lateral face:

$A = \frac{1}{2}bh \longrightarrow \frac{1}{2}(12)(17) = 102 \text{ cm}^2$

Area of each yellow triangular lateral face:

$A = \frac{1}{2}bh \longrightarrow \frac{1}{2}(30)(10) = 150 \text{ cm}^2$




*Lateral area* = *Sum of the areas of the lateral faces*
$= 2(102) + 2(150) = 504 \text{ cm}^2$

*Surface area* = *area of the base* + *lateral area*
$= 360 + 504 = 864 \text{ cm}^2$

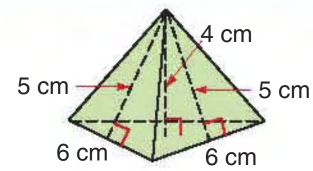
So Veronica will use 864 square centimeters of wood.

**Key Concept**

**Surface Area ($S$) of a Pyramid**

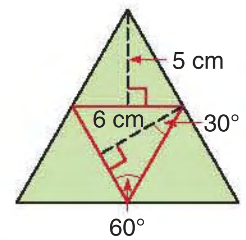
$S = B + LA$, where $S$ = surface area,
$B$ = area of the base, and $LA$ = lateral area.

▶ Some dimensions of a figure may not always be given or the dimensions need to be expressed as different units of measure.




Find the surface area of this tetrahedron (triangular pyramid) in square millimeters.

**Think**
The lateral faces are congruent isosceles triangles. The base is an equilateral triangle.




The dimensions of the pyramid are given in centimeters. The height of the equilateral triangular base is *not* given.

**1** Find the height of the equilateral triangular base. Use the Pythagorean Theorem. The height of the equilateral base divides the equilateral triangle into two congruent right triangles, each with a base of 3 cm and a hypotenuse of 6 cm.

**Remember:** The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the legs.

$6^2 = 3^2 + x^2$ ← Use the Pythagorean Theorem.

$6^2 - 3^2 = 3^2 + x^2 - 3^2$ ← Subtract $3^2$ from both sides to isolate $x^2$.

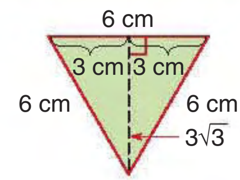
$6^2 - 3^2 = x^2$ ← Evaluate the exponential expressions.

$36 - 9 = x^2$ ← Simplify.

$27 = x^2$ ← Take the square root of both sides.

$\sqrt{27} = x$ ← $\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

$x = 3\sqrt{3} \approx 5.2$




So the height of the equilateral triangular base is about 5.2 cm.

**2** Convert each dimension from centimeters to millimeters.

$$6\ \cancel{\text{cm}} \cdot \frac{10\text{ mm}}{1\ \cancel{\text{cm}}} = 60\text{ mm}$$

$$5\ \cancel{\text{cm}} \cdot \frac{10\text{ mm}}{1\ \cancel{\text{cm}}} = 50\text{ mm}$$

$$5.2\ \cancel{\text{cm}} \cdot \frac{10\text{ mm}}{1\ \cancel{\text{cm}}} = 52\text{ mm}$$

**3** Calculate the surface area.

- Find the Lateral Area ($LA$).

$$LA = 3\left(\tfrac{1}{2}(60)(50)\right)$$
$$= 3(1500) = 4500\text{ mm}^2$$

- Find the Area of the Base

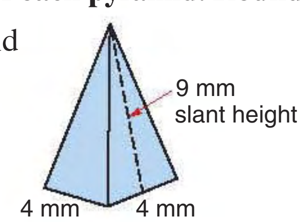
$$A \approx \tfrac{1}{2}(60)(52) = 1560\text{ mm}^2$$
$$S \approx 1560 + 4500 = 6060\text{ mm}^2$$

So the surface area of the tetrahedron is about 6060 square millimeters.

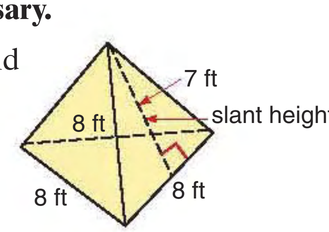
## Try These

**Find the surface of each pyramid. Round to the nearest tenth, if necessary.**

**1.** square pyramid




**2.** triangular pyramid



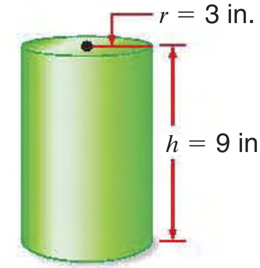

**3. Discuss and Write** Describe the difference between the slant height of a pyramid and the height of a pyramid.

Go to PRACTICE BOOK Lesson 11-4 for exercise sets.

Algebra

# 11-5 Surface Area of Cylinders and Cones

**Objective** To draw and use nets to find the surface area of cylinders and cones
• To use formulas to find the surface area of cylinders and cones



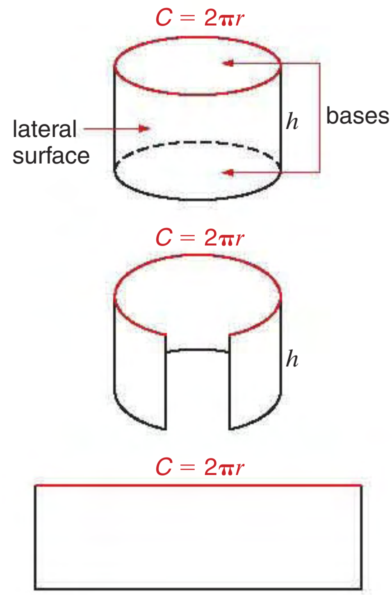

A designer is creating a new label that will completely cover a cylindrical carton. What is the surface area in square inches of the new label?

To find the surface area of the label, find the surface area of a cylinder.

► The surface area of a cylinder is the sum of the lateral area and the area of the two congruent circular bases. The lateral area of a cylinder is the area of the *lateral surface*. The lateral surface of a cylinder is the curved surface that is not a base.

When the cylinder's curved surface is completely open, it takes the shape of a rectangle. Its length is the circumference of the circular base and its width is the height of the cylinder.

**Lateral Area of a Cylinder**




The area of the lateral surface of a cylinder is:
$A = \ell w = Ch = (2\pi r)h = 2\pi rh$

The area of each circular bases is $A = \pi r^2$.

The total surface area $= \pi r^2 + \pi r^2 + 2\pi rh$
$= 2\pi r^2 + 2\pi rh$

**Key Concept**

**Surface Area ($S$) of a Cylinder**
$S = 2\pi r^2 + 2\pi rh$ where $S$ = surface area, $r$ = radius of the circular base, and $h$ = height of the cylinder.

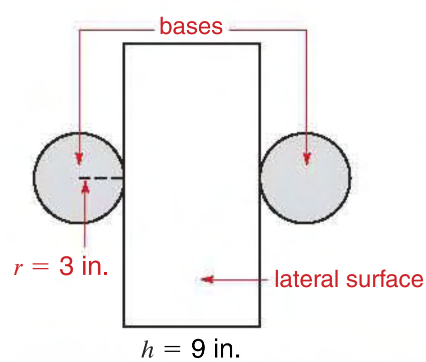
To find the surface area of the cylinder, use $\pi \approx 3.14$.

Total area of the circular bases: $A = 2\pi r^2$
$= 2\pi(3)^2$
$= 18\pi \approx 56.52$ in.$^2$

Lateral Area: $A = \ell w = (2\pi r)(h)$
$= 2\pi(3)(9)$
$= 54\pi \approx 169.56$ in.$^2$

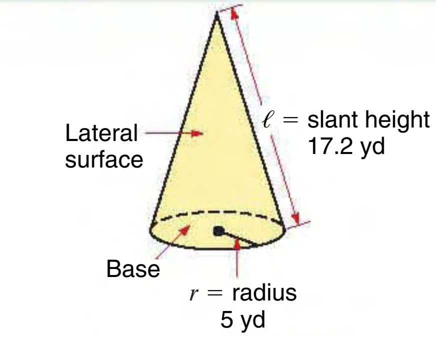
$S = 18\pi + 54\pi$
$= 72\pi$
or
$S \approx 56.52 + 169.56$
$= 226.08$ in.$^2$

So the new designed label will be about 226.08 square inches.



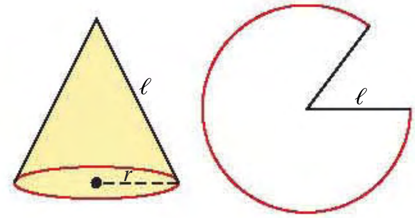

**Remember:** When you use either 3.14 or $\frac{22}{7}$ for $\pi$, your answer will be an approximation so you should use the $\approx$ symbol. You can use the $=$ symbol when you find surface area in terms of $\pi$.

► You can also find the surface area of a cone. Find the approximate surface area of this cone.




The surface area of a cone is the sum of the lateral area and the area of its circular base. Like a cylinder, the lateral area of a cone is the area of its *lateral surface.* Unlike a cylinder, the lateral surface of a cone is not a rectangle. The slant height of a cone is the altitude of the lateral surface.

When the cone is completely open, you have a piece of a larger circle. Look at the diagram at the right. The two red curves have exactly the same length.




Since you know the circumference of the base of the cone is $2\pi r$, the ratio of the circumference of the cone to the red arc of the circle is $\frac{2\pi r}{2\pi \ell}$, or $\frac{r}{\ell}$. Multiplying this ratio by the area of the circle gives the area of the lateral surface of the cone.

$\frac{r}{\ell} \bullet \pi \ell^2 = \pi r \ell$

The area of the lateral surface of a cone is $\pi r \ell$.
The area of the circular base is $\pi r^2$.
The total surface area is $\pi r^2 + \pi r \ell$.

**Key Concept**

**Surface Area ($S$) of a Cone**
$S = \pi r^2 + \pi r \ell$ where $S$ = surface area,
$r$ = radius of the circular base, and
$\ell$ = the slant height of the cone.

So, using $\pi \approx 3.14$, the approximate surface area of the cone above is:

Area of the base: $A = \pi r^2$
$= \pi(5)^2 = 25\pi \approx 78.5 \text{ yd}^2$

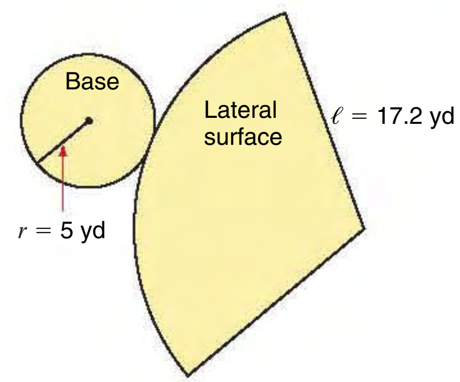
Lateral area ($LA$): $LA = \pi r \ell$
$= \pi(5)(17.2) = 86\pi \approx 270.04 \text{ yd}^2$

Total surface area ($S$): $S = 25\pi + 86\pi = 111\pi$

or

$S \approx 78.5 + 270.04 = 348.54 \text{ yd}^2$

So the area of the cone is approximately 348.54 square yards.




## Try These

**Find the surface area of the cylinder with given dimensions. Use $\pi \approx 3.14$.**

**1.** $h = 12$ m; $r = 4$ m  **2.** $h = 12$ ft; $r = 12$ ft  **3.** $h = 3.1$ in.; $r = 2$ in.

**Find the surface area of the cone with given dimensions. Use $\pi \approx 3.14$.**

**4.** $\ell = 14$ mm; $r = 3$ mm  **5.** $\ell = 100$ yd; $r = 10$ yd  **6.** $\ell = 9.9$ cm; $r = 7$ cm

**7. Discuss and Write** If you used the height of a cone instead of the slant height to find its surface area, would your result be greater or less than the actual surface area? Explain.

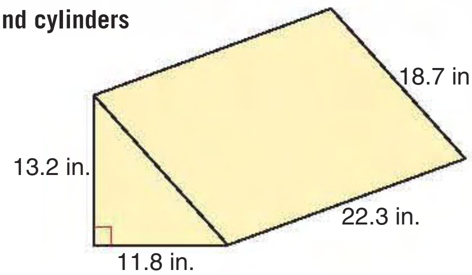
Go to PRACTICE BOOK Lesson 11-5 for exercise sets.

Algebra

# 11-6 Estimate Surface Area

**Objective** To estimate the surface area of prisms and cylinders

Inga is wrapping a very large present packed in a box like the one shown. Her measurements, made to the nearest quarter inch, are shown in the drawing. About how many square feet of wrapping paper are needed to wrap the gift?




► When computing surface area, sometimes an estimate is sufficient and an exact calculation is not needed.

There are different types of estimates. An overestimate is an estimate greater than the actual value. It is found by rounding a number up to a *greater* number. An underestimate is an estimate less than the actual value. It is found by rounding a number down to a *lesser* number.

Inga should *overestimate* to be sure she has enough wrapping paper.

To find about how many square feet of wrapping paper Inga needs, round each measurement to the nearest whole number. Then compute.

1. Round each dimension up to the nearest whole number.
   length of each rectangular face = 22.3 in. ≈ 23 in.
   base of the right triangular base = 11.8 in. ≈ 12 in.
   height of the right triangular base = 13.2 in. ≈ 14 in.
   side parallel to the hypotenuse of the triangular base = 18.7 in. ≈ 19 in.

2. Estimate the area of the triangular base.
   $A = \frac{1}{2}bh$
   $A \approx \frac{1}{2}(12)(14)$
   $\approx \frac{1}{2}(168) \approx 84 \text{ in.}^2$

3. Estimate the area of each lateral face.
   $A = \ell w$
   $A_1 \approx (12)(23) \approx 276 \text{ in.}^2$
   $A_2 \approx (19)(23) \approx 437 \text{ in.}^2$
   $A_3 \approx (14)(23) \approx 322 \text{ in.}^2$

4. Estimate the total surface area.
   $S = 2B + A_1 + A_2 + A_3$
   $S \approx 2(84) + 276 + 437 + 322$
   $\approx 1203 \text{ in.}^2$

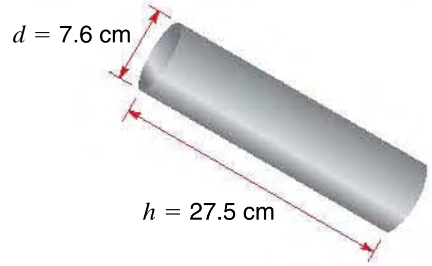
5. Rename square inches as square feet.
   Divide: $1203 \div 144 \approx 8.35$
   $1203 \text{ in.}^2 \approx 8 \text{ ft}^2$

**Remember:**
1 ft = 12 in. and 1 $\text{ft}^2$ = 12 in. × 12 in. or 144 $\text{in.}^2$

So Inga needs about 8 square feet of wrapping paper to cover the box.

## Example

**1** José wants to paint the entire surface of a solid metal rod. About how many square centimeters does he need to cover the rod with paint?




To find about how many square centimeters, estimate the surface area of the rod (a cylinder).

To estimate the surface area, round each measurement to the nearest whole number. Then compute.

**1** Round each dimension to the nearest whole number.

diameter = 7.6 cm ≈ 8 cm

height = 27.5 cm ≈ 28 cm

**2** First estimate the radius.

$r = \frac{d}{2} \rightarrow r \approx \frac{8}{2} \approx 4$ cm

**3** Estimate the area of the circular base.

$A = \pi r^2$

$A \approx 3(4^2)$

$A \approx 48 \text{ cm}^2$

Think $\pi \approx 3.14 \approx 3$

**4** Estimate the lateral area.

Lateral Area $= 2\pi rh$

$\approx 2(3)(4)(28)$

$\approx 672 \text{ cm}^2$

**5** Estimate the total surface area.

$S = 2\pi r^2 + 2\pi rh$

$S \approx 2(48) + 672$

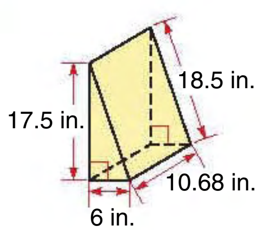
$\approx 768 \text{ cm}^2$

José needs about 768 $\text{cm}^2$ of paint to cover the rod.

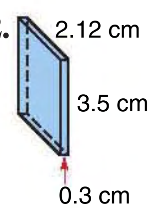
## Try These

**Estimate the surface area for each solid to the nearest whole unit.**

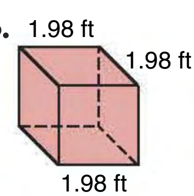
**1.**



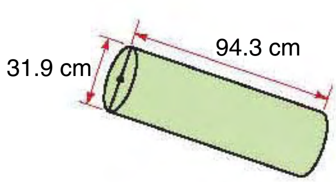

**2.**




**3.**

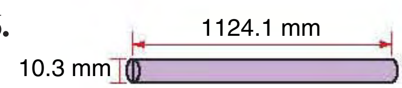



**4.**




**5.**


**6.**




**7. Discuss and Write** Explain how you could express your answer to exercise 5 to the nearest square yard. Show the steps to support your explanation.

Go to PRACTICE BOOK Lesson 11-6 for exercise sets.

Algebra

# 11-7 Volume of Prisms

**Objective** To use a formula to find the volume of a rectangular prism • To use a formula to find the volume of a triangular prism • To find an unknown dimension given the volume of a rectangular prism or a triangular prism • To rename volume units in equivalent forms

A fish tank in the shape of a rectangular prism is 30 inches long, 15 inches wide, and 16 inches high. How much water is needed to fill the tank completely?

To find how much water is needed to fill the tank, find the *volume* of the tank.

► The volume of a three-dimensional figure is the amount of space it occupies or contains. Volume is measured in cubic units—for example, cubic centimeters ($cm^3$) or cubic feet ($ft^3$).

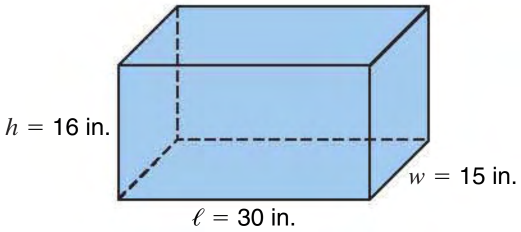
**Method 1** Use Unit Cubes

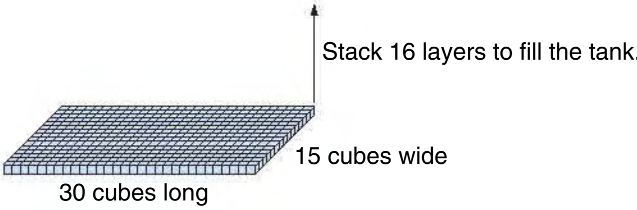
One way to find the volume is to count the number of unit cubes the tank could hold. Imagine making a layer of 1-inch cubes on the bottom of the tank. The layer would be 30 cubes long and 15 cubes wide. So the layer would contain 30 • 15 cubes, or 450 cubes. Since the height of the tank is 16 inches, you would need 16 layers to fill the tank for a total of 16 • 450 cubes, or 7200 cubes.

**Think**

The length, width, and height of a 1-inch cube each measure 1 inch. So the volume of a 1-inch cube is 1 cubic inch, or 1 $in.^3$

1 • 1 • 1 is "1 to the third power" or "1 cubed."




**Method 2** Use a Formula

To find the number of cubes needed to fill the tank, you multiplied the length (30 cubes), width (15 cubes), and height (16 cubes).

Use the formula: $V = \ell wh$

$= 30 \bullet 15 \bullet 16$

$= 7200$

So the volume of the tank is 7200 cubic inches, or 7200 $in.^3$

**Key Concept**

**Volume ($V$) of a Rectangular Prism**

$V = \ell wh$, where $V$ = volume, $\ell$ = length, $w$ = width, and $h$ = height

The formula can also be expressed as $V = Bh$, where $B$ represents the area of the base and $h$ represents the height.

► Another way to think about finding the volume of the rectangular prism is to think about multiplying the area of the base ($\ell w$) by the height ($h$). This process leads to the formula for the volume of a triangular prism.

**Key Concept**

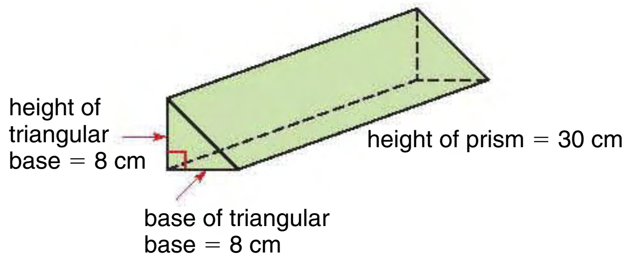
**Volume ($V$) of a Triangular Prism**

$V = Bh$, where $V$ = volume,
$B$ = area of the base, and
$h$ = height

To find the volume of the triangular prism at the right, use the formula.

**Remember:**
**Area of a Triangle**
$A = \frac{1}{2}bh$, where $b$ = base and $h$ = height

$V = Bh$
$= \left(\frac{1}{2}bh_{\text{base}}\right)h_{\text{prism}}$ ← Substitute the formula for the area of a triangle for $B$.
$= \left(\frac{1}{2} \bullet 8 \bullet 8\right)30$ ← Substitute known values.
$= (32)30$ ← Simplify within the parentheses.
$= 960$ ← Multiply.




So the volume of the triangular prism is 960 cubic centimeters, or 960 $\text{cm}^3$.

► You can rename the volume of a three-dimensional figure by finding an equivalent volume expressed in different units.

PROBLEM-SOLVING STRATEGY 7

Rename the volume, 960 $\text{cm}^3$, of the triangular prism above in cubic millimeters.

To find the volume in cubic millimeters, multiply:
960 • 1000 = 960,000

**Think**
1 cm • 1 cm • 1 cm = 1 $\text{cm}^3$
↓ ↓ ↓ ↓
10 mm • 10 mm • 10 mm = 1000 $\text{mm}^3$
So the number of cubic millimeters is the number of cubic centimeters multiplied by 1000.

So the volume of the triangular prism can also be written as 960 000 $\text{mm}^3$.

► If you know the volume of a three-dimensional figure, you can use a formula to solve for an unknown dimension of that figure.

Rita wants to store 378 $\text{in.}^3$ of soup in a container shaped like a rectangular prism. The base of the container measures 9 in. by 6 in. How tall must the container be to hold all of the soup?

$V = \ell wh$ ← Formula for volume of a rectangular prism
$378 = 9 \bullet 6 \bullet h$ ← Substitute the known values.
$378 = 54h$ ← Multiply.
$\frac{378}{54} = \frac{54h}{54}$ ← Divide both sides by 54 to isolate $h$.
$7 = h$ ← Simplify.

The container must be at least 7 in. tall in order to store the soup.

## Try These

**Find the volume of each rectangular prism ($\ell$ = length, $w$ = width, and $h$ = height).**

**1.** $\ell = 3$ m; $w = 18$ m; $h = 14$ m

**2.** $\ell = 5.5$ ft; $w = 4.5$ ft; $h = 10.5$ ft

**3.** $\ell = 10$ cm; $w = 100$ cm; $h = 200$ cm

**4. Discuss and Write** A rectangular prism measures 2 cm by 5 cm by 10 cm. Can you find another rectangular prism that has the same volume but has less surface area? Can you find another rectangular prism that has the same volume but has more surface area? Explain your answers.

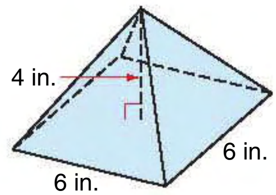
Go to PRACTICE BOOK Lesson 11-7 for exercise sets.

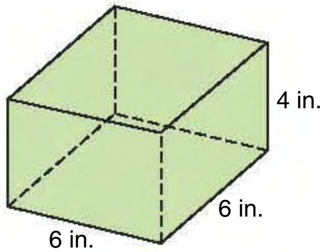
Algebra

# 11-8 Volume of Pyramids

**Objective** To use formulas to find the volumes of pyramids • To find unknown dimensions given the volumes of rectangular and triangular pyramids

Charmaine has two containers—one in the shape of a square pyramid and the other in the shape of a square prism. The bases of the containers are congruent and their heights are equal. As an experiment, she completely fills the pyramid with water and empties it into the prism. How many times does she have to do this in order to completely fill the prism?




► The volume of a pyramid is equal to $\frac{1}{3}$ the volume of a prism if the figures have equal heights and congruent bases. So to fill the prism with water, Charmaine has to fill and empty the pyramid three times.

**Remember:** The height of a pyramid is the perpendicular distance from the pyramid's base to its vertex.

**Key Concept**

**Volume ($V$) of a Pyramid**

$V = \frac{1}{3}Bh$, where $B$ = area of the base and $h$ = height

Find the volume of each of the two figures above.

**Volume of pyramid:** $V = \frac{1}{3}Bh$

$$V = \frac{1}{3}\ell wh$$
$$= \frac{1}{3}(6 \bullet 6)4$$
$$= \frac{1}{3}(36)4$$
$$= (12)4 = 48$$

**Volume of prism:** $V = \ell wh$ or $V = Bh$

$$V = (6)(6)(4)$$
$$= (36)4$$
$$= 144$$

**Check:** $48 \text{ in.}^3 \stackrel{?}{=} \frac{1}{3}(144 \text{ in.}^3)$

$48 \text{ in.}^3 = 48 \text{ in.}^3$ True

The volume of the pyramid is 48 in.$^3$, which is $\frac{1}{3}$ the volume of the prism.

► The formula also applies to triangular pyramids. Find the volume of the pyramid at the right.

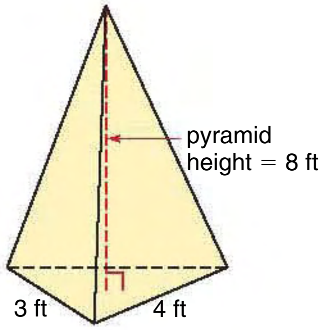
$V = \frac{1}{3}Bh$ ← Formula for the volume of a pyramid

$= \frac{1}{3}\left(\frac{1}{2}bh_{\text{base}}\right)h_{\text{pyramid}}$ ← Substitute the formula for the area of a triangle for $B$.

$= \frac{1}{3}\left(\frac{1}{2} \bullet 3 \bullet 4\right)8$ ← Substitute known values.

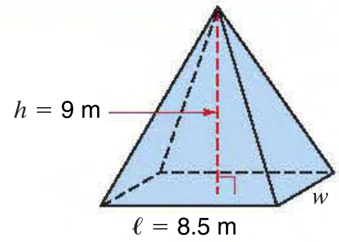
$= \frac{1}{3}(6)8$ ← Simplify within the parentheses.

$= (2)8 = 16$ ← Multiply.




So the volume of the triangular pyramid is 16 ft$^3$.

► If you know the volume of a pyramid, you can use formulas and algebra to solve for an unknown dimension of the pyramid.




PROBLEM-SOLVING STRATEGY 9

The rectangular pyramid at the right has a volume of 160.65 cubic meters. What is the width of its base?

$V = \frac{1}{3}Bh$ ← Formula for the volume of a pyramid

$= \frac{1}{3}(\ell w)h$ ← Substitute the formula for the area of a rectangle for $B$.

$160.65 = \frac{1}{3}(8.5w)9$ ← Substitute known values.

$160.65 = 3(8.5w)$ ← Use the Commutative Property to simplify.

$\frac{160.65}{25.5} = \frac{25.5w}{25.5}$ ← Divide both sides by 25.5 to isolate $w$.

$6.3 = w$

So the rectangular base of the pyramid has a width of 6.3 meters.

## Examples

**1** A square pyramid has a base that measures 10 ft on each side. Its height is $21\frac{3}{4}$ ft. What is the pyramid's volume?

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(\ell w)h$$
$$= \frac{1}{3}(10 \bullet 10)21\frac{3}{4}$$
$$= \frac{1}{3}(100)\frac{87}{4}$$
$$= \frac{29}{4}(100) = 725$$

So the pyramid's volume is 725 $\text{ft}^3$.

**2** A triangular pyramid has a volume of 1618.2 $\text{cm}^3$. Its base has a height of 15.5 cm and a base of 18 cm. What is the pyramid's height?

$$V = \frac{1}{3}Bh$$
$$1618.2 = \frac{1}{3}\left(\frac{1}{2}bh_{\text{base}}\right)h_{\text{pyramid}}$$
$$1618.2 = \frac{1}{3}\left(\frac{1}{2} \bullet 18 \bullet 15.5\right)h$$
$$1618.2 = \frac{1}{3}(139.5)h$$
$$1618.2 = 46.5h$$
$$34.8 = h$$

So the pyramid's height is 34.8 cm.

## Try These

**Find the volume of each pyramid.**

1. Rectangular pyramid:
   $\ell = 3$ m; $w = 18$ m; $h = 14$ m

2. Triangular pyramid:
   base of base = 6 yd; height of base = $4\frac{1}{2}$ yd;
   height of pyramid = $9\frac{1}{3}$ yd

**Find the unknown dimension.**

3. Triangular pyramid:
   $V = 140\ \text{m}^3$; height of pyramid = 15 m;
   base of base = 8 m; height of base = ?

4. Rectangular pyramid:
   $V = 12{,}852\ \text{in.}^3$; $\ell$ = ?; $w = 17$ in.; $h = 36$ in.

5. **Discuss and Write** Why is the slant height *not* a part of the formula to find the volume of a square pyramid?

Go to PRACTICE BOOK Lesson 11-8 for exercise sets.

Algebra

# 11-9 Volume of Cylinders and Cones

**Objective** **To use formulas to find the volumes of cylinders and cones • To find unknown dimensions given the volumes of cylinders and cones**

A cylindrical tank is 25-feet deep and has a diameter of 110 feet. How many cubic feet of water will the tank hold?

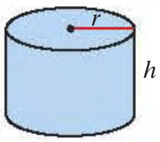
To find how much water, find the volume of the cylinder.

► When you found the volume of a prism, you calculated the area of the prism's base and then multiplied it by the prism's height. You can also think about the volume of a cylinder by considering its base and height.

**Key Concept**

**Volume ($V$) of a Cylinder**

$V = Bh$ or $V = \pi r^2 h$, where
$B$ = area of the base,
$r$ = radius of the base, and
$h$ = height of the cylinder.




To find the volume of the tank, use the formula. You can do this either in terms of $\pi$ or by using an approximation for $\pi$.

**Volume of the pool (in terms of $\pi$)**

$$\begin{aligned} V &= Bh \\ &= (\pi r^2)h \\ &= (\pi \bullet 55^2)25 \\ &= (3025\pi)25 \\ &= 75{,}625\pi \end{aligned}$$

**Think** The radius is half the diameter. So $r = 55$ ft.

**Volume of the pool (using 3.14 for $\pi$)**

$$\begin{aligned} V &= Bh \\ &= (\pi r^2)h \\ &\approx (3.14 \bullet 55^2)25 \\ &\approx (9498.5)25 \\ &\approx 237{,}462.5 \end{aligned}$$

So the tank will hold $75{,}625\pi$ ft$^3$, or about $237{,}462.5$ ft$^3$, of water.

## Example

**1** Find the volume, in terms of $\pi$, of a cylinder with a radius of 7 mm and a height of 22 mm.

$$\begin{aligned} V &= Bh \\ &= (\pi r^2)h \\ &= (\pi \bullet 7^2)22 \\ &= (49\pi)22 \\ &= 1078\pi \end{aligned}$$

**Remember:** When you use either 3.14 or $\frac{22}{7}$ for $\pi$, your answer will be an approximation so you should use the $\approx$ symbol. You can use the $=$ symbol when you find volume in terms of $\pi$.

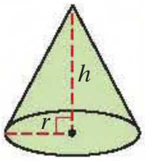
So the cylinder has a volume of $1078\pi$ cubic millimeters.

► The volume of a cone is related to a cylinder's volume in the same way a pyramid's volume is related to a prism's volume. If a cone and a cylinder have congruent bases and equal heights, the volume of the cone is $\frac{1}{3}$ that of the cylinder.

**Key Concept**

**Volume ($V$) of a Cone**

$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$, where $B$ = area of the base, $r$ = radius of the base, and $h$ = height of the cone.




Find the volume, to the nearest hundredth, of the cone below. Use 3.14 for $\pi$.

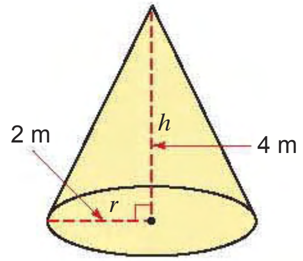
$V = \frac{1}{3}Bh$

$= \frac{1}{3}(\pi r^2)h$ ← Substitute the formula for the area of a circle for $B$.

$\approx \frac{1}{3}(3.14 \bullet 2^2)4$ ← Substitute known values.

$\approx \frac{1}{3}(12.56)4$ ← Simplify within the parentheses.

$\approx \frac{1}{3}(50.24) \approx 16.75$




The cone has a volume of approximately 16.75 $m^3$.

► If you know the volume of a cylinder or cone, you can use a formula and algebra to solve for the length of a radius or an unknown height.

A cylindrical grain silo needs to hold 3956 $yd^3$ of grain and have a radius of 6 yd. How tall must the silo be in order to hold the required amount of grain? Use 3.14 for $\pi$ and round your answer to the nearest yard.

$V = \pi r^2 h$

$3956 \approx (3.14 \bullet 6^2)h$ ← Use 3.14 for $\pi$.

$3956 \approx (113.04)h$

$\frac{3956}{113.04} \approx \frac{113.04h}{113.04}$

$35 \approx h$

So the grain silo must be at least 35 yd tall.

## Try These

**Find the volume of the cylinder or cone in terms of $\pi$.**

**1.** Cone: $r = 19$ in.; $h = 12$ in.

**2.** Cylinder: $r = 1.4$ km; $h = 19.6$ km

**Find the unknown dimension. Use 3.14 for $\pi$. Round to the nearest tenth.**

**3.** Cylinder: $V = 930.4$ $m^3$; $r = 5.25$ m; $h = ?$

**4.** Cone: $V = 2034.72$ $cm^3$; $r = ?$; $h = 24$ cm

**5. Discuss and Write** Use two identical $8\frac{1}{2} \times 11$ sheets of paper. Roll and tape one vertically and one horizontally to form two cylinders. Measure the dimensions and compute the volume of each. Which has the greater volume? Explain why.

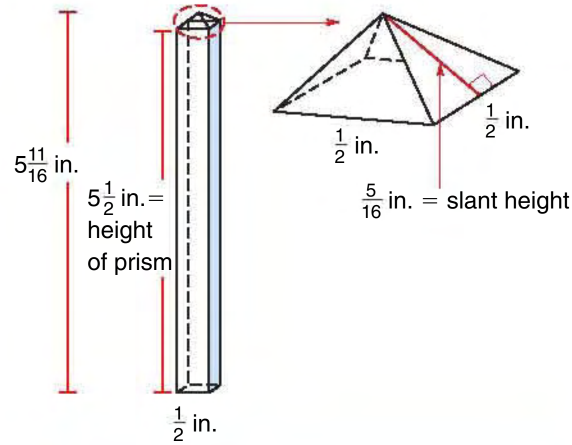
Go to PRACTICE BOOK Lesson 11-9 for exercise sets.

Algebra

# 11-10 Surface Area and Volume of Complex Three-Dimensional Figures

**Objective** To draw complex three-dimensional figures • To use nets to find the surface area and volume of complex three-dimensional figures • To use formulas to find the surface area and volume of complex three-dimensional figures

A souvenir manufacturer will produce replicas of the Washington Monument. Replicas will be created from plastic and then painted with a texture paint. For each replica, about how many square inches will be covered by paint?

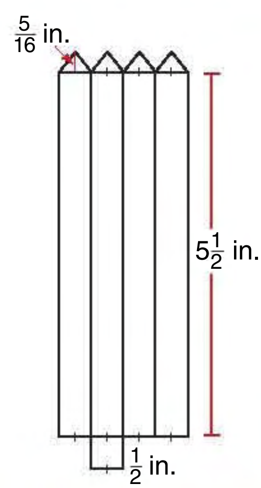



To answer the question, you need to find the surface area of *complex three-dimensional figures*. As with two-dimensional figures, complex three-dimensional figures can be constructed by placing two or more simpler three-dimensional shapes together.

**Remember: Surface Area Formulas**
Rectangular Prism: $S = 2\ell w + 2\ell h + 2hw$
Triangular Prism: $S = 2B + A_1 + A_2 + A_3$
Pyramid: $S = B + LA$
Cylinder: $S = 2\pi r^2 + 2\pi rh$
Cone: $S = \pi r^2 + \pi r\ell$

► To find the surface area of a complex figure, create a net. Then find the area of each two-dimensional figure and calculate the sum of the areas.

To find the surface area of the souvenir, create a net. Notice that all the figures in this net are polygons.




Area of the triangles: $A = 4\left(\frac{1}{2}bh\right) = 4\left[\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{5}{16}\right)\right] = \frac{5}{16}$ in.$^2$

Area of the rectangles: $A = 4\ell w = 4\left(\frac{1}{2}\right)\left(\frac{11}{2}\right) = 11$ in.$^2$

$\boxed{5\frac{1}{2} = \frac{11}{2}}$

Area of the square: $A = s^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ in.$^2$

Then find the sum of the areas ($S$) to the nearest 0.1 in.$^2$

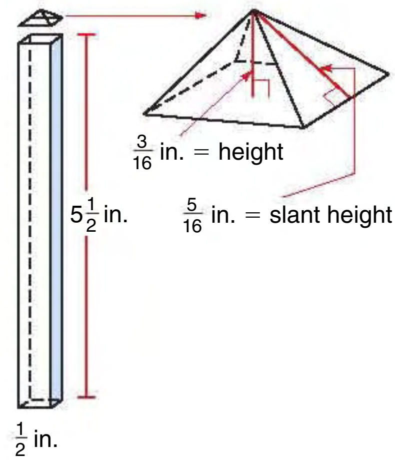
$$S = \frac{5}{16} + 11 + \frac{1}{4}$$
$$= 11\frac{9}{16}$$
$$\approx 11.6 \text{ in.}^2$$

So the manufacturer will cover about 11.6 square inches of each souvenir with paint.

► To find the volume of a complex figure, identify which figures helped to create it. A complex figure can be made from the *union* of two or more three-dimensional figures. Then, calculate the volume of the complex figure by finding the *sum of the volumes* of the figures used to construct it.

**Remember: Volume Formulas**
Rectangular Prism: $V = \ell wh$
Triangular Prism: $V = Bh$
Rectangular Pyramid: $V = \frac{1}{3}Bh$
Triangular Pyramid: $V = \frac{1}{3}Bh$
Cylinder: $V = Bh = \pi r^2 h$
Cone: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

About how many cubic inches of plastic will be used for each replica?




**1** The souvenir is created by placing a square pyramid on top of a rectangular prism. The height of the prism is given, and the height of the complex figure is given. So you need to subtract to find the height of the pyramid.

$h = 5\frac{11}{16} - 5\frac{1}{2} = 5\frac{11}{16} - 5\frac{8}{16} = \frac{3}{16}$ in.

**2** Now find the volume of each figure.

Volume of the pyramid: $V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2} \bullet \frac{1}{2}\right)\frac{3}{16} = \frac{1}{64}$ in.$^3$

Volume of the rectangular prism: $V = Bh = \left(\frac{1}{2} \bullet \frac{1}{2}\right)\frac{11}{2} = \frac{11}{8}$ in.$^3$

**3** Find the sum of the volumes to the nearest 0.1 in.$^3$

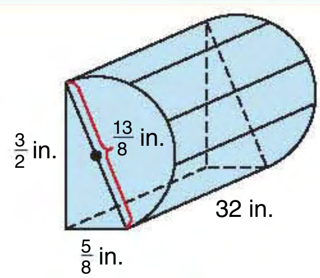
Volume of the Souvenir = Volume of Pyramid + Volume of Rectangular Prism

$= \frac{1}{64} + \frac{11}{8} = \frac{89}{64} \approx 1.4$ in.$^3$

So the manufacturer requires about 1.4 cubic inches of plastic for each souvenir.

## Example

**1** This figure was formed by placing half of a cylinder along one lateral face of a triangular prism.




Find the volume of this figure. Use 3.14 to approximate $\pi$. Where necessary, round to the nearest 0.1 in.$^3$

Volume of triangular prism: $V = Bh = \left[\frac{1}{2}\left(\frac{5}{8}\right)\left(\frac{3}{2}\right)\right]32 = 15$ in.$^3$

Volume of half of cylinder: $V = \frac{1}{2}Bh = \frac{1}{2}(\pi r^2)h = \frac{1}{2}\left(\pi\left(\frac{13}{16}\right)^2\right)32 \leftarrow r = \frac{1}{2} \bullet \frac{13}{8} = \frac{13}{16}$

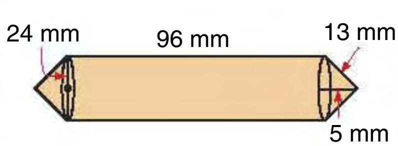
$= (3.14)\frac{169}{16} \approx 33.2$ in.$^3$

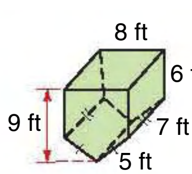
Volume of the figure: $V \approx 15 + 33.2 = 48.2$ in.$^3$

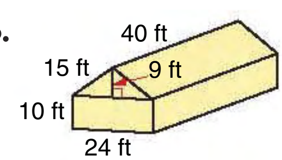
So the volume of this figure is about 48.2 cubic inches.

## Try These

**Find the surface area and volume for each figure. Use 3.14 to approximate $\pi$. Round to the nearest tenth, if necessary.**

1. 


2. 


3. 


4. **Write and Discuss** Why is the slant height of a pyramid always greater than the height of the pyramid?

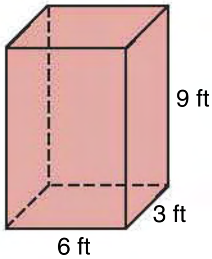
Go to PRACTICE BOOK Lesson 11-10 for exercise sets.

Algebra

# 11-11 Changing Dimensions of Three-Dimensional Figures

**Objective** To understand how change of scale relates to change in dimensions • To understand how changes in scale and dimension relate to changes in volume and surface area

A sculpture is planned to be put in front of a civic center. The shape of the sculpture will be a rectangular prism with length 6 feet, width 3 feet, and height 9 feet. The sculptor built a model with a scale factor of $\frac{2}{3}$. How does the surface area of the scale model compare to the surface area of the actual sculpture?




To compare surface areas, first find the dimensions of the model. Then compute the surface area of each and compare.

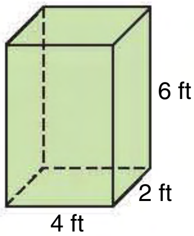
► The scale factor for comparing a dimension of the model to the corresponding dimension of the sculpture is $\frac{2}{3}$. So multiply each dimension of the sculpture by $\frac{2}{3}$ to find the corresponding dimension of the model.

**Remember:**
scale factor $= \dfrac{\text{scale model dimension}}{\text{actual object dimension}}$

$$\text{model length} = \frac{2}{3} \bullet 6 \text{ feet} = 4 \text{ feet}$$

$$\text{model width} = \frac{2}{3} \bullet 3 \text{ feet} = 2 \text{ feet}$$

$$\text{model height} = \frac{2}{3} \bullet 9 = 6 \text{ feet}$$




Find the surface areas of the sculpture and the model.

$$\begin{aligned} S_{\text{model}} &= 2\ell w + 2\ell h + 2wh \\ &= 2(8) \text{ ft} + 2(24) \text{ ft} + 2(12) \text{ ft} \\ &= 16 \text{ ft}^2 + 48 \text{ ft}^2 + 24 \text{ ft}^2 \\ &= 88 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} S_{\text{sculpture}} &= 2\ell w + 2\ell h + 2wh \\ &= 2(18) \text{ ft} + 2(54) \text{ ft} + 2(27) \text{ ft} \\ &= 36 \text{ ft}^2 + 108 \text{ ft}^2 + 54 \text{ ft}^2 \\ &= 198 \text{ ft}^2 \end{aligned}$$

You can determine if a figure is a scale model of the original by checking to see if *each* and *every* dimension of the scale figure has been transformed by the same scale factor.

The ratio of the surface areas $= 88 : 198 = 4 : 9$. $\frac{4}{9}$ is the square of the scale factor $\frac{2}{3}$.

So the ratio of surface areas equals the square of the scale factor.

• How does the volume of the scale model compare to the volume of the actual sculpture?

$$\begin{aligned} V_{\text{model}} &= \ell wh \\ &= 4 \text{ ft} \bullet 2 \text{ ft} \bullet 6 \text{ ft} \\ &= 48 \text{ ft}^3 \end{aligned}$$

$$\begin{aligned} V_{\text{sculpture}} &= \ell wh \\ &= 6 \text{ ft} \bullet 3 \text{ ft} \bullet 9 \text{ ft} \\ &= 162 \text{ ft}^3 \end{aligned}$$

Since $48 : 162 = 8 : 27$, the ratio of the volumes equals the cube of the scale factor.

**Think**
$\frac{8}{27}$ is the cube of the scale factor $\frac{2}{3}$.

► Changes in dimensions affect changes in volume.

How would a cube's volume change if you doubled each of its dimensions? How would its volume change if you halved each of its dimensions?

A cube with edge length 6 mm has a volume of $6^3$ or 216 $mm^3$.

**Doubling each dimension**

The new cube has edges of length 12 mm. Its volume is $12^3 = 1728$ $mm^3$.

Because $1728 = 8 \bullet 216$, the new volume is 8 times the original.

So multiplying each dimension by 2 multiplies the volume by $2^3$, or 8.

**Halving each dimension**

The new cube has a side length of 3 mm. Its volume is $3^3 = 27$ $mm^3$.

Because $27 = \frac{1}{8} \bullet 216$, the new volume is $\frac{1}{8}$ the original. So multiplying each dimension by $\frac{1}{2}$ multiplies the volume by $\left(\frac{1}{2}\right)^3$, or $\frac{1}{8}$.

► To understand how volume changes when only *one* or *two* dimensions are changed, try using a volume formula and substituting different values for each dimension.

A triangular prism has a height of 10 cm and a triangular base with a base of 4 cm and a height of 6 cm. Its volume is $Bh = \left(\frac{1}{2} \bullet 4 \bullet 6\right) \bullet 10 = 120$ $cm^3$.

**Triple *only* the prism's height**

So $h = 30$ cm.

$V = Bh = \left(\frac{1}{2} \bullet 4 \bullet 6\right) \bullet 30$

$= 360$ $cm^3$

The new prism's volume is 3 times as large as the original.

**Triple *only* the height of the base of the prism**

So $h_{base} = 18$ cm.

$V = Bh = \left(\frac{1}{2} \bullet 4 \bullet 18\right) \bullet 10$

$= 360$ $cm^3$

The new prism's volume is 3 times as large as the original.

**Triple the height of the prism *and* the height of the base**

$h_{prism} = 30$ cm and $h_{base} = 18$ cm

$V = Bh = \left(\frac{1}{2} \bullet 4 \bullet 18\right) \bullet 30$

$= 1080$ $cm^3$

The new prism's volume is 9 times, or $3^2$, as large as the original.

**Key Concept**

**How Changes in Dimensions Affect Volume**

- When one dimension of a three-dimensional polyhedron is tripled, the volume is multiplied by 3.
- When two dimensions of a three-dimensional polyhedron are tripled, the volume is multiplied by $3^2$.
- When all three dimensions of a three-dimensional polyhedron are tripled, the volume is multiplied by $3^3$.

## Try These

1. Make a conjecture: If you triple all three dimensions of a prism, how does the surface area of the new prism compare to the surface area of the original? Use these dimensions to verify your conjecture: $\ell = 2$ in.; $w = 3$ in.; $h = 1$ in.

2. **Discuss and Write** Explain why doubling the length and halving the height does *not* change the volume of the rectangular prism. Use the formula for a rectangular prism's volume to help you confirm your explanation.

Go to PRACTICE BOOK Lesson 11-11 for exercise sets.

# 11-12 Problem-Solving Strategy: Work Backward

**Objective** To solve problems using the strategy *Work Backward*

**Problem 1:** Before going to work, Dad took half the money that Mom left in an envelope on the kitchen table. Later, Susan took half of what was in the envelope when she left for school. Still later, Wally took half of what was left when he went to school. When Mom retrieved the envelope upon returning home from work that afternoon, she found exactly $12 in it. How much money was originally in this envelope?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Three members of the family successively took half the money in an envelope. After this, $12 remained.

**Question:** How much did the envelope initially hold?

**Problem-Solving Strategies**
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Plan** **Select a strategy.**
By using the strategy *Work Backward*, you can start from the amount that remains at the end of the day and figure out how much was in the envelope at the beginning of the day.

**Solve** **Apply the strategy.**
- Because Wally left $12 in the envelope, he must have taken $12. So there was $24 in the envelope before he got to it.
- When Susan saw the envelope, it must have held exactly $48 (twice $24). She would have grabbed half of the money, or $24, and left half, $24, in the envelope.
- It follows that Dad would have seen twice this $48, or $96, in the envelope. He took $48 and left $48 in the envelope.

So Mom must have originally left $96 in the envelope.

**Check** **Check to make sure your answer makes sense.**
You can check your solution by running through the scenario from start to finish.
- Mom left $96 in an envelope.
- Dad takes half of $96, or $48, and leaves $48 in the envelope.
- Susan takes half of $48, or $24, and leaves $24 in the envelope.
- Wally takes half of $24, or $12, and leaves $12 in the envelope.
- Mom returns home and finds $12 in the envelope.

The answer checks.

**Problem 2:** You want to cook a hamburger for exactly 7 minutes. You have only a 4-minute timer and a 5-minute timer. (These are like hourglass sand timers. They empty in 4 or 5 minutes, but they cannot be used for marking other times.) How can you use these timers to time exactly 7 minutes?

**Read** **Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** You have only a 4-minute timer and a 5-minute timer.

**Question:** How can you use these timers to time 7 minutes?

**Plan** **Select a strategy.**

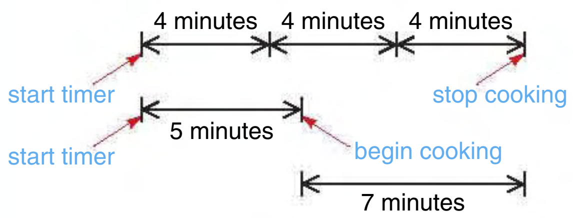
You can try to work backward.

Start with the 7-minute time frame and try to find a way to represent it in terms of 4 and 5 minutes.

**Solve** **Apply the strategy.**

You know that $(3 \bullet 4) - 5 = 7$. This suggests that you can run the 4-minute timer three consecutive times and "take away" the 5-minute timer once.

The diagram below shows how this can be done.




So here's the plan:

- Begin both timers at the same time.
- When the 4-minute timer finishes, restart it.
- When the 5-minute timer finishes, begin cooking the burger. (The 4-minute timer has 3 minutes left on it.)
- When the 4-minute timer ends (at which time the burger has been cooking for 3 minutes), restart it.
- Stop cooking when the 4-minute timer ends again. (The 4 additional minutes give a total of 7 minutes of cooking time.)

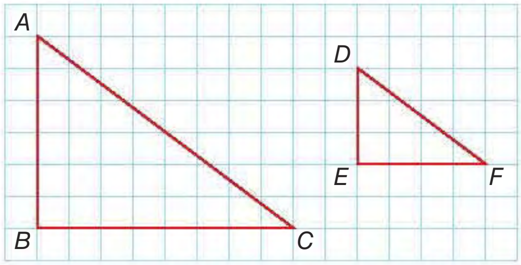
**Check** **Check to make sure your answer makes sense.**

You can verify this in 12 minutes by acting out the action while two of your classmates use their watches as 4- and 5-minute timers.

Go to PRACTICE BOOK Lesson 11-12 for exercise sets.

# Enrichment:
## Three-Dimensional Figures and the Ratio of Similarity

**Objective** **To explore the relationship between the ratio of similarity and the ratios of the area and volumes of similar solids**




You know that if two 2-dimensional figures are similar, then the ratio of their areas is equal to the square of the ratio of similarity.

For example, triangle $ABC$ and triangle $DEF$ are similar. The ratios of the lengths of the corresponding sides is 2 : 1, so the ratio of similarity is 2 : 1.

| Triangle $ABC$ | |
|---|---|
| **Side** | **Length** |
| $AB$ | 6 units |
| $CA$ | 10 units |
| $BC$ | 8 units |

| Triangle $DEF$ | |
|---|---|
| **Side** | **Length** |
| $DE$ | 3 units |
| $FD$ | 5 units |
| $EF$ | 4 units |

The area of triangle $ABC$ is $\frac{1}{2}(8)(6) = 24$ square units.

The area of triangle $DEF$ is $\frac{1}{2}(4)(3) = 6$ square units.

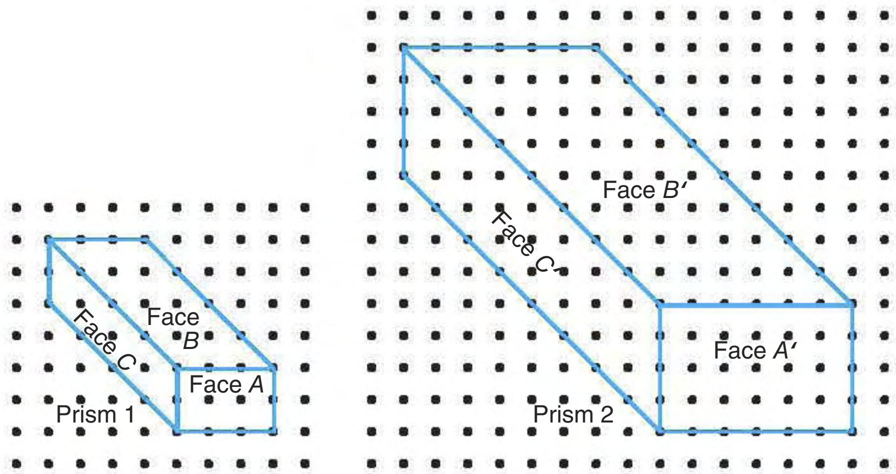
The ratio of the areas is 24 : 6, which is equivalent to 4 : 1; and 4 : 1 is equivalent to $2^2 : 1^2$. So the ratio of the areas of the triangles is the square of the ratio of similarity.

► Now you will explore the ratio of similarity and the surface areas and volumes of similar 3-dimensional figures.

The ratio of similarity for similar solids is the ratio of the corresponding linear parts or linear segments.

Use dot paper to draw two similar rectangular prisms.

You may use dimensions other than those in the drawings below.



Prism 1 in the drawing has edges that are half as long as those in Prism 2, so the ratio of similarity is 1 : 2.

Calculate the surface areas of the prisms you drew. Remember, the surface area is the sum of the areas of the faces. Then find the ratio of the surface areas. For Prisms 1 and 2, the results are shown below.

| Prism | Surface Area | Ratio |
|---|---|---|
| 1 | 52 units$^2$ | 52 : 208 or 1:4 |
| 2 | 208 units$^2$ | |

How does 1 : 4 relate to the ratio of similarity, 1 : 2?

1 : 4 can be written as $1^2 : 2^2$, the square of the ratio of similarity. For these two similar prisms (and for all similar prisms), the ratio of surface areas is the square of the ratio of similarity.

Now, calculate the volumes of your prisms. Then find the ratio of the volumes. For Prisms 1 and 2, the results are shown at the right.

| Prism | Volume | Ratio |
|---|---|---|
| 1 | 24 units$^3$ | 24 : 192 or 1 : 8 |
| 2 | 192 units$^3$ | |

How does this ratio relate to the ratio of similarity, 1 : 2?

The ratio is 1 : 8 and can be written as $1^3 : 2^3$. For these two similar prisms (and for all similar prisms), the ratio of volumes is the cube of the ratio of similarity.

## Try These

**In exercises 1–3, the pair of solids is similar.**
**Find each of the following:**

- **a. the ratio of similarity of the first solid to the second solid**
- **b. the surface areas and the ratio of the surface areas (first solid to second solid)**
- **c. the volumes and the ratio of the volumes (first solid to second solid)**

**1.** Cylinder E: radius 7 cm, height 14 cm
Cylinder F: radius 14 cm, height 28 cm

**2.** Square pyramid G: length 6 ft, face height 5 ft
Square pyramid H: length 30 ft, face height 25 ft

**3.** Cone J: radius 20 in., height 48 in., slant height 52 in.
Cone K: radius 5 in., height 12 in., slant height 13 in.

**4.** **Discuss and Write** Refer to your results from exercises 1–3. How does the ratio of the surface areas of each pair of solids relate to the ratio of similarity? How does the ratio of the volumes of each pair of solids relate to the ratio of similarity for the two solids?

Go to PRACTICE BOOK pages 365–366 for exercise sets.

# Test Prep: Multiple-Choice Questions

## Strategy: Apply Mathematical Reasoning

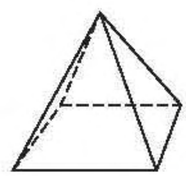
Often you can draw a model or diagram to *visualize the problem*. Check that your sketch matches the given information.

**Read the whole test item, including the answer choices.**

- Underline important words.
  How many edges does a rectangular pyramid have?
  A *rectangular pyramid* is a solid that has 1 rectangular and 4 triangular faces. The *edges* are where two faces meet.
- Restate the question in your own words.
  How many edges are there in a rectangular pyramid?

**Solve the problem.**

- Sketch a diagram of a rectangular pyramid.
- Count the number of edges.



**Item Analysis**

**Choose the answer.**

- Analyze and eliminate answer choices. Watch out for distractors.

**A.** 4 ← This does not count the edges of the base. Eliminate this choice.

**B.** 5 ← This is the number of vertices, not the number of edges. Eliminate this choice.

**C.** 6 ← This is the number of edges of a triangular pyramid. Eliminate this choice.

**D.** 8 ← This is the correct choice!

**Sample Test Item**

How many edges does a rectangular pyramid have?

**A.** 4
**B.** 5
**C.** 6
**D.** 8

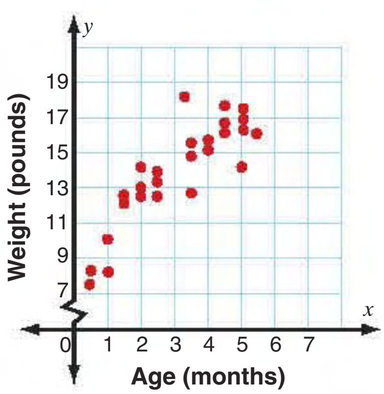
**Test-Taking Tips**

- Underline important words.
- Restate the question.
- Apply appropriate rules, definitions, or properties.
- Analyze and eliminate answer choices.

**Try These**

**Choose the correct answer. Explain how you used strategies.**

**1.** The scatter plot shows the weights of male infants at various ages.




Which is the best estimate for the weight of a 6-month-old male infant?

**A.** 6 pounds　**C.** 18 pounds

**B.** 14 pounds　**D.** 24 pounds

**2.** Amber made a scale drawing of her bedroom. One half inch on her drawing represents 4 feet. The length of her bedroom on the drawing is 3 inches. What is the actual length of her bedroom?

**F.** $1\frac{1}{2}$ feet

**G.** 6 feet

**H.** 12 feet

**J.** 24 feet

Go to PRACTICE BOOK page 367 for exercise sets.

CHAPTER 12

# Probability

## In This Chapter You Will:

- Use a tree diagram to find the number of possible outcomes in a sample space
- Use the Fundamental Counting Principle to find the size of a sample space
- Use factorials to find the size of a sample space
- Find the theoretical probability of an event
- Find the experimental probability of an event
- Compute odds in favor and odds against an event
- Compute the probability of independent and dependent events
- Determine the number of permutations
- Determine the number of combinations
- Review problem-solving strategies
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- To rename a fraction as a decimal, divide the numerator by the denominator.
- To rename a decimal as a percent, multiply the decimal by 100.
- $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$, where $b \neq 0$ and $d \neq 0$
- A ratio is a comparison of two numbers, $a$ and $b$, by division.

**For Practice Exercises:**

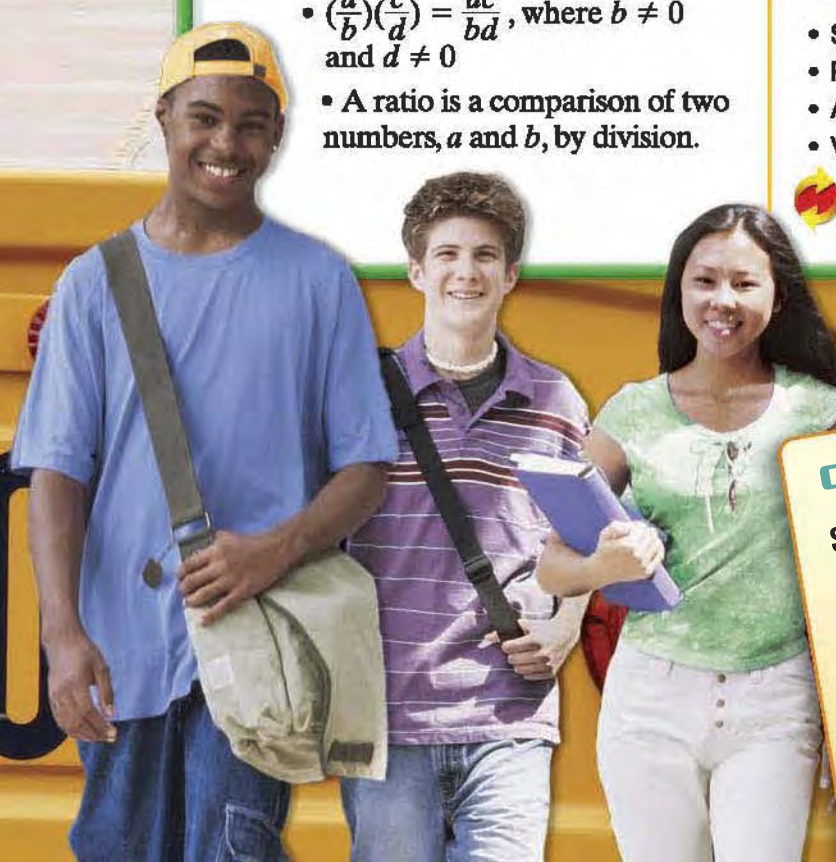
Go to PRACTICE BOOK, pp. 373–398

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES



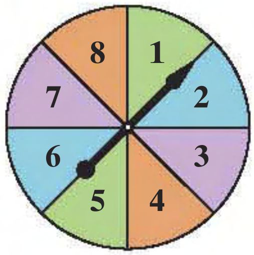
## Critical Thinking

Simone randomly surveyed 80 of her 400 classmates to see about how far each of them must travel to school. Fifteen of the classmates she surveyed travel between 4 and 5 miles from home to school. How many of her remaining 320 classmates would you predict also travel between 4 and 5 miles to school?

# 12-1 Sample Space

**Objective** To determine the sample space of an experiment • To determine the likelihood of an event • To use a tree diagram to find the sample space for two events • To use a tree diagram to determine the likelihood of an event

This spinner is divided into 8 equal sections. Francisco spins it once. Is it more likely that the spinner will land on an even number or an odd number?




► An experiment is the performance of an action, such as spinning a spinner or tossing a coin. An outcome is the result of an experiment. For example, any number that comes up from spinning the spinner is an outcome. A sample space is the collection of *all* possible outcomes in an experiment.

The sample space for Francisco's experiment is {1, 2, 3, 4, 5, 6, 7, 8}.

An event is any grouping of one or more outcomes from the sample space. The likelihood of an event is the chance of an event happening. Events can be either impossible, certain, equally likely, or not equally likely.

- Events that *cannot* happen are impossible.
- Events that *must* happen are certain.
- Events that are just as likely to happen as other events are equally likely.
- Events that are *not* as likely to happen as other events are not equally likely. If two events are not equally likely, one event will be *more likely* to occur, and the other event will be *less likely* to occur.

Here are some examples of events based on Francisco's spinner:

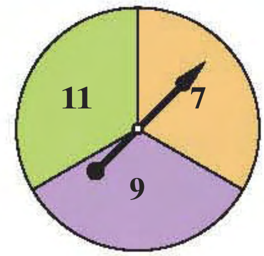
| Event | Likelihood |
|---|---|
| spinning a 9 | impossible |
| spinning a natural number less than 9 | certain |
| spinning 1, 2, 3, 4 or spinning 5, 6, 7, 8 | equally likely |
| spinning 1, 2 or spinning 4, 5, 6 | not equally likely; spinning 4, 5, 6 will be more likely than spinning 1, 2. |

For the sample space in Francisco's experiment, the even numbers are 2, 4, 6, and 8. The odd numbers are 1, 3, 5, and 7. There are the same number of possible outcomes for spinning an even number as for spinning an odd number.

So for Francisco's experiment, the events of Francisco spinning an even number and spinning an odd number are equally likely.

► When all outcomes of an experiment are equally likely to occur and an event has two or more stages, it is helpful to draw a tree diagram. A tree diagram is a visual representation that shows all possible outcomes of one or more events. Each possible event needs a branch so the number of branches equals the number of possible outcomes.

- Francisco spins this spinner, which is divided into 3 equal sections. How many elements are in the sample space? List the sample space.

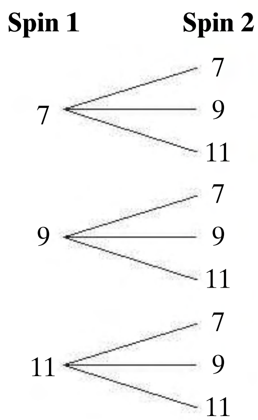



  Sample space = {7, 9, 11}
  There are 3 elements in the sample space.

- Suppose Francisco spins the spinner twice.

  Which possible outcome is more likely: that the sum of the spins equals 18 or that the sum will not be 18?

  Use a tree diagram to identify the sample space. Notice that the tree diagram gives all the possible outcomes for this experiment.




| Spin 1 | Spin 2 | Outcome | Sum of Numbers |
|---|---|---|---|
| 7 | 7 | {7, 7} | 14 |
| 7 | 9 | {7, 9} | 16 |
| 7 | 11 | {7, 11} | 18 |
| 9 | 7 | {9, 7} | 16 |
| 9 | 9 | {9, 9} | 18 |
| 9 | 11 | {9, 11} | 20 |
| 11 | 7 | {11, 7} | 18 |
| 11 | 9 | {11, 9} | 20 |
| 11 | 11 | {11, 11} | 22 |

Sample space = {(7, 7); (7, 9); (7, 11); (9, 7); (9, 9); (9, 11); (11, 7); (11, 9); (11, 11)}
There are 9 outcomes in the sample space, and 3 of these outcomes are sums of 18. So since there are 6 outcomes that are *not* sums of 18, a sum that is *not* 18 is more likely to occur than a sum of 18.

► Another way to find a sample space is to make an organized list or table.

Frank flips two nickels at the same time.
What is the sample space for this experiment?

Each nickel will land heads up (H) or tails up (T).
The table at the right shows the possible combinations.

Sample space = {HH, HT, TH, TT}

So there are 4 possible outcomes in the sample space.

Nickel 2

| Nickel 1 | | H | T |
|---|---|---|---|
| | H | HH | HT |
| | T | TH | TT |

## Try These

**Draw a tree diagram or make a table to find the number of possible outcomes. List the sample space.** (*Note:* A number cube has a different number from 1 to 6 on each of its six faces.)

**1.** Ian tosses a number cube twice.

**2.** Jan flips a nickel and spins the spinner at the top of the page.

**3. Discuss and Write** Is Ian more likely to roll an even sum or an odd sum? Support your answer with a diagram.

Go to PRACTICE BOOK Lesson 12-1 for exercise sets.

# 12-2 Fundamental Counting Principle and Factorials

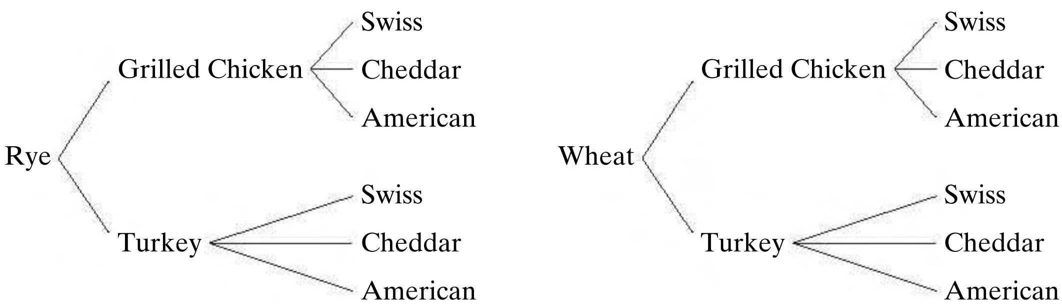
**Objective** To use the Fundamental Counting Principle to find the size of a sample space • To use factorials to find the size of a sample space

PROBLEM-SOLVING STRATEGY 7

A diner offers a variety of made-to-order sandwiches. Customers can choose from rye (R) or wheat (W) bread; grilled chicken (G) or turkey (T); and Swiss (S), cheddar (C), or American (A) cheese. On her first day of work at the diner, Althea wants to determine how many different kinds of sandwiches she might be asked to make.

► To find the total number of different kinds of sandwiches, Althea can draw a tree diagram or use multiplication.

**Method 1** Make a Tree Diagram




Sample space = {(R, G, S); (R, G, C); (R, G, A); (R, T, S); (R, T, C); (R, T, A); (W, G, S); (W, G, C); (W, G, A); (W, T, S); (W, T, C); (W, T, A)}

There are 12 elements in the sample space.

**Method 2** Use Multiplication

$$\text{Total kinds of sandwiches} = \underbrace{2}_{\text{bread choices}} \bullet \underbrace{2}_{\text{meat choices}} \bullet \underbrace{3}_{\text{cheese choices}} = 12$$

So Althea might be asked to make any one of 12 different kinds of sandwiches.

► Using multiplication to find the sample space is known as the *Fundamental Counting Principle*.

**Key Concept**

**Fundamental Counting Principle**

If an event can occur in $m$ ways and a second event can occur in $n$ ways, then the total number of possible ways that the events can occur together equals $m \bullet n$. This principle can be extended to any number of events.

A travel company offers 1-day vacation packages to Seattle. Customers can choose from 6 museums, 3 restaurants, 2 parks, and 4 shopping districts. How many different vacation packages are possible if customers choose one attraction from each category?

Total possible vacation packages:

$$\underset{\text{museum choices}}{6} \bullet \underset{\text{restaurant choices}}{3} \bullet \underset{\text{park choices}}{2} \bullet \underset{\text{shopping choices}}{4} = 144$$

So the travel company offers 144 different 1-day vacation packages.

► You can also use the Fundamental Counting Principle to find the number of different ways a group of items can be ordered.

How many different ways can you arrange four DVDs on a shelf?

Let A, B, C, and D represent the four DVDs. Note that the choice you make for each position affects the choices that are available for the next position. For example, if you choose A as the first DVD, only B, C, and D are available for the second DVD, and so on.

With this in mind, you can multiply to find the total number of possible arrangements.

| 1st Position | | 2nd Position | | 3rd Position | | 4th Position | | |
|---|---|---|---|---|---|---|---|---|
| 4 choices | • | 3 choices | • | 2 choices | • | 1 choice | = | 24 arrangements |

You can arrange four DVDs on a shelf 24 different ways.

► The product 4 • 3 • 2 • 1 can be represented in shorthand as 4!, which is read as "4 factorial." A factorial of a given integer is the product of all positive integers less than or equal to that integer.

So 6! is equal to 6 • 5 • 4 • 3 • 2 • 1, or 720.

**Key Concept**

**Factorial**

The expression $n$! is the product of all positive integers that are less than or equal to $n$.

Note that 1! = 1 and 0! = 1.

## Try These

**Find the value of each expression. Be sure to use the order of operations.**

**1.** 9! **2.** 5! • 0! **3.** 4! • 3! **4.** 6! − 5! **5.** $\frac{12!}{10!}$

**6.** In how many ways can a group of 10 friends stand in a row for a picture?

**7.** A bank gives out 10-digit account numbers using only the numbers 0–9. If numbers can repeat in a given account number, how many different account numbers could the bank issue?

**8. Discuss and Write** Compare the advantages and disadvantages of using the Fundamental Counting Principle versus using a tree diagram. Give examples to support your answer.

Go to PRACTICE BOOK Lesson 12-2 for exercise sets.

Algebra

# 12-3 Theoretical Probability

**Objective** **To write probabilities as fractions, decimals, and percents • To represent probabilities as fractions, decimals, and percents on a number line from 0 to 1 • To define the theoretical probability of an event and use a formula to find theoretical probability • To find the theoretical probability of complementary events**

Dee can choose from a grab bag of 20 small packages. There are 5 packages of raisins, 5 packages of nuts, 5 packages of trail mix, and 5 packages of dried fruit. Dee randomly selects one package. What is the probability that she will select a package of raisins?

► The probability of an event, $P(E)$, is a measure of how likely it is that an event will occur. Probability is represented by a ratio that is equal to 0 or 1, or that lies between 0 and 1.

If an event is *impossible*, its probability is 0.
If an event is *certain*, its probability is 1.
Otherwise, the probability of an event is between 0 and 1.
The *more likely* an event, the closer its probability will be to 1.
The *less likely* an event, the closer its probability will be to 0.

Probabilities can be expressed as fractions, decimals, and percents. You can use a number line to visualize probabilities.

| Impossible | Less Likely | Equally Likely | More Likely | Certain |
|---|---|---|---|---|
| 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
| 0.0 | 0.25 | 0.5 | 0.75 | 1.0 |
| 0% | 25% | 50% | 75% | 100% |

**Remember:** To rename a fraction as a decimal, divide the numerator by the denominator.
To rename a decimal as a percent, multiply the decimal by 100, and write the percent symbol.

*Theoretical probability* is calculated by analyzing possible outcomes rather than by conducting an experiment. It is based on known information, such as the number of equal sections in a spinner. It tells what *should* happen in an experiment. The outcomes that you are looking for in an event are called favorable outcomes. The theoretical probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.

**Key Concept**

**Theoretical Probability Formula**

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Note that the formula is based on all possible outcomes being equally likely.

To find the probability of Dee selecting a package of raisins, use the formula for theoretical probability.

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \rightarrow P(\text{raisins}) = \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} = \frac{1}{4}$$

So the probability of Dee selecting a package of raisins is $\frac{1}{4}$ or 0.25 or 25%.

► Two events, E and *not* E, are complementary events if both events cannot occur at the same time. The sum of their likelihood of occurring is 1. The complement of E can be written as E′.

**Key Concept**

**Complementary Events**

$P(\text{E}) + P(\text{not E}) = 1$

or

$P(\text{E}) + P(\text{E}') = 1$

There are 6 blue marbles, 4 red marbles, 7 green marbles, and 3 yellow marbles in a bag. Find the probability of choosing a blue marble. Then find the probability of choosing a marble that is *not* blue.

$$P(\text{blue}) = \frac{\text{number of blue marbles}}{\text{total number of marbles}} = \frac{6}{6+4+7+3} = \frac{6}{20} = \frac{3}{10} = 0.3 = 30\%$$

$$P(\text{not blue}) = 1 - P(\text{blue}) = 1 - \frac{3}{10} = \frac{10}{10} - \frac{3}{10} = \frac{7}{10} = 0.7 = 70\%$$

► Two events, A and B, that have no outcomes in common are called mutually exclusive events or disjoint events.

**Key Concept**

**Mutually Exclusive Events**

$P(\text{A or B}) = P(\text{A}) + P(\text{B})$

Refer to the marbles in the bag above. Find the probability of choosing a blue or a red marble.

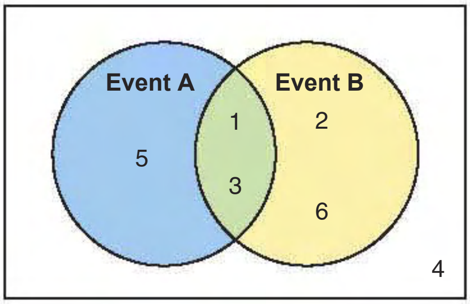
$$P(\text{blue or red}) = P(\text{blue}) + P(\text{red}) = \frac{6}{20} + \frac{4}{20} = \frac{10}{20} = \frac{1}{2} = 0.5 = 50\%$$

► Two events, A and B, that have one or more outcomes in common are called overlapping events. These events are not mutually exclusive.

A 1–6 number cube is rolled.
Are the following events mutually exclusive or overlapping?

Event A: Roll an odd number
Event B: Roll a factor of 6

Since the two events have outcomes in common, the events are overlapping. The Venn diagram at the right shows the overlapping outcomes.




## Try These

**A pair of numbered 1–6 number cubes is rolled. Find the theoretical probability of each event. Write each as a fraction, decimal, and percent.**

**1.** $P$(multiple of 3) **2.** $P$(odd) **3.** $P$(not 4) **4.** $P$(factor of 15 or 6)

**5. Discuss and Write** You and your 8 friends decide to draw straws for a prize. There are 8 long straws and 1 short straw. Describe how the theoretical probability of drawing the short straw changes as each friend picks a straw.

Go to PRACTICE BOOK Lesson 12-3 for exercise sets.

Algebra

# 12-4 Experimental Probability

**Objective** **To find the experimental probability of an event • To find, record, and predict outcomes of probability experiments • To simulate events to predict probability**

Jay, Bailey, and Celeste are playing a board game with a 1–6 number cube. As each player rolls the cube, Jay keeps track of which number is rolled. Based on the results shown in the table below, about how many times in 210 trials will any player roll a 6?

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| \|\| | 𝍸 | \|\| | 𝍸 \| | 𝍸 \|\| | \|\|\| |

To predict the number of times a 6 will be rolled in 210 trials, find the *experimental probability*. Then write and solve a proportion.

► Experimental probability is the ratio of the number of times an event occurs to the total number of trials. It serves as an estimate that an event will happen based on how often the event occurs after collecting data from an experiment. Unlike theoretical probability, which analyzes the outcomes in a sample space, experimental probability analyzes the results of one or more experiments. Each time an experiment is performed, it is called a trial. As the number of trials increases, the closer each experimental probability will be to the theoretical probability.

**Key Concept**

**Experimental Probability Formula**

Exp $P(\text{E}) = \dfrac{\text{number of times an event occurs}}{\text{number of trials}}$

Theoretical probability is the number of times an event *should* occur in an experiment.
Experimental probability is the number of times an event *actually* occurs in an experiment.

- Find the number of times a 6 was rolled: 3 times
  Add the tally marks to find the number of trials: $2 + 5 + 2 + 6 + 7 + 3 = 25$
  Write the experimental probability of the event:

$$P(\text{rolling a 6}) = \frac{\text{number of times an event occurs}}{\text{number of trials}} = \frac{3}{25}$$

- Write and solve a proportion to predict the number of times Jay, Bailey, and Celeste will roll a 6 in 210 trials.
  Let $x$ = the number of times 6 will be rolled.

The number of times 6 was rolled. → $\dfrac{3}{25} = \dfrac{x}{210}$ ← The number of times 6 is expected to be rolled.
original number of trials → ← new number of trials

$3 \bullet 210 = 25 \bullet x$ ← Cross multiply.

$630 = 25x$ ← Simplify.

$\dfrac{630}{25} = \dfrac{25x}{25}$ ← Divide both sides by 25.

$25.2 = x$

$25 \approx x$ ← Round to the nearest whole number.

So Jay, Bailey, and Celeste should expect to roll the number 6 about 25 times in 210 trials.

► Sometimes you can simulate events to find experimental probabilities and predict outcomes.

A simulation is a mathematical experiment that is used to approximate the results of a real-life situation. Models, such as coins, color cubes, and spinners, can be used to simulate a situation.

## Example

**1** PROBLEM-SOLVING STRATEGY 6

A dog has 4 puppies. What is the probability that exactly 2 puppies in the litter of 4 are male?

To find the probability, you can simulate the problem by tossing 4 coins. This assumes that the probability of a puppy being a male or a female is equally likely.

Let H = a male puppy.
Let T = a female puppy.

The sample space below shows the results for 15 trials.

| | | |
|---|---|---|
| H H T T | H T H H | H T T T |
| T H T T | H T T H | T T H H |
| T T H T | H H H T | T H T H |
| T T T T | H H H H | T T T H |
| T H H T | H T H T | H H H H |

$$P(\text{exactly 2 male puppies}) = \frac{\text{number of times there are exactly 2 males}}{\text{number of trials}} = \frac{\cancel{6}^{\,2}}{\cancel{15}_{\,5}} = \frac{2}{5}$$

So, according to the simulation, the experimental probability that exactly 2 puppies in a litter of 4 are male is $\frac{2}{5}$.

## Try These

**Find each experimental probability. Write each probability as a fraction, a decimal, and a percent. Write each fraction in simplest form.**

**1.** 100 coin tosses, 30 tails
$P(\text{tails}) = \underline{?}$

**2.** 500 coin tosses, 230 tails
$P(\text{heads}) = \underline{?}$

**3.** 45 coin tosses, 30 tails
$P(\text{tails}) = \underline{?}$

**4.** Theresa is tallying the number of students who wear glasses at her school. Of the first 25 students that enter the school, 11 are wearing glasses. There are 800 students at Theresa's school. Predict how many students at the school wear glasses.

**5.** **Discuss and Write** Toss a coin 10 times, and record the number of heads and the number of tails. Repeat the experiment, increasing the number of trials to 25. Repeat the experiment again, increasing the number of trials to 50. What do you notice about the results of these experiments?

Go to PRACTICE BOOK Lesson 12-4 for exercise sets.

Algebra

12-5

# Odds and Fairness

**Objective** To compute odds in favor and odds against • To distinguish between the two forms of odds and the language of probability • To identify fair and unfair games

Odds are another way of expressing the likelihood of an event. Odds in favor of an event are represented by a ratio that compares the number of favorable outcomes to the number of *unfavorable outcomes*. Unfavorable outcomes are any outcomes that are *not* represented by the event. They are the complement of the event. Unfavorable outcomes are calculated by subtracting the number of favorable outcomes from the total number of possible outcomes.

**Key Concept**

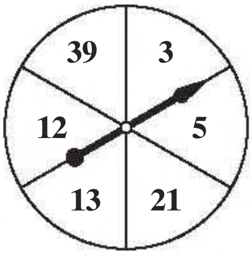
**Odds**

odds in favor of an event $= \dfrac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$

odds against an event $= \dfrac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}}$

**Think**
If the odds in favor of an event are represented by the ratio $a : b$, then the odds against that event are represented by the ratio $b : a$.

To find the odds in favor of and against the spinner landing on a multiple of 3, write a list of the possible outcomes. Count the number of favorable outcomes. Then subtract to find the number of unfavorable outcomes.




number of possible outcomes: 6 ← {3, 5, 21, 13, 12, 39}

number of favorable outcomes: 4 ← {3, 21, 12, 39}

number of unfavorable outcomes: 2 ← number of possible outcomes − number of favorable outcomes

- Calculate the odds in favor of the event by writing the ratio of favorable to unfavorable outcomes. Write each ratio in simplest form, if necessary.

  Odds in favor of a multiple of 3 $= \dfrac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$

  $= \dfrac{4}{2} = \dfrac{2}{1}$ or $2 : 1$ or 2 to 1

  The odds in favor of the spinner landing on a multiple of 3 are 2 to 1.

- Calculate the odds against the event happening by writing a ratio of unfavorable outcomes to favorable outcomes.

  Odds against a multiple of 3 $= \dfrac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}}$

  $= \dfrac{2}{4} = \dfrac{1}{2}$ or $1 : 2$ or 1 to 2

  The odds against the spinner landing on a multiple of 3 are 1 to 2.

Odds are commonly expressed as a ratio $a : b$, where $b \neq 0$; but they may also be expressed in fractional, decimal, or percent form.

Note that when odds in favor of an event happening are $2 : 1$, there are a total of 3 possible outcomes with 2 favorable outcomes and 1 unfavorable outcome. In this case, the probability of the event is $\frac{2}{3}$. So odds of $2 : 1$ are the same as a probability of $\frac{2}{3}$.

► You can use odds to determine if a game is *fair*. A game is said to be fair if the number of favorable outcomes is equal to the number of unfavorable outcomes. Players are equally likely to win the game.

The odds of the events in fair games are 1 : 1. Odds that take the form 1 : 1 in simplest form are called *even* odds, or the same odds. This means that the odds in favor of an event happening are the same as the odds against the event happening.

**Think**

Odds of 1 : 1 mean that of 2 possible outcomes, 1 is favorable and 1 is not favorable.

$P(\text{E}) = \frac{1}{2}$ means that 1 of 2 of the possible outcomes are favorable.

So odds of 1 : 1 is the same as a probability of $\frac{1}{2}$.

• Frank has two coins. On one coin, he will tape a piece of blue paper to both sides. On the other coin, he will tape a piece of blue paper to one side and a piece of red paper to the other side.

Frank proposes the following game to Lena.

Frank tosses both coins. If both sides are the same color, Frank wins a point. If the sides are different colors, Lena wins a point. Lena then tosses the coins with the same rules. Play proceeds until Frank and Lena have each tossed the coins 15 times. The player with the most points is the winner.

Is this an example of a fair game?

Make a table to see all possible outcomes of the game.
Let B = blue.
Let R = red.

| Coin 1 \ Coin 2 | B | B |
|---|---|---|
| B | BB | BB |
| R | RB | RB |

From the table, you can see that there are 4 outcomes in the sample space.

Two outcomes (BB, BB) favor Frank, and two outcomes (RB, RB) favor Lena.

The *odds in favor* of Lena winning are 2 : 2 or 1 : 1. Similarly, the *odds against* Lena winning are 1 : 1.

So the game is fair because it has the same odds.

## Try These

**Find the odds in favor of and the odds against each outcome for rolling a fair 1–6 number cube. Write each pair of odds in simplest form.**

**1.** an odd number **2.** a composite number **3.** a number divisible by 2

**4.** Jim and Dan play a game. If a coin lands on heads, Jim wins. If a coin lands on tails, Dan loses and Jim wins. Are both players equally likely to win the game? Explain.

**5. Discuss and Write** Write a formula for converting odds to theoretical probability. Explain how you found your formula. (*Hint:* Think about the number of favorable outcomes, the number of unfavorable outcomes, and the number of possible outcomes in the sample space, and how they relate.)

Go to PRACTICE BOOK Lesson 12-5 for exercise sets.

Algebra

# 12-6 Compound Events

Update your skills. See page 414 XVII.

**Objective** To find the probability of independent events
• To find the probability of dependent events

In a language arts class, 10 students must do individual oral presentations. The class meets each day Monday through Friday. The teacher determines that two presentations would be presented each day. The teacher places cards in two jars. The first jar determines the order of the presentations (first or second) on the specific day. The second jar determines the day (Monday through Friday) of the presentation. For each jar, the possibilities are equally likely. Joy is the first student to choose from the jars. What is the probability that Joy will present first on Friday?

To determine the probability of presenting first on Friday, find the probability of the compound event.

► A compound event is an event that consists of two or more events considered as a single event.

A compound event can be an *independent event* or a *dependent event.* For an independent event, the occurrence of one event does *not* affect the probability or likelihood that the other event will occur. For a dependent event, the occurrence of one event affects the probability or likelihood that the other will occur.

Since Joy's pick from jar 1 does not affect the pick from jar 2 and her pick from jar 2 does not affect the pick from jar 1, the events are independent.

To find the probability of independent events, multiply the probability of each event.

**Key Concept**

**Probability of Independent Events**
If A and B are independent events, then $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$.

First identify the events.
Let event A = Joy presenting first.
Let event B = Joy presenting on Friday.

$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$
$P(\text{first and Friday}) = P(\text{first}) \times P(\text{Friday})$

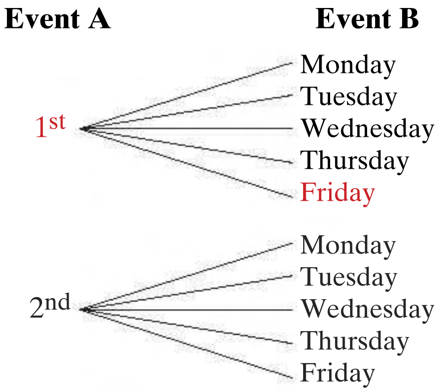
$$= \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

The probability that Joy will present first on Friday is $\frac{1}{10}$ or 0.1 or 10%.

**Check:** Draw a tree diagram to check your answer.

Sample space = {(1st, M); (1st, T); (1st, W); (1st, Th); (1st, F); (2nd, M); (2nd, T); (2nd, W); (2nd, Th); (2nd, F)}

Using the sample space, you can confirm that the probability of Joy presenting first on Friday is $\frac{1}{10}$, 0.1 or 10%.



▶ To find the probability of dependent events, multiply the probability of the first event by the probability of the second event, after the first event has occurred.

**Key Concept**

**Probability of Dependent Events**
If A and B are dependent events, then
$P(\text{A, then B}) = P(\text{A}) \times P(\text{B after A})$.

- A box holds the following letter cards: B L U E.
  Mia picks a card at random and does not replace it in the box. Without looking, she picks another card at random. What is the probability that Mia will pick a consonant and then a vowel?

**Think**
The first event affects the probability of the second event, so the events are dependent.

Identify the events: A = picking a consonant
B = picking a vowel

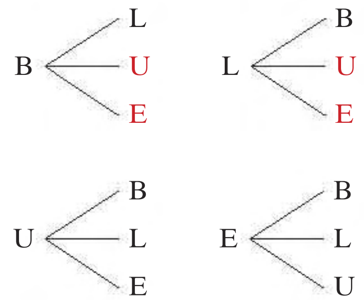
$P(\text{A, then B}) = P(\text{A}) \times P(\text{B after A})$

$P(\text{A}) = P(\text{a consonant}) = \frac{2}{4}$ ← number of consonant cards / ← total number of cards

$P(\text{B after A}) = P(\text{vowel, after a consonant}) = \frac{2}{3}$ ← number of vowel cards / ← number of remaining cards

$P(\text{consonant, then a vowel}) = \frac{2}{4} \times \frac{2}{3} = \frac{4}{12} = \frac{1}{3} = 33\frac{1}{3}\%$

**Check:** Draw a tree diagram to check your answer.




Sample space = {(B, L); (B, U); (B, E); (L, B); (L, U); (L, E); (U, B); (U, L); (U, E); (E, B); (E, L); (E, U)}

Using the sample space, you can confirm that the probability of picking a vowel after a consonant is $\frac{4}{12} = \frac{1}{3}$ or $33\frac{1}{3}\%$.

## Try These

**Explain why the events are either dependent or independent.**

1. Rolling an even number on both the 1st and 2nd roll of a 1–6 number cube
2. Choosing a red rose, not replacing it, and then choosing a white rose from a vase of 3 white, 2 red, and 6 yellow roses

**Find each probability.**

A box contains 2 red blocks (R), 1 yellow block (Y), and 1 green block (G). A block is chosen at random and not replaced. Then another block is chosen.

3. $P(\text{R, then Y})$
4. $P(\text{R, then R})$
5. $P(\text{G, then G})$

A box contains 2 striped blocks (S) and 1 checkered block (C). A block is chosen at random and replaced. Then another block is chosen.

6. $P(\text{S and S})$
7. $P(\text{S and C})$
8. $P(\text{one of each})$

9. **Discuss and Write** Describe two events that are independent and two that are dependent.

Go to PRACTICE BOOK Lesson 12-6 for exercise sets.

Algebra

12-7

# Permutations

**Objective** To determine the number of permutations of objects

Four students are standing in line at lunch. In how many different ways can the four students stand in line?

To find how many different ways, find the number of permutations for four students.

► A permutation is an arrangement of items or objects in which order is important.

To determine the number of possible permutations, you can use an organized list, a tree diagram, or the Fundamental Counting Principle. Sometimes, but not always, you can use factorials.

Here are two ways to solve the problem above.

**Method 1** Make an Organized List

Let A, B, C, and D represent the four students.

| *Student A Is First* | *Student B Is First* | *Student C Is First* | *Student D Is First* |
|---|---|---|---|
| ABCD | BCDA | CABD | DABC |
| ABDC | BCAD | CADB | DACB |
| ACBD | BDAC | CBAD | DBCA |
| ACDB | BDCA | CBDA | DBAC |
| ADBC | BACD | CDAB | DCAB |
| ADCB | BADC | CDBA | DCBA |

**Remember:** In a permutation, the order matters!

There are 24 different orders.

**Method 2** Use the Fundamental Counting Principle

There are four different students standing in line at lunch. Any of the four students could be first in line. After the first person in line is determined, there are three students left who could be second in line. After the second person in line is established, there are two students left who could be third in line, and finally one student left to be last in line.

| 1st Position | | 2nd Position | | 3rd Position | | 4th Position |
|---|---|---|---|---|---|---|
| ↓ | | ↓ | | ↓ | | ↓ |
| 4 choices | • | 3 choices | • | 2 choices | • | 1 choice |

Number of ways to stand in line: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

So there are 24 different ways for the four students to line up at lunch.

**Remember:** A *factorial* of a number, $n$, is the product of all positive integers less than or equal to $n$. It is symbolized by $n!$, which is read as "*$n$ factorial.*"

## Example

**1** How many different 3-digit security codes can be made if no digit repeats?

Digits: 0–9 ← 10 digits in all    $10 \bullet 9 \bullet 8 = 720$

**Think**
There are 10 choices for the first digit.
There are 9 choices for the second digit.
There are 8 choices for the third digit.

720 different 3-digit security codes can be made if no digit repeats.

► Sometimes a permutation concerns only *part* of a list to be ordered.

Jill received five new books for her birthday. She decided to read two books immediately and to save the other three books to read during the summer. In how many different ways can Jill read two of the five books?

**Think**
You cannot use factorials to solve this problem.

Here are two possible methods for solving the problem.

**Method 1** Make a Tree Diagram

Let 1, 2, 3, 4, and 5 represent the five different books.

| 1st choice | 2nd choice | 1st choice | 2nd choice | 1st choice | 2nd choice | 1st choice | 2nd choice | 1st choice | 2nd choice |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | | 1 | | 1 | | 1 | | 1 |
| 1 | 3 | 2 | 3 | 3 | 2 | 4 | 2 | 5 | 2 |
| | 4 | | 4 | | 4 | | 3 | | 3 |
| | 5 | | 5 | | 5 | | 5 | | 4 |

There are 20 different possibilities.

**Method 2** Use the Fundamental Counting Principle

There are 5 choices when deciding which book to read first.
There are 4 choices that remain for the second book.

Number of book choices $= 5 \bullet 4 = 20$

So there are 20 different ways for Jill to read two of the five books she received.

## Try These

1. There are nine players on a baseball team. How many different batting lineups are possible?

2. How many different ways can you arrange 5 DVDs on a shelf?

3. How many four-letter arrangements can you make from the letters in the word *NUMBER*?

4. Rachel is in charge of selling yearbooks. She has nine student volunteers to help her, and she needs to fill three positions: collecting money, unpacking yearbooks from the boxes, and handing out yearbooks. In how many different ways can the jobs be assigned to any three of the nine volunteers?

5. **Discuss and Write** Explain why 9! is *not* the solution to problem 4 above. Describe a problem that would have 9! as the solution.

Go to PRACTICE BOOK Lesson 12-7 for exercise sets.

Algebra

# 12-8 Combinations

**Objective** To determine the number of combinations of objects • To find permutations and combinations using a graphing calculator

PROBLEM-SOLVING STRATEGY 7

You and your friend Jesse are going to paint his basement. His parents have six cans of leftover paint in different colors that you can use. Jesse's parents suggest that you combine two colors to make a more unusual color for the walls. How many different combinations of two colors can be made from the six ready-made colors?

► To solve this problem, you need to find the number of different 2-color combinations.

A combination is a collection of objects or items in which order *does not* matter. In this problem, a blue-green paint color is the same as a green-blue paint color. It does not matter in which order the paints are chosen. They are considered to be the same pair.

Here are two possible ways to solve this problem.

**Method 1** Make an Organized List

Suppose the paint colors are Red (R), Yellow (Y), Blue (B), Green (G), Violet (V), and White (W).

All possible outcomes are:

| | | | | | |
|---|---|---|---|---|---|
| RY | ~~YR~~ | ~~BR~~ | ~~GR~~ | ~~VR~~ | ~~WR~~ |
| RB | YB | ~~BY~~ | ~~GY~~ | ~~VY~~ | ~~WY~~ |
| RG | YG | BG | ~~GB~~ | ~~VB~~ | ~~WB~~ |
| RV | YV | BV | GV | ~~VG~~ | ~~WG~~ |
| RW | YW | BW | GW | VW | ~~WV~~ |

In the organized list above, cross-out any repeated paint colors. So there are 15 different 2-color combinations of paints.

**Method 2** Use Permutations

To find the number of combinations,

- First find the number of permutations of picking two paints from a group of six.
- Then divide this number by the number of permutations of 2 paints to eliminate any duplicate combinations.

$6 \bullet 5 = 30$ permutations

number of permutations of picking 2 paints from a group of 6

$$\frac{6 \bullet 5}{2!} = \frac{30}{2 \bullet 1} = \frac{30}{2} = 15$$

number of permutations of 2 paints

So there are 15 different ways to combine two paint colors from six possible choices.

**Key Concept**

**Combinations**

When you have a list of $n$ items and you want to know the number of combinations with only $r$ items, where $r \leq n$:

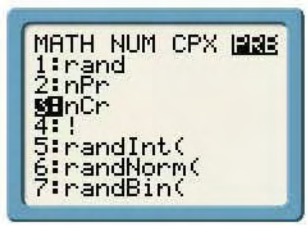
- Find the number of permutations of picking $r$ objects from a group of $n$ objects.
- Then divide by the number of permutations of $r$ objects.

## Technology

When the numbers involved in finding the number of combinations (and permutations) become very great, you can use a calculator.

Your Uncle Joe is taking family photographs at a picnic. He randomly selects 6 people at a time from the 51 gathered and takes their picture. How many different groups of 6 people could he select to photograph?

Here are the steps to find the number of different combinations by using a graphing calculator.





1. First enter the total number of objects, 51.
2. Then press MATH. Use the right arrow key to highlight **PRB**.
3. Press 3. This will select **nCr**.

> To find a permutation, press 2 to select **nPr**.
> To find a factorial, press 4 to select **!**, the factorial symbol.

4. Then press the number of objects being considered. In this case, enter 6 for the six people chosen at random.
5. Press ENTER.

So there are 18,009,460 different combinations possible.

## Try These

**1.** In how many ways can you choose 4 types of fish from an aquarium of 10 different fish?

**2.** Five people in a room decide to shake each other's hands once. How many handshakes will there be altogether?

**3.** The school orchestra has 10 flute players and 6 violinists. In how many different ways can 2 flute players be selected to play at a concert?

**4.** **Discuss and Write** Give an example of a combination and a permutation.

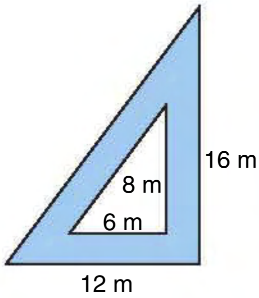
Go to PRACTICE BOOK Lesson 12-8 for exercise sets.

# 12-9 Problem Solving: Review of Strategies

Read · Plan · Solve · Check

**Objective** To solve problems using a variety of strategies

**Problem:** The corresponding sides of the right triangles below are parallel. All three pairs of sides are the same distance apart. What is the area of the shaded region?




**Problem-Solving Strategies**

1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Read to understand what is being asked.**

List the facts and restate the question.

**Facts:** A right triangle with legs of length 6 meters and 8 meters is inside a right triangle with legs of length 12 meters and 16 meters. Corresponding sides of the two triangles are parallel and the same distance apart.

**Question:** What is the area of the region between the triangles?

**Select a strategy.**

You can use the strategy *Reason Logically* and regard the area of the region as the difference in areas of the two triangles. You can also use the strategy *Adopt a Different Point of View* and consider moving the inside triangle to a new location.

**Apply the strategy.**

### ► Method 1: Reason Logically

A right triangle with legs of length $a$ and $b$ has area $A$ given by $A = \frac{1}{2}ab$.

Therefore, the area of the larger right triangle is

$$A_L = \tfrac{1}{2}(12\text{ m})(16\text{ m}) = 96\text{ m}^2.$$

The area of the smaller right triangle is

$$A_S = \tfrac{1}{2}(6\text{ m})(8\text{ m}) = 24\text{ m}^2.$$

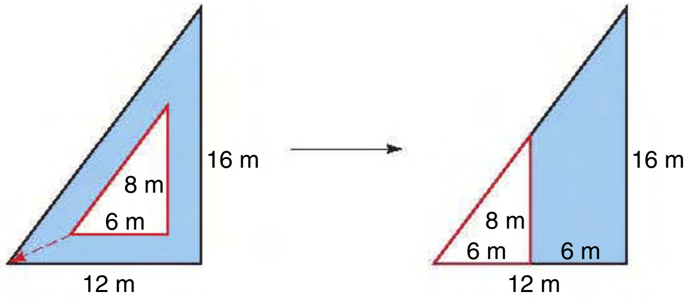
So the area of the shaded region is

$$A = 96\text{ m}^2 - 24\text{ m}^2 = 72\text{ m}^2.$$

### ► Method 2: Adopt a Different Point of View

You can move the smaller triangle around within the larger triangle without changing the area of the region between the triangles.

As shown in the figure below, if you move the smaller triangle down so that its lower left vertex corresponds with the lower left vertex of the outside triangle, the shaded region becomes a trapezoid with $b_1 = 16$ m, $b_2 = 8$ m, and $h = 6$ m.




To find the area of the trapezoid, use the area formula for a trapezoid.

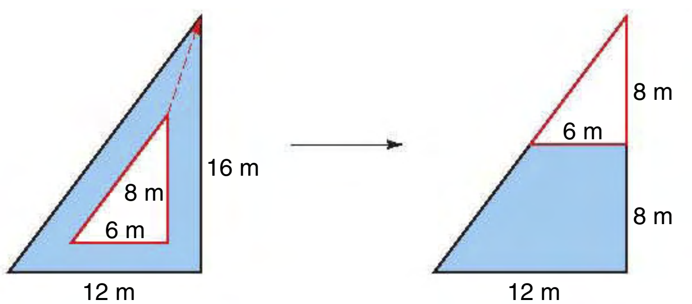
$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{\cancel{2}_{1}}(\overset{3}{\cancel{6}}\text{ cm})(16\text{ cm} + 8\text{ cm}) = (3\text{ cm})(24\text{ cm})$

$= 72\text{ cm}^2$

**Check to make sure your answer makes sense.**

You can move the inside triangle as shown in the figure below to create a different trapezoid.




The area of the trapezoid is

$A = \frac{1}{2}(8\text{ m})(6\text{ m} + 12\text{ m}) = 72\text{ m}^2.$

This is the same answer as the one above.

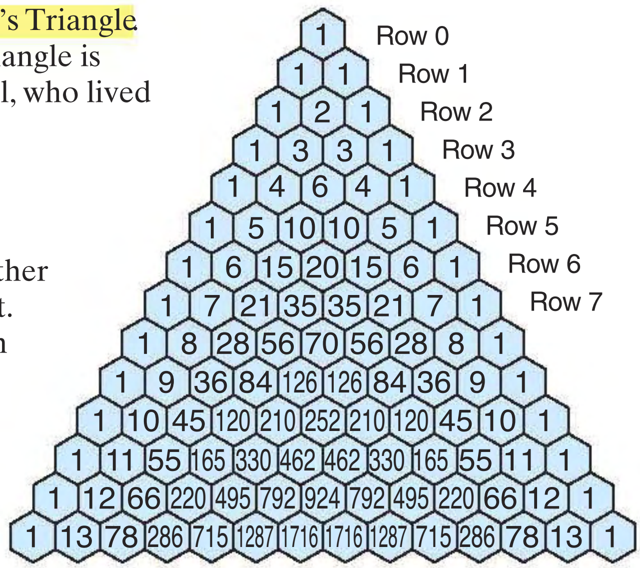
So the answer checks.

Go to PRACTICE BOOK Lesson 12-9 for exercise sets.

# Enrichment: Pascal's Triangle

**Objective** **To use Pascal's Triangle to find numbers of combinations**

The pattern of numbers at the right is part of Pascal's Triangle. (The full triangle continues with more rows.) The triangle is named after the French mathematician Blaise Pascal, who lived from 1623–1662.




► Pascal's Triangle has many interesting patterns. Here are just a couple. See if you can find others.

- The numbers on the edges are 1s. Each of the other numbers is the sum of the two numbers above it. For example, the number 10 in Row 5 is the sum of 4 and 6 above it.
- The sums of the numbers in each row are successive powers of 2.

  Row 0: $1 = 2^0$

  Row 1: $1 + 1 = 2 = 2^1$

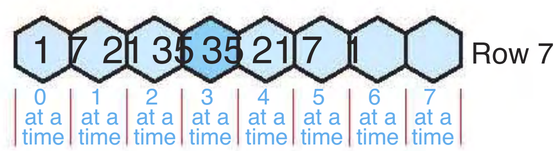
  Row 2: $1 + 2 + 1 = 4 = 2^2$

  Row 3: $1 + 3 + 3 + 1 = 8 = 2^3$ and so on.

► You can use Pascal's Triangle to find numbers of combinations. For example, suppose you have three extra tickets to a concert, and seven of your friends want to go with you. How many different combinations of three friends are possible?

To find the number of combinations of 7 things taken 3 at a time, use Pascal's Triangle.

- Choose the row that corresponds to the number of items you are choosing from. In this case, choose Row 7. (Notice Row 7 is actually the eighth row.)
- The first number, 1, is the number of combinations of 7 things taken 0 at a time. The second number, 7, is the number of combinations of 7 things taken 1 at a time, and so on.
- You want to find the number of combinations of 7 things taken 3 at a time. This is the fourth number in the row, which is 35.




There are 35 possible combinations of three friends.

## Try These

1. Sawyer has six pairs of shorts. He wants to take four pairs on vacation. Find the number of combinations for four pair of shorts.

2. An ice-cream parlor offers 10 flavors. Akila wants to order a sundae with three different flavors of ice cream. How many different three-scoop sundaes are possible?

Go to PRACTICE BOOK page 391 for exercise sets.

# Test Prep: Extended-Response Questions

## Strategy: Organize Information

One way to organize the information needed to answer an extended-response question is to *make an organized list*.

**Sample Test Item**

The soccer team is choosing new uniforms. They can choose red, navy, or yellow jerseys; the shorts can be navy, black, or white; and the socks can be black or white.

**Part A**
How many different combinations of uniforms are possible?

**Part B**
The team decides on red shirts. What are the possible choices of uniforms they could order? ***Show all your work.***

**Read the whole test item carefully.**

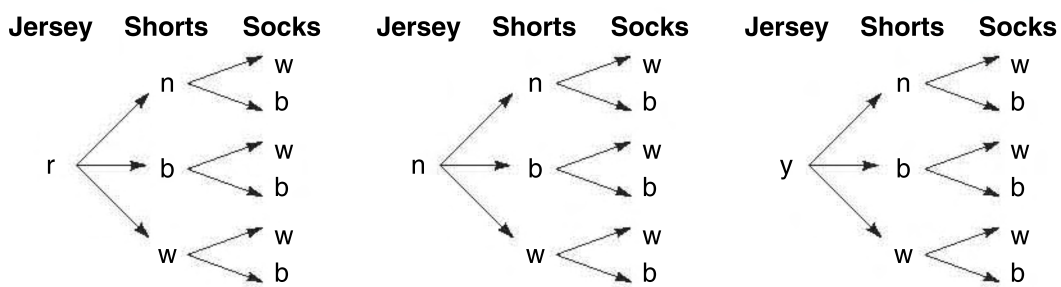
- Reread the test item.
- Create an organized list.
  1. Make a tree diagram to find all possible combinations.
  2. Use the list to identify which combinations have red shirts.

### Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply an appropriate strategy.
  To solve **Part A**, make a tree diagram to list the possible combinations of jerseys, shorts, and socks. Use *r* for red, *n* for navy, *y* for yellow, *b* for black, and *w* for white.




**Answer:** There are 18 different possible uniform combinations.

To solve **Part B**, list the combinations from the tree diagram that contain a red jersey.

**Answer:** red jersey, navy shorts, white socks; red jersey, navy shorts, black socks;
red jersey, black shorts, white socks; red jersey, black shorts, black socks;
red jersey, white shorts, white socks; red jersey, white shorts, black socks

**Item Analysis**

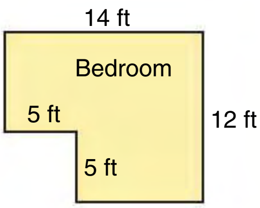
**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Do they make sense?
  Use the counting principle. Multiply to find the total number of choices.
  Number of different uniforms: $3 \cdot 3 \cdot 2 = 18$
  Number of different uniforms with red jersey: $1 \cdot 3 \cdot 2 = 6$

**Try These** Item 1 is partially worked out for you.

**Solve. Create an organized list.**

**1.** Grace plans to paint the walls of her bedroom. The bedroom has the dimensions shown below. The ceiling is 10 feet high.




**Part A**
What is the area Grace will paint?
***Show all your work.***

**Part B**
One gallon of paint covers about 400 square feet and costs $15.99. How many gallons of paint will Grace need to buy, and how much money will she spend?
***Show all your work.***

**Read the test item for a general idea of the problem.**

- Reread the test item.
- Create an organized list.
  1. Find the area of each wall.
  2. Determine the number of gallons of paint Grace will need.

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

To solve **Part A**, you will need to find the area of each wall.

**Think**
How many walls are there in Grace's room?

To solve **Part B**, use your answer from **Part A**.

**Think**
Can Grace buy part of a can of paint?

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the items.**

- Analyze your answers. Do they make sense?
  Use estimation. Find the perimeter of the room and multiply by the height.

**2.** Drew, Elise, and Randy are running for student body office positions. The student with the greatest number of votes will become president; the student with the second greatest number will become vice-president; and the student with the third greatest number will become class representative.

**Part A**
What are the different possible ways Drew, Elise, and Randy can win the different positions?
***Show all your work.***

**Part B**
If Tory also runs for office, how many different ways can these students be elected president and vice-president?
***Show all your work.***

Go to PRACTICE BOOK pages 393–394 for exercise sets.

# Patterns, Relations, and Functions

CHAPTER 13

## In This Chapter You Will:

- Recognize, describe, and extend patterns in sequences
- Form a conjecture and prove that it is false or demonstrate its truth
- Graph a linear function from a table of values and identify solutions to the related linear equation
- Identify different forms of slope and find the slope of a line from two given points
- Identify linear functions and nonlinear functions using equations and graphs
- Identify a graphical representation of a real-world situation
- Identify and graph transformations of figures on the coordinate plane
- Apply the strategy: *Consider Extreme Cases*
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- An ordered pair is a pair of numbers used to locate a point in a coordinate plane. The first number is the *x*-coordinate, and the second number is the *y*-coordinate.
- A rate is a ratio that compares two unlike quantities.
- A translation slides a figure in a direction. A reflection flips a figure over a line. A rotation turns a figure around a point.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 399–432

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES


## Critical Thinking

Jai takes the train to work every morning. He catches an 8:40 A.M. train that arrives at his destination 10 miles away at 9:00 A.M. In the evening Jai takes the bus home. He catches a bus at 6:35 P.M. It arrives at his stop at 6:50 P.M. How much faster in miles per hour does the bus travel than the train?

Remember: 1 hour = 60 minutes

Algebra

# 13-1 Arithmetic Sequences and Geometric Sequences

**Objective** To recognize, describe, and extend simple patterns in sequences • To recognize and continue a number sequence • To identify number sequences as arithmetic, geometric, or neither • To find missing terms in a sequence

PROBLEM-SOLVING STRATEGY 7

Franco used 4 toothpicks to form the first shape and 7 toothpicks to form the second shape in the pattern below. If he continues the pattern, how many toothpicks will he need to make the 8th shape in the pattern?

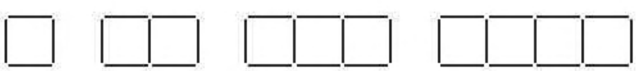
To find the number of toothpicks, list the *sequence* for the number of toothpicks used to make each shape. Then identify and extend the pattern.

► A sequence is a list of numbers in a specific order. A sequence may or may not follow a pattern. Each number in the sequence is a called a term.

An arithmetic sequence is a sequence of numbers that follow a pattern. Each term is found by adding a fixed number from one term to the next. This fixed number is called the constant difference. It is the difference between each pair of consecutive numbers in the sequence.

The constant difference of an arithmetic sequence can be a positive or negative number.

In order to determine if a sequence is an arithmetic sequence, examine the consecutive terms. If all consecutive terms have a constant difference, the sequence is arithmetic.



4, 7, 10, 13

$7 - 4 = 3$ $10 - 7 = 3$ $13 - 10 = 3$

**Think**
The constant difference is 3.

The difference, $d$, is always 3, so the sequence is arithmetic. The pattern rule is: Start at 4, and add 3 repeatedly.

To find the 8th shape in the pattern, add 3 to each succeeding term.

$13 + 3 = 16$ $16 + 3 = 19$ $19 + 3 = 22$ $22 + 3 = 25$

4, 7, 10, 13, 16, 19, 22, 25

+ 3 + 3 + 3 + 3 + 3 + 3 + 3

So Franco will need 25 toothpicks to make the 8th shape in the pattern.

► For the sequence above, you found the next term first by finding a pattern and then by using the pattern to make a conjecture about the next term. A conjecture is a prediction that suggests what you expect to happen.

► A geometric sequence is a sequence of numbers in which each term is found by multiplying the preceding term by a fixed number, called the constant ratio. In order to determine if a sequence is a geometric sequence, examine the consecutive terms. If all consecutive terms have a constant ratio, the sequence is geometric.

The constant ratio of a geometric sequence can be a positive or negative number.

Find the next three terms in the sequence: 3, 12, 48, 192, . . .

$$\frac{12}{3} = \frac{48}{12} = \frac{192}{48} = \frac{4}{1}$$ ← The constant ratio is $\frac{4}{1}$ or 4.

The ratio is always 4, so the sequence is geometric.
The pattern rule is: Start at 3, and multiply by 4 repeatedly.

So to find the next three terms in the pattern, multiply each succeeding term by 4.

$3 \xrightarrow{\times 4} 12 \xrightarrow{\times 4} 48 \xrightarrow{\times 4} 192 \xrightarrow{\times 4} 768 \xrightarrow{\times 4} 3072 \xrightarrow{\times 4} 12{,}288$

**Think**
192(4) = 768
768(4) = 3072
3072(4) = 12,288

► Some sequences are neither arithmetic nor geometric, and some are *both*. For the sequence 6, 5, 10, 9, 18, 17, 34, . . . , there is neither a constant difference nor a constant ratio. The sequence 1, 1, 1, 1, 1, . . . is both arithmetic and geometric, since the rules *add 0 to each term* and *multiply each term by 1* both apply.

► You can find missing terms in an arithmetic sequence or geometric sequence.

## Examples

**1** Find the missing term for the arithmetic sequence. 111, ?, 84.6, 71.4, 58.2, . . .

- Find the constant difference. 71.4 − 84.6 = −13.2
- Then add the constant difference to 111. 111 + (−13.2) = 97.8

**2** Find the missing term for the geometric sequence. 512 , 64, ?, 1, $\frac{1}{8}$, $\frac{1}{64}$, . . .

- Find the constant ratio. $\frac{64}{512} = \frac{1}{8}$
- Then multiply 64 by the constant ratio. $64 \bullet \frac{1}{8} = 8$

## Try These

**Describe each sequence as *arithmetic*, *geometric*, or *neither*.**
**If the sequence is arithmetic or geometric, determine its 7th term.**

**1.** −10, −3, 4, 11, . . . **2.** 625, −125, 25, −5, . . . **3.** 1, 1, 2, 3, 5, . . . **4.** 7, 1, 3, 2, 10, . . .

**Find the missing term for each sequence.**

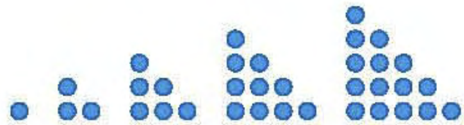
**5.** 1, −1.1, ?, −1.331, . . . **6.** $\frac{256}{27}$, $\frac{64}{9}$, $\frac{16}{3}$, ?, 3, . . . **7.** ?, $11\frac{3}{10}$, $8\frac{3}{5}$, $5\frac{9}{10}$, . . .

**8. Discuss and Write** Describe an arithmetic sequence and a geometric sequence.

Go to PRACTICE BOOK Lesson 13-1 for exercise sets.

# Algebraic Patterns and Sequences

**Objective** To recognize, describe, and extend patterns with more than one constant • To recognize, describe, and extend number patterns • To recognize patterns related to iterations



Danilo drew the visual pattern at the right. Danilo's pattern represents *triangular numbers*. Triangular numbers are a sequence of whole numbers in which each number corresponds to an arrangement of dots in the shape of a triangle. How many dots are in the 8th term in this pattern?

To find the 8th term, list the sequence represented by the dots, then identify and extend the pattern.

1, 3, 6, 10, 15, . . .

► Some sequences do not have a constant difference or a constant ratio.

$3 - 1 = 2 \qquad 6 - 3 = 3 \qquad 10 - 6 = 4 \qquad 15 - 10 = 5$

Notice that for this sequence, the difference increases by consecutive numbers.

The pattern rule is: Start at 1. Add consecutive numbers.

+ 2 + 3 + 4 + 5 + 6 + 7 + 8

1, 3, 6, 10, 15, 21, 28, 36, . . .

So the 8th term in the pattern above will have 36 dots in the shape of a triangle.

$1$
$1 + 2 = 3$
$(1 + 2) + 3 = 6$
$(1 + 2 + 3) + 4 = 10$
$(1 + 2 + 3 + 4) + 5 = 15$
$(1 + 2 + 3 + 4 + 5) + 6 = 21$
$(1 + 2 + 3 + 4 + 5 + 6) + 7 = 28$
$(1 + 2 + 3 + 4 + 5 + 6 + 7) + 8 = 36$

## Example

**1** Find the next three terms in this pattern: 2, 4, 3, 6, 5, 10, 9, 18, 17, . . .

× 2 × 2 × 2 × 2

2, 4, 3, 6, 5, 10, 9, 18, 17, . . .

+ + + + (−1)

These terms have a constant ratio of 2. The rule is *multiply by 2*.

These terms have a constant difference of −1. The rule is *add −1*.

The pattern rule is: Start at 2. Multiply by 2. Then add −1.

× 2 × 2 × 2 × 2 × 2 × 2

2, 4, 3, 6, 5, 10, 9, 18, 17, 34, 33, 66

+ + + + (−1) + (−1)

► Some sequences have terms that relate directly to the value of their positions in the sequence rather than to the value of preceding and succeeding terms. Tables help show this relationship.

## Examples

**1** What is the 11th term in the sequence: 1, 4, 9, 16, 25, . . . ?

Relate each term to its position in the sequence using a table.

| Position | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Term | 1 | 4 | 9 | 16 | 25 |

**Think**
The position number squared equals the term.
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$, and so on

The value of each term is the square of its position in the sequence.

So the 11th term is $(11)^2 = 121$.

**2** The sequence of factorials is 1, 2, 6, 24, 120, . . .
What is the next term in the sequence?

Relate each term to its position in the sequence using a table.

| Position | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Term | 1 | 2 | 6 | 24 | 120 |

**Think**
The factorial of the position number equals the term.
$1! = 1$
$2! = 2$
$3! = 6$, and so on

The value of each term is the factorial of its position in the sequence.

So the next term is $6! = 720$.

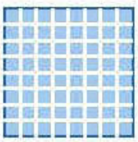
► The repeating step in a visual pattern is called an iteration.

What is the next term in this visual pattern?

Pattern rule: Start with 1 square, and then divide each square in each succeeding term into 4 squares.

The iteration is the division of each square into 4 squares.

The next term in the pattern is:



## Try These

**Identify and extend each pattern by 3 terms.**

**1.** $\frac{1}{2}, 1\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, 6\frac{1}{2}, \ldots$

**2.** 1, 8, 27, 64, . . .

**3.** 4, 3, 0, −1, 0, −1, . . .

**4.** Identify the sequence represented by the visual pattern of squares above.

**5.** **Discuss and Write** Explain why $3^1, 3^2, 3^3, 3^4, 3^5, \ldots$ is a geometric sequence.

Go to PRACTICE BOOK Lesson 13-2 for exercise sets.

Algebra 13-3

# Conjectures and Counterexamples

**Objective** To verify a conjectures or provide a counterexample • To form a conjecture and prove that it is false or demonstrate its truth

Problem-Solving Strategy 8

Rory and his group must prove or disprove the following conjecture:

The square of a number is *always* greater than the original number.

Is this conjecture true or false?

► A conjecture is a statement that appears to be true but that has not yet formally been proven to be true. It is usually made based on observations of patterns and what you predict will happen for future cases. A counterexample is a case that proves that the conjecture is false. Only one counterexample is needed to prove a conjecture false.

To see if the conjecture is true or false, test various cases.

Let $n$ = a number

**Case 1:** The original number, $n$, is greater than 1.

If $n = 2$, then $n^2 = 4$. $\quad 4 > 2$

If $n = 9.1$, then $n^2 = 82.81$. $\quad 82.81 > 9.1$

If $n = 20\frac{1}{2}$, then $n^2 = \left(20\frac{1}{2}\right)^2 = 420\frac{1}{4}$. $\quad 420\frac{1}{4} > 20\frac{1}{2}$

Case 1 does not lead to a counterexample.

**Case 2:** The original number, $n$, is less than 0.

If $n = -1$, then $n^2 = 1$. $\quad 1 > -1$

If $n = -\frac{1}{2}$, then $n^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$. $\quad \frac{1}{4} > -\frac{1}{2}$

Case 2 does not lead to a counterexample.

**Case 3:** The original number, $n$, is greater than or equal to 0 and less than or equal to 1.

If $n = \frac{1}{2}$, then $n^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. $\quad \frac{1}{4} < \frac{1}{2}$ ← Disproved. The square is less than the original number.

So when $n$ is greater than or equal to 0 and less than or equal to 1, the conjecture is false.

Case 3 is a counterexample to the conjecture.

**Remember:** Only one counterexample is necessary to prove that a conjecture is false.

► To test a conjecture, you need to think of all possibilities (cases). If a conjecture is false, you need to provide at least one counterexample. If a conjecture is true, you should explain why it is true for all cases.

## Examples

**1** **Conjecture:** The sum of two odd numbers is always an odd number.

To test the conjecture, add the first two odd numbers: $1 + 1 = 2$
Immediately, a counterexample was found, so your work is done.
The conjecture is false.
The sum of two odd numbers is *not* always an odd number.

**2** **Conjecture:** All prime numbers are odd.

To test the conjecture, list prime numbers: 2, 3, 5, 7, 11, 13, 15, . . .
The first prime number, 2, is *not* odd.
The conjecture is false.
Not all prime numbers are odd.

► To prove a conjecture is true, you cannot simply try a large number of cases and conclude that since no counterexamples could be found, the conjecture must be true. Sometimes you need to make generalizations to exhaust all possibilities.

**Conjecture:** A multiple of 6 is always also a multiple of 3.

Start with a few random tests and extreme cases.

| Choose a multiple of 6. | Express the number as a multiple of 6. | Can you express the number as a multiple of 3? | Can you express the number as a multiple of 6 and 3? |
|---|---|---|---|
| Try 36. | $6 \bullet 6$ | $3 \bullet 12$ | Yes |
| Try 60. | $6 \bullet 10$ | $3 \bullet 20$ | Yes |
| Try −336. | $6(-56)$ | $3(-112)$ | Yes |

Think more generally: Any multiple of 6 can be written as $6x$, where $x$ is any nonzero whole number.

Rewrite $6x$ as the equivalent expression $(2 \bullet 3)x$.

So $6x = (2 \bullet 3)x$
$= 3(2x)$ ← Apply the Commutative Property of Multiplication.

$3(2x)$ is a multiple of 3.
So any multiple of 6 is also a multiple of 3.
This proves the conjecture is true.

## Try These

**Test the conjecture to decide whether it is true or false.**
**If true, explain why. If false, provide a counterexample.**

**1.** The sum of two even numbers is always an even number.

**2.** **Discuss and Write** Explain a systematic approach you can use to test the conjecture that every real number is an integer.

Go to PRACTICE BOOK Lesson 13-3 for exercise sets.

Algebra

13-4

# Relations and Functions

**Objective** To identify the domain and range of a relation • To identify a given relation as a function or not a function • To write an equation to represent a function from a table of values

In 1963, there were only 417 male-female pairs of bald eagles in the United States. Since then, the bald eagle population has grown. In 1999, the bald eagle was removed from the list of endangered and threatened species. The table below at the right shows the increase in the number of bald eagle pairs since 1996.

► The data in the table demonstrate a *relation*. A relation is a set of ordered pairs that associates two quantities in a specific order. In this case, the relation is the ordered pairs, $(x, y)$, that associate a year with a number of pairs of bald eagles.

A relation consists of a *domain* and a *range*. The domain is the set of input values, or $x$-values, in the ordered pairs. The range is the set of output values, or $y$-values, in the ordered pairs.

The domain is {1996, 1998, 2000, 2002, 2004, 2006}.
The range (in thousands) is {5.1, 5.7, 6.5, 6.7, 7.0, 9.8}.

| Year | Number of Bald Eagle Pairs (thousands) |
|---|---|
| 1996 | 5.1 |
| 1998 | 5.7 |
| 2000 | 6.5 |
| 2002 | 6.7 |
| 2004 | 7.0 |
| 2006 | 9.8 |

► There are different types of relations.

| Type of Relation | Description | Example |
|---|---|---|
| *One to one* | For each input value, there is only one output value. This one-to-one type of relation is called a function. | The amount earned depends on the amount of time worked.<br>(work 1 hour, earn $20) |
| *One to many* | One input value corresponds to many output values.<br>This relation is *not* a function. | One date corresponds to many people's birthdays.<br>(date, person's birthday) |
| *Many to one* | Many input values correspond to the same output value.<br>This relation is a function. | Several students may have the same grade.<br>(student, test grade) |
| *Many to many* | Many input values correspond to many output values.<br>This relation is not a function. | Many teachers can have the same student, and many students can have the same teacher.<br>(teacher, student) |

**Key Concept**

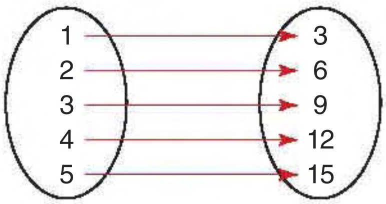
**Functions**
A function is a set of ordered pairs, $(x, y)$, in which there is only one $y$-value for each $x$-value.

► One way to determine whether a relation is a function is to pair or map each input ($x$) value with its corresponding output ($y$) value.

Determine if each relation is a function.

The table below shows the total number of cans of food Zoe's cats ate over 5 days.

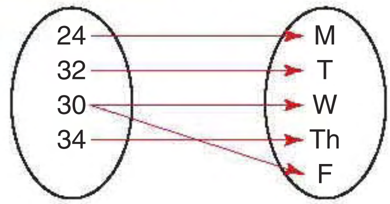
| Number of Days ($x$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cans of Cat Food ($y$) | 3 | 6 | 9 | 12 | 15 |




The relation is one-to-one. Each $x$-value is mapped to exactly one $y$-value.
The relation is a function.

The table below shows the average temperature over a five-day period.

| Avg. Temp. (°F) ($x$) | 24 | 32 | 30 | 34 | 30 |
|---|---|---|---|---|---|
| Day ($y$) | M | T | W | Th | F |




The relation is one-to-many.
The $x$-value 30 is mapped to two $y$-values.
The relation is not a function.

► A function rule relates the input ($x$) values to the corresponding output ($y$) values. To write a function rule, compare the output values to their corresponding input values.

Write a function rule for the relationship between the number of hours and the number of miles traveled.

| Number of Hours | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of Miles | 55 | 110 | 165 | 220 | 275 |

Look for a pattern between the input and output values.
Let $x$ = the number of hours. ← The number of hours is the input value.
Let $y$ = the number of miles traveled. ← The number of miles is the output value.

Each output value is 55 times the corresponding input value.
Write the function rule as $y = 55x$.

## Try These

**Identify the domain and range of each relation. Is the relation a function? Explain your answer.**

1. (25, 6), (20, 8), (15, 6), (25, 8), (15, 7)
2. (8, 5), (6, 3), (10, 5), (9, 5), (3, 3)

**Write a function rule for the relation.**

3.

| Number of Hours ($x$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Amount Earned ($y$) | \$15.50 | \$31 | \$46.50 | \$62 | \$77.50 |

4. **Discuss and Write** Explain how you would write a function rule to describe the relationship of hours to minutes. Write the function rule.

Go to PRACTICE BOOK Lesson 13-4 for exercise sets.

Algebra 13-5

# Functions

**Objective** To write a function rule to represent a situation • To evaluate a function given a specific domain • To evaluate a function when the domain is not specified

A jet airplane climbs 20 feet in altitude for each second after takeoff. Write an equation, or a function rule, that describes the relationship between elapsed time and the altitude.

To write a function rule, first define the variables. Then write a function rule comparing the relationship between time and altitude.

Create a function table to help see the relationship. A function table helps organize and display the $x$- and $y$-values of a function.

Let $x$ = time in seconds.
Let $y$ = altitude in feet.

| $x$ | $y$ |
|---|---|
| 1 | 20 |
| 2 | 40 |
| 3 | 60 |
| 4 | 80 |
| 5 | 100 |

+20 +20 +20 +20

The constant difference is 20.

Write the function rule as $y = 20x$.

## Example

**1** Manny earns $45 an hour tutoring. For each tutoring session, he donates $5 to charity and saves the remaining amount. Write a function rule to describe the relationship between the number of hours per session and the amount he saves. Create a function table to help see the relationship.

Let $x$ = the number of hours per tutoring session.
Let $y$ = the amount Manny saves.

| Length of Session (hours) | Amount Saved (dollars) |
|---|---|
| 1 | $45 - 5 = 40$ |
| 2 | $90 - 5 = 85$ |
| 3 | $135 - 5 = 130$ |
| 4 | $180 - 5 = 175$ |
| 5 | $225 - 5 = 220$ |

+45 +45 +45 +45

The constant difference is 45.

Write the function rule as $y = 45x - 5$.

► To evaluate a function for a given domain, substitute the given $x$-values into the function, and simplify.

For the function $y = 2x - 3$, find the $y$-values for a domain of 0, 1, and 2. Then identify the range of the function.

| Input $x$ | $y = 2x - 3$ | Output $y$ |
|---|---|---|
| 0 | $y = 2(0) - 3$ | $-3$ |
| 1 | $y = 2(1) - 3$ | $-1$ |
| 2 | $y = 2(2) - 3$ | 1 |

The range of the function $y = 2x - 3$ for the given domain is $\{-3, -1, 1\}$.

## Example

**1** Evaluate the function $y = -2.5x + 6$ for a domain of $\{-2, -1, 0, 1, 2\}$. Then identify the range of the function.

| $x$ | $y = -2.5x + 6$ | $y$ |
|---|---|---|
| $-2$ | $y = -2.5(-2) + 6$ | 11 |
| $-1$ | $y = -2.5(-1) + 6$ | 8.5 |
| 0 | $y = -2.5(0) + 6$ | 6 |
| 1 | $y = -2.5(1) + 6$ | 3.5 |
| 2 | $y = -2.5(2) + 6$ | 1 |

The range of the function $y = -2.5x + 6$ for the given domain is $\{11, 8.5, 6, 3.5, 1\}$.

► When the domain of a function is not given, the domain is considered to be the set of real numbers.

Evaluate the function $y = \frac{1}{2}x + 4$ by choosing several input values, and finding the corresponding output values of the function.

| $x$ | $y = \frac{1}{2}x + 4$ | $y$ |
|---|---|---|
| $-2$ | $y = \frac{1}{2}(-2) + 4$ | 3 |
| 0 | $y = \frac{1}{2}(0) + 4$ | 4 |
| 2 | $y = \frac{1}{2}(2) + 4$ | 5 |

## Try These

**1.** Bailey goes to an amusement park. The admission fee is \$6. Tickets for rides cost \$5 each. One ticket is needed for each ride. Write a function to show the total amount of money Bailey spends at the amusement park.

**2.** Swati starts a checking account with \$100. Each month, she deposits \$125 into the account. Write a function to show how much money Swati has in her checking account in any given month.

**3.** **Discuss and Write** Explain how you found the function for exercise 1.

Go to PRACTICE BOOK Lesson 13-5 for exercise sets.

# Graph Linear Functions

**Objective** To graph a linear function • To identify ordered pairs of a linear function from its graph • To use a graphing calculator to graph linear functions


The EZCar Company charges $10 per hour plus a $30 fee to rent a car. You can show the relationship between the number of hours a car is rented and the total cost of renting the car by using a function, a function table, and a graph.

► To write the function, first define the variables. Then write a function expressing the relationship between the two variables.

Let $x$ = the number of hours a car is rented.
Let $y$ = the total cost of renting a car.

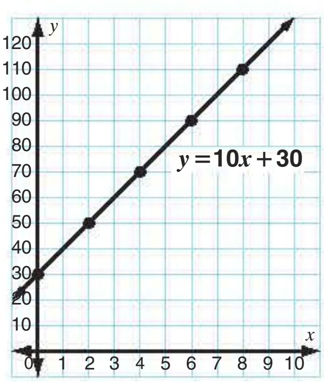
$$\underset{\text{Total cost}}{y} = \underset{\text{Cost per hour / Hours rented}}{10x} + \underset{\text{Fee}}{30}$$

► Use the function to make a function table. Choose at least two input values. Then substitute the values into the function to find the corresponding output values.

| $x$ | $y = 10x + 30$ | $y$ |
|---|---|---|
| 0 | $10(0) + 30$ | 30 |
| 2 | $10(2) + 30$ | 50 |
| 4 | $10(4) + 30$ | 70 |
| 6 | $10(6) + 30$ | 90 |
| 8 | $10(8) + 30$ | 110 |

► Use the input and output values from the function table to write a list of ordered pairs. Then plot and connect the points.

| $(x, y)$ |
|---|
| (0, 30) |
| (2, 50) |
| (4, 70) |
| (6, 90) |
| (8, 110) |




Notice that all the points lie on a straight line.
The line represents *all* the ordered pairs of the function $y = 10x + 30$.

► The function $y = 10x + 30$ is a linear function. A linear function is a function whose graph is a nonvertical line. Any ordered pair on the graph of a linear function is a solution of the related equation.

Refer to the graph on page 362. You can use the graph to find the total cost of renting a car for any given number of hours.

Use the graph to determine how much it would cost to rent a car for 3 hours.

To find the cost for 3 hours, find the point on the graph where $x = 3$. From that point, move up to locate the corresponding $y$-coordinate, 60, on the graph.

The cost of renting a car for 3 hours is \$60, which is represented by the ordered pair (3, 60).

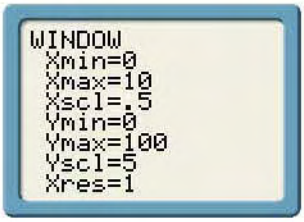
## Technology

► The distance formula, $d = rt$, is an example of a linear function, where $d$ = distance, $r$ = rate (or average speed), and $t$ = time.

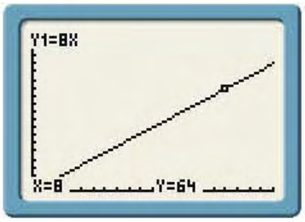
Omar rides his bicycle at an average speed of 8 miles per hour.

You can use a graphing calculator to show the relationship between the time (how long Omar rides his bike) and the distance he travels.


1. Press Y=, and then enter the function $d = 8t$, or $y = 8x$.
2. Press WINDOW, and select the scales and increments for your graph.




3. Press GRAPH to see the graph of the linear function.
4. Press 2ND TRACE ENTER to see specific ordered pair solutions. You can type in a value for $x$ and press ENTER. The calculator will show you the corresponding $y$-value on the graph.




The entire graph shows all ordered pairs of the linear function $d = 8t$.

Use the graph to confirm that if Omar rides his bike for 8 hours at a rate of 8 miles per hour, he will travel 64 miles.

## Try These

**Graph each linear function. Then use the graph to find $y$ when $x = 4$.**

**1.** $y = -x$ **2.** $y = x - 7$ **3.** $y = \frac{1}{2}x$ **4.** $y = -x + 10$ **5.** $y = 2.5x$

**6.** **Discuss and Write** Explain why only two points are needed to graph a linear function.

Go to PRACTICE BOOK Lesson 13-6 for exercise sets.

Algebra
13-7

# Slope

**Objective** To find the slope of a line
• To identify the four kinds of slope

Ms. Noriyama wants to build a wheelchair ramp. From researching safety guidelines, she knows that the ramp should only rise 1 inch for every 12 inches of ramp length. What will the slope of the wheelchair ramp be?

► **Slope** is the slant, or steepness, of a nonvertical line expressed as a ratio. This ratio compares the vertical change in any two points on the line to the horizontal change in the same two points. The vertical change is called the **rise**, and the horizontal change is called the **run**. On the coordinate plane, for any two points, $(x_1, y_1)$ and $(x_2, y_2)$, this ratio compares the change in the $y$-coordinates, $y_2 - y_1$, to the change in the $x$-coordinates, $x_2 - x_1$.

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

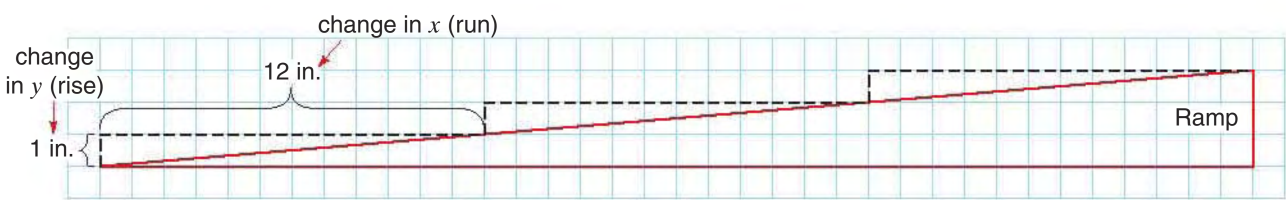
So using the diagram below, Ms. Noriyama can find the slope of the wheelchair ramp.

**Key Concept**

**Slope**

The slope of a line is the rate of change.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$




$$\text{Slope} = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{1}{12}$$

The slope of the wheelchair ramp will be $\frac{1}{12}$.

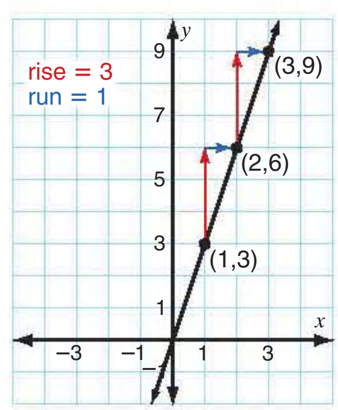
► Since the slope of a straight line is constant, you can calculate the slope of a line using the coordinates of any two points on the line.

The slope of the graph $y = 3x$ at the right is:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{2 - 1} = \frac{3}{1} = 3$$

Check by using another pair of points on the line.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 9}{2 - 3} = \frac{-3}{-1} = 3$$




Slope is a rate of change and is constant for a line. When a linear function is in the form of $y = kx$, the term $k$ is called the **constant of variation**. You can say that $y$ varies directly with $x$.

The graph of a direct variation is a line through the origin.

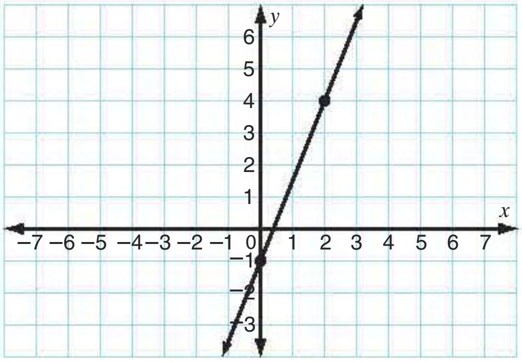
**Key Concept**

**Direct Variation**

$y = kx$ or $\frac{y}{x} = k$, where $x, k \neq 0$

► The slope of a line may be positive, negative, zero, or undefined.

### Positive Slope



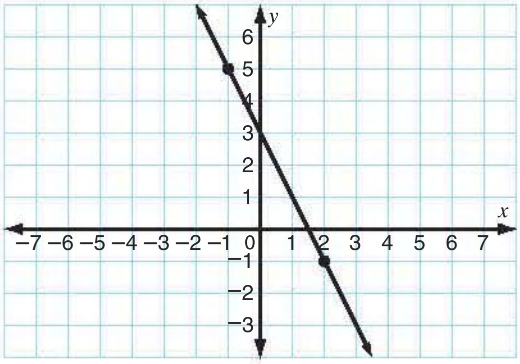
The slope of this line is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{0 - 2} = \frac{-5}{-2} = 2.5$$

2.5 is a positive number, so this line is said to have a *positive* slope.

**Think**
A line with a positive slope *rises* from left to right.

### Negative Slope



The slope of this line is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - (-1)} = \frac{-6}{3} = -2$$

$-2$ is a negative number, so this line is said to have a *negative* slope.

**Think**
A line with a negative slope *falls* from left to right.

### Zero Slope

A *horizontal line* is a line whose points all have the same $y$-coordinate. The line $y = -3$ is a horizontal line. Two points on the line $y = -3$ are $(-6, -3)$ and $(0, -3)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-3)}{-6 - 0} = \frac{0}{-6} = 0$$

The slope of a horizontal line is zero.

### Undefined Slope

A *vertical line* is a line whose points all have the same $x$-coordinate. The line $x = 4$ is a vertical line. Two points on the line $x = 4$ are $(4, 7)$ and $(4, -1)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{4 - 4} = \frac{8}{0}$$

The slope of a vertical line is undefined.

**Remember:** Division by 0 is undefined.

## Try These

**Find the slope of the line that passes through each pair of points. Then classify the slope as *positive*, *negative*, *zero*, or *undefined slope*.**

**1.** $(7, 4), (-2, 4)$ **2.** $(1, 3), (9, -1)$ **3.** $(-5, -1), (6, 10)$ **4.** $(-8, 0), (-8, -9)$

**Tell whether the function represents a direct variation. If the function represents a direct variation, identify the constant of variation.**

**5.** $y = -2x$ **6.** $y = 3x - 2$ **7.** $y = 5.5x$ **8.** $y = \frac{3}{4}x$

**9. Discuss and Write** Look at the graph with the positive slope at the top of the page. Describe the change in slope if point $(2, 4)$ is changed to point $(-1, 4)$.

Go to PRACTICE BOOK Lesson 13-7 for exercise sets.

Algebra 13-8

# Nonlinear Functions

**Objective** To differentiate linear functions from nonlinear functions • To use a table of values to graph a nonlinear function on a coordinate plane • To use technology to graph a simple quadratic equation

Joelle's mother is a microbiologist. Over a number of days, she records the number of cells in a sample to track the sample's growth. Joelle examines the data and decides to help her mother by making a table and plotting the data on a coordinate plane. Does the data represent a linear function or a nonlinear function?

| *t*, time (days) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| *c*, number of cells | 1 | 2 | 4 | 8 | 16 | 32 | 64 |

► A nonlinear function is a function that does *not* have a constant rate of change. The graph of a nonlinear function is *not* a straight line.

Nonlinear functions may involve equations like $y = |x|$, $y = \frac{1}{x}$, $y = x^2$, and $y = \sqrt{x}$.

To determine if a function is linear or nonlinear, analyze the rate of change (the slope) or the graph of the function.

**Method 1** Find the Slope

Find the slope to determine if the function has a constant rate of change or a variable rate of change.

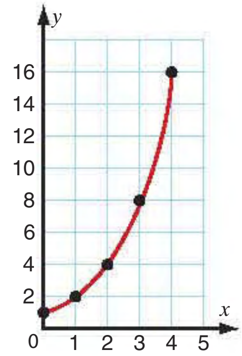
Choose any two points from the data, and find the slope. Then to determine if the slope is constant or variable, find the slope using another pair of points.

Use (0, 1) and (1, 2). $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{1-0} = \frac{1}{1} = 1$

Use (1, 2) and (2, 4). $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{2-1} = \frac{2}{1} = 2$

Since the slopes are different, the data represents a nonlinear function.

**Method 2** Examine the Graph

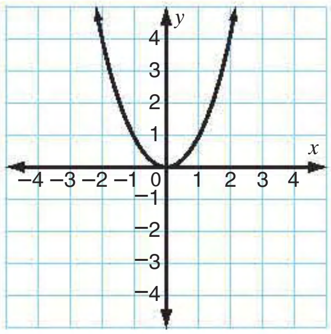



The graph is not a straight line. The graph gets steeper and steeper as the *x*-values increase.

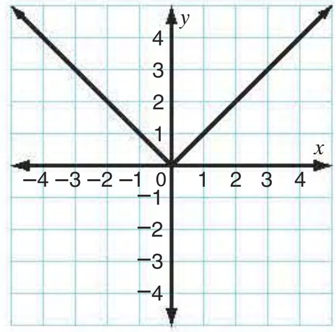
This is a graph of a nonlinear function.

► The quadratic function and the absolute-value function are examples of nonlinear functions.

A quadratic function, when graphed on a coordinate plane, takes the form of a parabola—a U-shaped curve—that can open either up or down. The graph at the right shows the quadratic function $y = x^2$.



An absolute-value function, when graphed on a coordinate plane, takes a V-shaped form. The graph at the right shows the absolute-value function $y = |x|$.



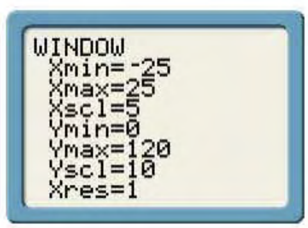

## Technology

You can use graphing calculators to examine the graph of a quadratic function. Graph the function $y = x^2 + 4$. Using the graph, find the value of $y$ when $x = 10$.

1. Press Y= to enter the function.
   Press X,T,Θ,n $x^2$ + 4.
2. Select WINDOW, and choose the range of the $x$- and $y$-values shown below.
3. Select GRAPH to see the graph of the quadratic function.
4. Select 2ND TRACE. Then select **1: value**.
   Press 10 ENTER to calculate the value of $y$ when $x = 10$.

1



2
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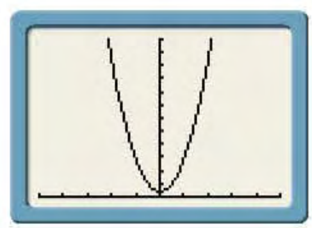

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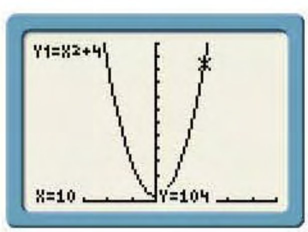



When $x = 10$ in the equation $y = x^2 + 4$, $y = 104$.

## Try These

**Does the data table or function represent a linear function or a nonlinear function? Explain.**

**1.**

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y$ | 15 | 12 | 9 | 6 | 3 | 0 |

**2.**

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | −11 | −8 | −7 | −8 | −11 | −16 |

**3.** $y = -x^2$

**4.** $y = |x + 1|$

**5.** $y = -2x^2$

**6.** $y = x - 3$

**7.** **Discuss and Write** Without using the slope or graphing ordered pairs, explain how you can determine whether or not a function is nonlinear. Give examples to support your answer.

Go to PRACTICE BOOK Lesson 13-8 for exercise sets.

Algebra

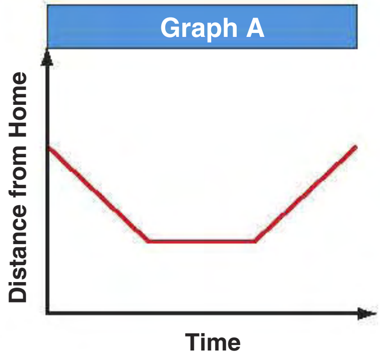
13-9

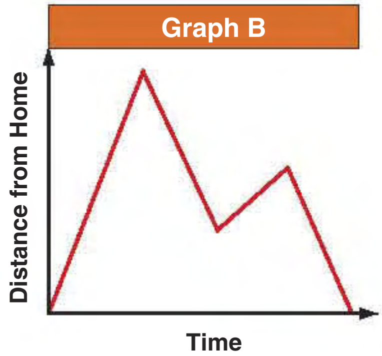
# Graph a Situation

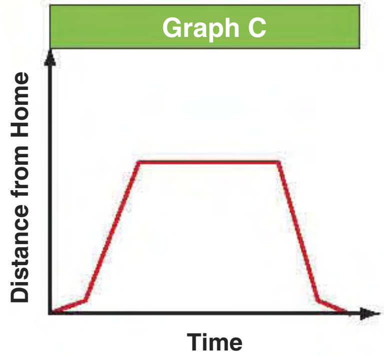
**Objective** To choose a representation for a situation • To sketch a graph for a situation • To interpret a graph or a specific part of a graph in terms of the situation it represents

Rafael leaves home and drives to the airport. He takes an airplane to Mexico City, stays for 1 week, and then flies home again. Rafael drives back home from the airport.

Which graph best represents this situation?




► Sometimes a graph does not show a function, but a situation. In the graphs above, a line slanting up or down represents a varying rate of change. The steeper the slant, the faster the rate of change. The less steep the slant, the slower the rate of change. A horizontal line represents a constant (zero) rate of change.

To see if a graph and a situation match, break the situation into a sequence of events. Then see if each event fits the graph and if the events are shown in the correct order.

**Event 1:** Rafael drives from home to the airport.
**Event 2:** Rafael takes a plane to Mexico City.
**Event 3:** Rafael stays in Mexico City for 1 week.
**Event 4:** Rafael flies home.
**Event 5:** Rafael drives home from the airport.

**Graph A**

The story begins and ends at home. Therefore, the graph should begin at (0, 0) and end on the $x$-axis. This graph cannot describe the situation.

**Graph B**

While this graph shows a departure and a return home, it does not show Rafael staying in one place (Mexico City) for a period of time. So this graph does not describe the situation.

**Graph C**

This graph begins and ends at home. As Rafael's distance from home increases, the line on the graph steepens. The horizontal line represents the time Rafael spent in Mexico City. Then as Rafael's distance from home decreases, the steepness of the line lessens.

Graph C matches the situation.

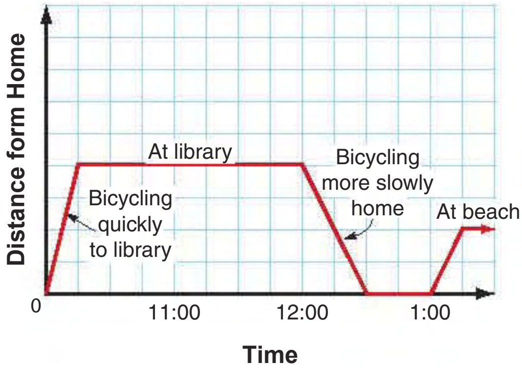
So Graph C best represents the situation.

► You can sketch a graph based on a situation.

Jeanette leaves home on her bicycle at 10 A.M. and travels quickly to the library for a study session with friends. She stays at the library from 10:15 A.M. until noon. At noon, loaded down with books, Jeanette slowly rides her bicycle home again. She stays at home from 12:30 until 1:00 P.M. and then rides her bicycle to the beach, which is closer to her house, in 15 minutes. She then stays there for the rest of the afternoon.

Sketch a graph that relates time to the distance Jeanette is from home.

To graph this situation, first break the situation into different events. Then graph the events so that each event starts where the preceding event finishes.

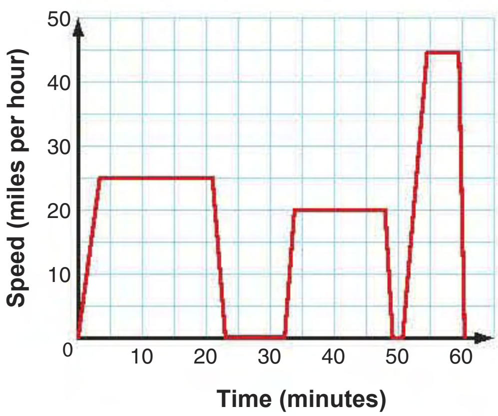



**Event 1:** Jeanette quickly bicycles to the library at 10:00.
**Event 2:** Jeanette studies from 10:15 to 12:00.
**Event 3:** Jeanette bicycles home slowly.
**Event 4:** Jeanette stays at home from 12:30 to 1:00.
**Event 5:** Jeanette leaves at 1:00 for a 15-minute ride to the beach.

► You can also create a verbal or written description of a situation based on a graph.

Mr. Satay is a city truck driver. In the course of 1 hour, he makes several deliveries. Suppose his company made a graph to show his activities for that hour and how the activities relate to the truck's speed and the time he traveled.

Use the graph to tell a story about the hour graphed. In the graph, time is related to speed (miles per hour).




Mr. Satay's hour starts with him accelerating to 25 miles per hour and driving at that constant speed for about 20 minutes. He then stops his truck for about 10 minutes to make a delivery. Mr. Satay gets back in his truck to go to his next delivery. He accelerates again, this time to 20 miles per hour and drives at this constant speed for about 15 minutes. He stops for about 5 minutes, makes his delivery, and then continues on his route. Now, Mr. Satay accelerates to 45 miles per hour. He drives at that constant speed for about 5 minutes and then stops to make his last delivery of the hour.

## Try These

1. Write a situation for each of graphs A and B on page 368.

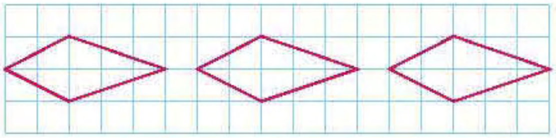
2. **Discuss and Write** Zelma sets her car's cruise control at 60 miles per hour. Sketch and describe a graph that shows her speed related to time. Now sketch a graph that shows her distance traveled related to time. Explain the differences in the graphs.

Go to PRACTICE BOOK Lesson 13-9 for exercise sets.

# Graph Translations and Reflections

**Objective** To demonstrate the reflection or translation of points on a coordinate plane • To identify the reflection or translation of polygons on a coordinate plane • To graph the reflection or translation of polygons on a coordinate plane

Ms. Choi is stenciling a simple border of a repeating kite-pattern along the wall near the ceiling. Does the border represent a translation or a reflection pattern?



► A transformation is a change in orientation (position), shape, or size of a figure. The figure that results from a transformation is called the image. Using prime notation, the image of a point $P$ is identified as $P'$ and is read as "$P$ prime."

A translation is a transformation that slides every point of a figure the same distance and in the same direction along a straight line without turning. In a translation, the figures are congruent and the orientation stays the same.

A reflection is a transformation that flips a figure over a line. This line is called the line of reflection. In a reflection, the image is congruent to the original, but has a different orientation.

In a reflection, the figure and the image are mirror images of each other and the line of reflection is a line of symmetry.

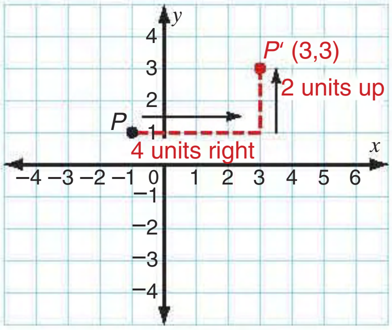
Ms. Choi's pattern represents a translation, because the figure slides along a straight line without turning and all corresponding angle measures and segment lengths in the image are congruent to the original.

► Points can also be transformed on a coordinate plane.

- Translate point $P(-1, 1)$ four units to the right and two units up. What are the coordinates of $P'$?

  Locate point $P$. Then from point $P$ move 4 units right and 2 units up. Plot $P'$.

  $P' = (3, 3)$



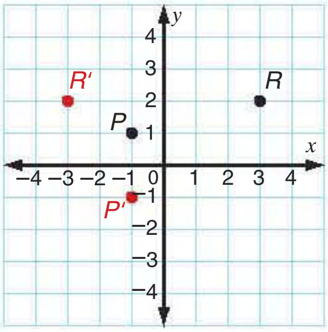

- Reflect point $P(-1, 1)$ over the $x$-axis.
  Reflect point $R$ (3, 2) over the $y$-axis.
  What are the coordinates of $P'$ and $R'$?

  To reflect a point over the $x$-axis, use the same $x$-coordinate and multiply the $y$-coordinate by $-1$. Plot $P'$.

  $P' = (-1, -1)$

  To reflect a point over the $y$-axis, use the same $y$-coordinate and multiply the $x$-coordinate by $-1$. Plot $R'$.

  $R' = (-3, 2)$



► Polygons can also be transformed on a coordinate plane.

To transform a polygon, first transform the vertices.
Then connect the images of the vertices to form the image of the polygon.

## Examples

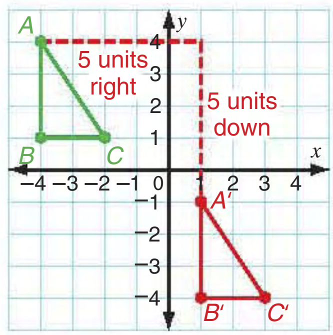
**1** Translate $\triangle ABC$ 5 units to the right and 5 units down.

Each vertex is moved 5 units to the right and 5 units down.

Give the coordinates of the vertices of the original figure and its image.

Original: $A(-4,4)$, $B(-4,1)$, $C(-2,1)$

Image: $A'(1,-1)$, $B'(1,-4)$, $C'(3,-4)$




**2** Identify the transformation as a translation or a reflection. Then describe how the figure was transformed.

$J(1,2) \rightarrow J'(-1,2)$

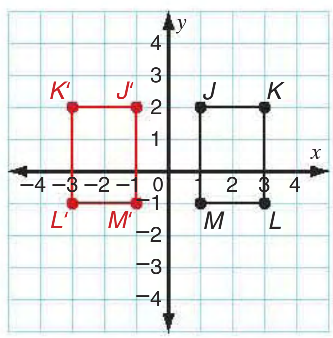
$K(3,2) \rightarrow K'(-3,2)$

$L(3,-1) \rightarrow L'(-3,-1)$

$M(1,-1) \rightarrow M'(-1,-1)$

**Think**
$(x, y) = (-1 \bullet x, y)$
$= (-x, y)$

This transformation is a reflection.
Rectangle $JKLM$ was reflected over the $y$-axis to form rectangle $J'K'L'M'$.

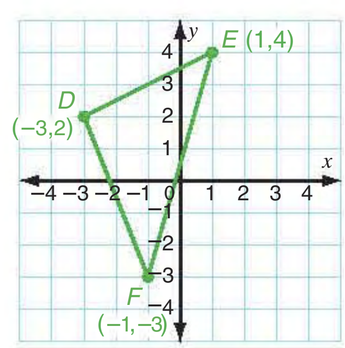



## Try These

**Graph each point and its image.**

1. $P(1,8)$
   left 6 units, up 4 units
2. $B(0,0)$
   right 3 units, down 1 unit
3. $M(-2,-1)$
   reflect over the $y$-axis

**Graph each transformation of $\triangle DEF$. Give the coordinates of the vertices of the original figure and its image.**



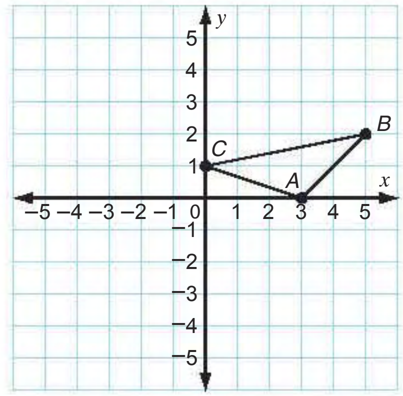

4. translate left 6 units
5. translate right 2 units and up 3 units
6. reflect over the $y$-axis
7. reflect over the $x$-axis
8. **Discuss and Write** How can you identify the coordinates of an image reflected over the $x$- or $y$-axis without graphing? Give examples to support your answer.

Go to PRACTICE BOOK Lesson 13-10 for exercise sets.

# Graph Rotations

**Objective** To demonstrate rotation of points on a coordinate plane in clockwise and counterclockwise directions • To identify graphs of rotations of figures • To graph rotations of polygons on a coordinate plane

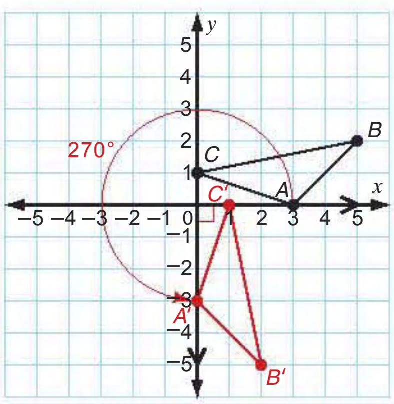
Jaime is creating a design from a triangle. He creates one design rotating the figure around the origin 270° counterclockwise. He creates another design rotating the figure around the origin 90° counterclockwise and yet another rotating the figure around the origin 180° counterclockwise. What are the coordinates for each image of the triangle?




**Think** Counterclockwise refers to the direction opposite to the way the hands on a clock rotate.

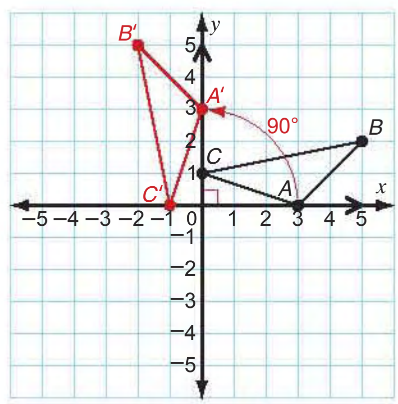
► A rotation is a transformation that turns a figure around a point, usually in a counterclockwise direction. The point around which the figure rotates is called the center of rotation. As with reflections and translations, the image of a rotation is congruent to the original figure.

A 90° rotation is a quarter turn.
A 180° rotation is a half turn.
A 270° rotation is a three-quarter turn.



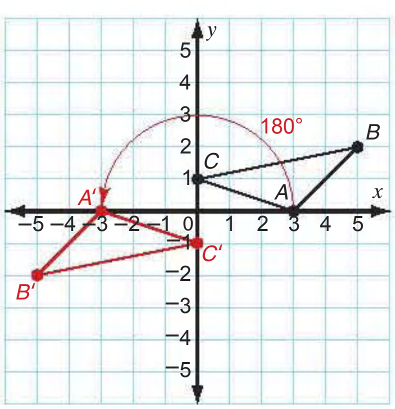

270° counterclockwise around the origin

So $A(3, 0) \rightarrow A'(0, -3)$
$B(5, 2) \rightarrow B'(2, -5)$
$C(0, 1) \rightarrow C'(1, 0)$




90° counterclockwise around the origin

So $A(3, 0) \rightarrow A'(0, 3)$
$B(5, 2) \rightarrow B'(-2, 5)$
$C(0, 1) \rightarrow C'(-1, 0)$




180° counterclockwise around the origin

So $A(3, 0) \rightarrow A'(-3, 0)$
$B(5, 2) \rightarrow B'(-5, -2)$
$C(0, 1) \rightarrow C'(0, -1)$

► You can write rules to describe the coordinates of an image under 90°, 180°, and 270° rotations around the origin.

For each rotation on page 372, examine the coordinates of the original figure with the coordinates of its image. What do you notice?

- To rotate a figure 270° counterclockwise around the origin, switch the coordinates of each point and then multiply the new second coordinate by $-1$.

**270° Counterclockwise Rotation:**
$A(3, 0), B(5, 2), C(0, 1) \rightarrow A'(0, -3), B'(2, -5), C'(1, 0)$

- To rotate a figure 90° counterclockwise around the origin, switch the coordinates of each point and then multiply the new first coordinate by $-1$.

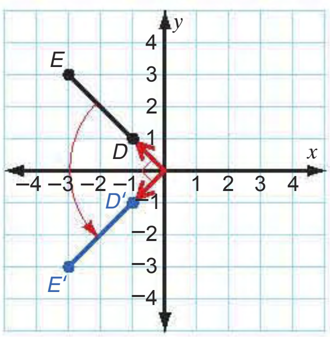
**90° Counterclockwise Rotation:**
$A(3, 0), B(5, 2), C(0, 1) \rightarrow A'(0, 3), B'(-2, 5), C'(-1, 0)$

- To rotate a figure 180° around the origin, multiply both coordinates by $-1$.

**180° Counterclockwise Rotation:**
$A(3, 0), B(5, 2), C(0, 1) \rightarrow A'(-3, 0), B'(-5, -2), C'(0, -1)$

## Example

**1** Graph line segment $DE$ with endpoints $D(-1, 1)$ and $E(-3, 3)$. Rotate the line segment 90° counterclockwise. Then write the coordinates of the image.




Use the rule for rotating a figure 90° counterclockwise.
$x \rightarrow -y$
$y \rightarrow x$

$D(-1, 1) \rightarrow D'(-1, -1)$
$E(-3, 3) \rightarrow E'(-3, -3)$

The coordinates of line segment $D'E'$ are $D'(-1, -1)$ and $E'(-3, -3)$.

## Try These

1. Draw $\triangle MNO$ with vertices $M(0, 0)$, $N(1, 4)$, $O(2, 2)$. Rotate the triangle 180° around the origin. What are the coordinates of the vertices of the image?

2. Draw rectangle $RSTU$ with vertices $R(1, 1)$, $S(5, 1)$, $T(5, 6)$, $U(1, 6)$. Rotate the rectangle 90° counterclockwise around the origin. What are the coordinates of the vertices of the image?

3. **Discuss and Write** Suppose you rotate line segment $DE$ (see the graph above) 180° counterclockwise around the origin. How would this transformation compare to reflecting line segment $DE$ over the line that includes line segment $D'E'$? Explain your answer.

Go to PRACTICE BOOK Lesson 13-11 for exercise sets.

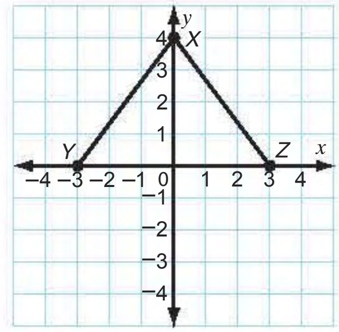
Algebra

13-12

# Graph Dilations

**Objective** To identify graphs of dilations of figures • To graph dilations of polygons on the coordinate plane • To compute length and area on the coordinate plane

To advertise an art show, Beth uses a triangle. She dilates the triangle to be three times as large so she can print it on posters. She dilates the pattern to be half as large so she can print it on postcards. What are the coordinates of the enlargement and the reduction images?




► A dilation is a transformation that reduces or enlarges the size of a figure. A dilation does not change the shape of a figure; its image is similar to the original figure. An enlargement is a dilation that is larger than the original figure. A reduction is a dilation that is smaller than the original figure.

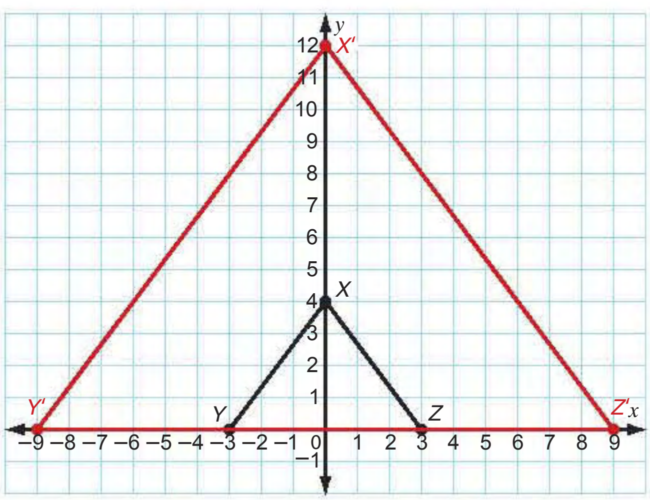
To dilate a polygon on a coordinate plane, multiply the $x$- and $y$-coordinates of each vertex by the same positive number, the scale factor. Then connect the vertices to form the image.

**Remember:** Scale factor is the ratio of the lengths of two corresponding sides of two similar polygons.

To find the coordinates of both images, dilate the triangle according to the scale factor.

**Enlargement**

Multiply both coordinates of each vertex by the scale factor of 3. Graph the images of each vertex and connect them to form the image.




$\mathbf{P(x, y)} \rightarrow \mathbf{P'(3x, 3y)}$

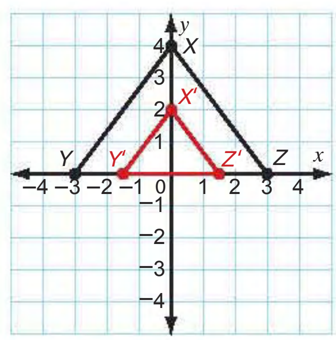
$X(0, 4) \rightarrow X'(3 \bullet 0, 3 \bullet 4) \rightarrow X'(0, 12)$

$Y(-3, 0) \rightarrow Y'(3 \bullet (-3), 3 \bullet 0) \rightarrow Y'(-9, 0)$

$Z(3, 0) \rightarrow Z'(3 \bullet 3, 3 \bullet 0) \rightarrow Z'(9, 0)$

**Reduction**

Multiply both coordinates of each vertex by the scale factor of $\frac{1}{2}$. Graph the images of each vertex and connect them to form the image.




$\mathbf{P(x, y)} \rightarrow \mathbf{P'\left(\frac{x}{2}, \frac{y}{2}\right)}$

$X(0, 4) \rightarrow X'\left(0 \bullet \frac{1}{2}, 4 \bullet \frac{1}{2}\right) \rightarrow X'(0, 2)$

$Y(-3, 0) \rightarrow Y'\left(-3 \bullet \frac{1}{2}, 0 \bullet \frac{1}{2}\right) \rightarrow Y'(-1.5, 0)$

$Z(3, 0) \rightarrow Z'\left(3 \bullet \frac{1}{2}, 0 \bullet \frac{1}{2}\right) \rightarrow Z'(1.5, 0)$

If the scale factor is between 0 and 1, the dilation is a reduction.
If the scale factor is greater than 1, the dilation is an enlargement.

Using the coordinates of a dilation, you can find its area. The length of a vertical line segment with endpoints $(x_1, y_1)$ and $(x_1, y_2)$ is $|y_1 - y_2|$. The length of a horizontal line segment with endpoints $(x_1, y_1)$ and $(x_2, y_1)$ is $|x_1 - x_2|$.

What is the area of each dilated figure on page 374?

**Enlargement**

**1 Find the base.**
The endpoints of the base are $(-9, 0)$ and $(9, 0)$.
So the base is: $|-9 - 9| = |-18| = 18$

**2 Find the height.**
The endpoints of the height are $(0, 12)$ and $(0, 0)$.
So the height is: $|12 - 0| = |12| = 12$

**3 Use the area formula.**
$A = \frac{1}{2}bh = \frac{1}{2}(18)(12) = 108 \text{ units}^2$

So the area of the enlarged triangle is 108 units$^2$

**Reduction**

**1 Find the base.**
The endpoints of the base are $(-1.5, 0)$ and $(1.5, 0)$.
So the base is: $|-1.5 - 1.5| = |-3| = 3$

**2 Find the height.**
The endpoints of the height are $(0, 2)$ and $(0, 0)$.
So the height is: $|2 - 0| = |2| = 2$

**3 Use the area formula.**
$A = \frac{1}{2}bh = \frac{1}{2}(3)(2) = 3 \text{ units}^2$

So the area of the reduced triangle is 3 units$^2$

## Example

**1** Rectangle $WXYZ$ has vertices $W(3, -2)$, $X(6, -2)$, $Y(6, -3)$, $Z(3, -3)$. Rectangle $PQRS$ has vertices $P(-8, 6)$, $Q(7, 6)$, $R(7, 1)$, $S(-8, 1)$. Is rectangle $PQRS$ a dilation of rectangle $WXYZ$? If it is a dilation, tell whether it is an enlargement or a reduction.

Compare the lengths of corresponding line segments.

**Rectangle $WXYZ$**

$WX = ZY = |3 - 6| = 3$

$WZ = XY = |-2 - (-3)| = 1$

**Rectangle $PQRS$**

$PQ = SR = |-8 - 7| = 15$

$PS = QR = |6 - 1| = 5$

The lengths of the corresponding sides of $PQRS$ and $WXYZ$ are in a ratio of 5 : 1. So $PQRS$ is a dilation of $WXYZ$. It is an enlargement with a scale factor of 5.

**Think**
$\frac{15}{3} = 5$ and $\frac{5}{1} = 5$

## Try These

1. Draw a triangle with vertices $(-1, 0)$, $(2, -1)$, $(2, 2)$. Use a coordinate grid to create two dilations: one with a scale factor of 4 and one with a scale factor of $\frac{1}{4}$.

2. **Discuss and Write** Explain how to determine whether one figure is a dilation of another figure.

Go to PRACTICE BOOK Lesson 13-12 for exercise sets.

13-13

# Problem-Solving Strategy: Consider Extreme Cases

**Objective** To solve problems using the strategy *Consider Extreme Cases*

**Problem 1:** After taking six tests worth 100 points each, Jacqueline's test average is 88. What is the lowest possible test result Jacqueline could have received?

**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Jacqueline's average on six 100-point tests is 88.

**Question:** What is the lowest test score Jacqueline could have received?

**Plan** **Select a strategy.**
You can use the strategy *Consider Extreme Cases*. Is it possible that Jacqueline could have received a 0 on one test?

**Solve** **Apply the strategy.**
Suppose Jacqueline had received a 0 on one of the tests.

Let $a$, $b$, $c$, $d$, and $e$ represent Jacqueline's scores on the other five exams. Then the average of the scores would be:

$$\frac{a + b + c + d + e + 0}{6} = 88$$

Multiplying both sides by 6 gives $a + b + c + d + e + 0 = 528$, or equivalently, $a + b + c + d + e = 528$. However, this is impossible because even if she had received 100 on each of the other five tests, the total score would be only 500, which is 28 short of 528. This reasoning leads to the answer.

The lowest possible score Jacqueline could have received is 28, not 0. That is, had Jacqueline scored 100 on five of her tests, she would have had to score exactly 28 on the sixth test to have an average of 88.

So 28 is the lowest possible test result Jacqueline could have received.

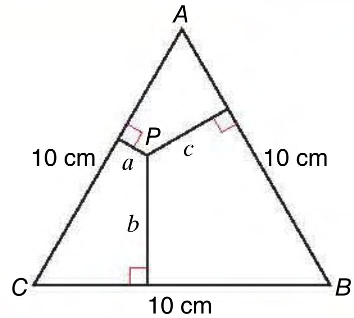
**Check** **Check to make sure your answer makes sense.**
Suppose that Jacqueline's scores were 100, 100, 100, 100, 100, and 28. Her average score would then be

$$\frac{100 + 100 + 100 + 100 + 100 + 28}{6} = \frac{528}{6} = 88.$$

So the answer checks.

**Problem-Solving Strategies**
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. **Consider Extreme Cases**

**Problem 2:** Equilateral triangle $ABC$ has sides of length 10 cm. Suppose a point $P$ is chosen inside the triangle. No matter where $P$ is located, the sum of the distances from $P$ to the three sides ($a + b + c$ in the diagram at the right) will be the same. What is this common sum?




**Remember:** The distance from a point to a line is the length of the perpendicular line segment from the point to the line.

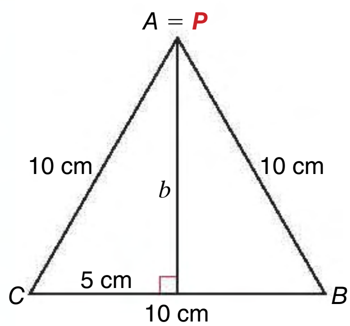
**Read** **Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** A point $P$ is located inside an equilateral triangle with 10-cm sides.
No matter where $P$ is located, the sum of the distances from $P$ to the three sides will be the same.

**Question:** What is this common sum?

**Plan** **Select a strategy.**
Use the strategy *Consider Extreme Cases*.

**Solve** **Apply the strategy.**
Because the sum of the three distances does not depend on the particular point, you can put $P$ in an extreme position to make your work easy. Put $P$ at vertex $A$.




This makes two of the distances very easy to find. The distance $a$ from $P$ to the left side is 0 cm. The distance $c$ from $P$ to the right side is 0 cm. The distance $b$ from $P$ to the base is the length of a leg of a right triangle with hypotenuse length 10 cm and leg length 5 cm. This distance can be found by using the Pythagorean Theorem.

$b^2 + 5^2 = 10^2$ ← Use the Pythagorean Theorem.
$b^2 + 25 = 100$ ← Compute the squares.
$b^2 = 75$ ← Subtract 25 from both sides, and simplify.
$b = \sqrt{75}$ ← Take the square root of both sides.
$b = 5\sqrt{3}$ ← Simplify.

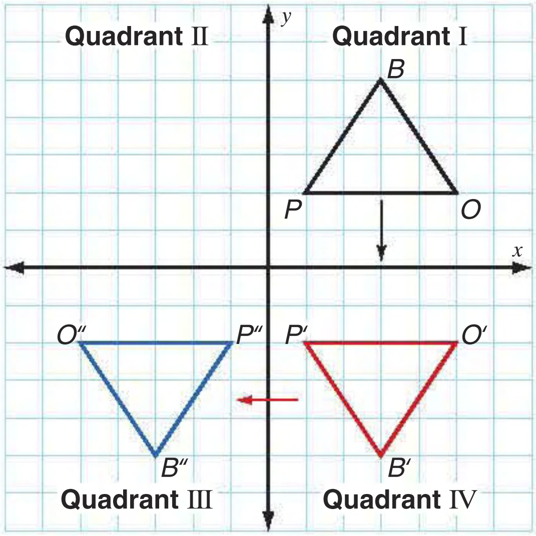
So $a + b + c = 0 + 5\sqrt{3} + 0 = 5\sqrt{3}$. The sum is $5\sqrt{3}$ cm, or about 8.66 cm.

**Check** **Check to make sure your answer makes sense.**
Draw several equilateral triangles with side lengths of 10 cm. Choose a different point inside of each. Measure the distance from the point to each side. Add each set of distances to verify that the sum is close to 8.66 cm.

Go to PRACTICE BOOK Lesson 13-13 for exercise sets.

# Enrichment: Combining Transformations

**Objective** **To apply two or more transformations to a figure on the coordinate plane**




You have learned about several kinds of transformations: translations, reflections, rotations, and dilations. The first three are sometimes called *congruence transformations*, because the image is congruent to the original figure (the pre-image). Note that a dilation does not result in a congruent image, but rather in an image that is similar to the pre-image.

In this lesson, you will explore combinations of transformations. That is, you will investigate what happens when you apply a transformation to a figure and then apply another transformation to the image.

What happens when equilateral $\triangle BOP$ at the right is reflected first over the $x$-axis and then over the $y$-axis?

Draw your own triangle on a coordinate plane and perform these two reflections on it. Label the first image $\triangle B'O'P'$ and the final image $\triangle B''O''P''$.

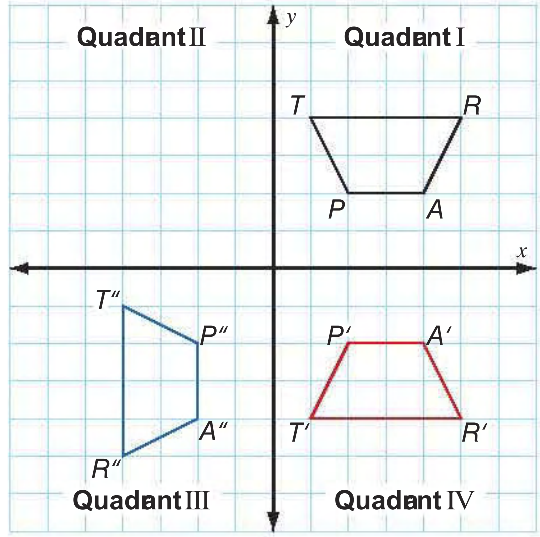
You can see that $\triangle BOP$ is congruent to $\triangle B'O'P'$ and to $\triangle B''O''P''$. The only change has been in the positions of the triangles.

► There are other combinations of transformations that will transform the pre-image $\triangle BOP$ into $\triangle B''O''P''$. Which of the following combinations results in the image shown in Quadrant III?

- Reflect $\triangle BOP$ first over the $y$-axis and then again over the $x$-axis.
- Reflect $\triangle BOP$ over the $x$-axis and then translate the image 6 units to the left.
- Translate $\triangle BOP$ six units to the left and then reflect the image over the $x$-axis.
- Rotate $\triangle BOP$ 180° about the origin.

An easy way to test all these combinations of transformations is to cut out a tracing of the pre-image triangle and move it around on the grid above. Include the labels for the vertices in your tracing. You will find that the second and third combinations have the letters $O''$ and $P''$ reversed, so they do not produce a match.

(You might try testing these same transformation combinations with a triangle that is not isosceles. Which ones still "work"?)




What happens when isosceles trapezoid *TRAP* is first reflected over the *x*-axis and then rotated 270° counterclockwise about the origin?

Draw a trapezoid on a grid and perform these two transformations.

You can see that *TRAP* is congruent to trapezoid $T'R'A'P'$ and to trapezoid $T''R''A''P''$. The only change has been in the position of trapezoid *TRAP*.

► There are other ways that isosceles trapezoid *TRAP* can be transformed into the image $T''R''A''P''$ like the following:

- Rotate *TRAP* counterclockwise 90° about the origin and then reflect over the *x*-axis.
- Reflect *TRAP* over the *y*-axis and then rotate it 90° counterclockwise about the origin.

(You might try these transformation combinations, starting with a trapezoid that is not isosceles.)

You can perform as many transformations as you like on a figure.

Suppose that you now want $T''R''A''P''$ to move to a position in Quadrant II. To do this you can choose from translating it up at least 5 units, rotating it 270° counterclockwise about the origin, or reflecting it over the *x*-axis.

## Try These

**Use grid paper to make a coordinate plane for each problem.**

1. Draw a quadrilateral in Quadrant I with a vertex at (1, 1). Starting with your quadrilateral, perform the following translations in order: reflect over the *x*-axis, translate left 10 units, reflect over the *x*-axis, and translate right 10 units. How does the position of the final image relate to the position of the pre-image?

2. Draw a scalene triangle in Quadrant III. If it is rotated 360° about the origin, it will return to its original position. Find a combination of two or more transformations that will also return the triangle to its original position.

3. Draw a pre-image square in Quadrant II. Rotate it 180° about the origin to obtain an image. Find a combination of two transformations that will give the same image.

4. **Discuss and Write** Choose a figure and a transformation. Then tell how to find a combination of transformations that result in the same image.

Go to PRACTICE BOOK pages 425–426 for exercise sets.

# Test Prep: Gridded-Response Questions

## Strategy: Apply Mathematical Reasoning

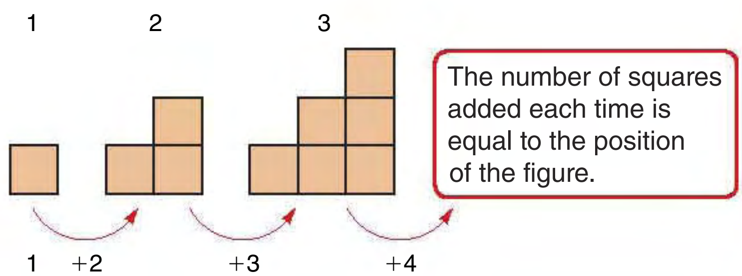
Some tests require that you fill in your answers on a gridded-response answer sheet. For a question that seems complex, you can first try to *solve a simpler problem*.

**Read the whole test item carefully.**

- Underline important words.
  If the pattern continues, how many squares will make up the eighth figure?
  *Pattern* suggests that the figures will keep changing in a predictable way.
- Restate the question in your own words.
  How many squares will the eighth figure have?

**Solve the problem.**

- Solve a simpler problem by looking for a pattern.




Continue the pattern.
There will be 36 squares in the eighth figure.

**Record your answer on the grid.**

- Print your answer in the answer boxes.
- Print only one number or symbol in each answer box.
- Fill in one bubble for every answer box you have written in.
  Do not fill in a bubble under a blank answer box.

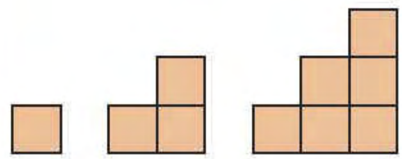
### Item Analysis

**Check your work.**

- Analyze your answer. Does it make sense? Sketch the eighth figure and count the squares.

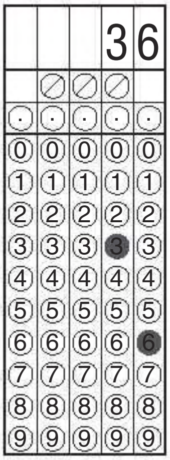
### Sample Test Item

If the pattern continues, how many squares will make up the eighth figure?



### Test-Taking Tips

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.




### Try These

**Record your answer on the gridded-response answer sheet provided in the Practice Book. Explain how you used strategies.**

**1.** What is the slope of a line that contains a point located at $(-3, -2)$ and another point located at $(2, 1)$?

**2.** Drew is wrapping a box 8 inches long, 3 inches wide, and 6 inches tall. What is the surface area of the box in square inches?

Go to PRACTICE BOOK page 427 for exercise sets.

# Polynomials, Equations, and Inequalities

CHAPTER 14

## In This Chapter You Will:

- Classify and evaluate polynomials
- Model polynomials
- Add and subtract polynomials
- Multiply and divide monomials
- Multiply a polynomial by a monomial
- Divide a polynomial by a monomial
- Solve multistep linear equations with a variable on both sides of the equation
- Solve and graph inequalities involving rational numbers
- Review problem-solving strategies
- Look for new vocabulary words **highlighted** in each lesson

## Do You Remember?

- A variable is a symbol, usually a letter, used to represent a number.
- Terms are the elements of a mathematical expression that are separated by addition or subtraction.
- A term that does not contain a variable is called a constant.
- A coefficient is the numerical factor of a term containing a variable.
- An exponent tells how many times a number, the base, is used as a factor.

**For Practice Exercises:**

Go to PRACTICE BOOK, pp. 433–462

**For Chapter Support:** ONLINE

Go to www.progressinmathematics.com

- Skills Update Practice
- Practice Activities
- Audio Glossary
- Vocabulary Activities
- Calculator Activities
- Enrichment Activities
- Electronic SourceBook

VIRTUAL MANIPULATIVES

## Critical Thinking

Shannon's sailboard has a sail in the shape of an isosceles right triangle. The hypotenuse of that triangle is 14 feet long. About how long are the other sides of the sail? You may use a calculator.

# Polynomials

**Objective** To classify a polynomial as a monomial, binomial, or trinomial • To identify the degree of a polynomial • To write a polynomial in standard form • To evaluate polynomials

PROBLEM-SOLVING STRATEGY 5

A pizza shop adds a \$3 delivery charge to each order. Each pizza costs \$10 and each topping costs \$1.50. What polynomial can represent the total bill for each delivery?

To find the *polynomial*, write the *monomial* that represents each cost.

► A polynomial is an algebraic expression that is the sum or difference of terms, called monomials.

**Remember:** A coefficient is the numerical factor of a term that contains a variable.

Let $p$ = the number of pizzas.
Let $q$ = the number of toppings.

| | cost of $p$ pizzas | plus | cost of $q$ toppings | plus | delivery charge |
|---|---|---|---|---|---|
| polynomial → | $\$10p$ | $+$ | $\$1.50q$ | $+$ | $\$3$ |

3 monomials

So the polynomial that represents the total bill for each delivery is $\$10p + \$1.50q + \$3$.

► If a polynomial has two terms, then it is called a binomial. If it has three terms, then it is called a trinomial. The chart gives some examples of types of polynomials.

| | |
|---|---|
| **Monomial** | $a$, $2x$, $\frac{k}{4}$, $856p^4$, $x^3yz^2$ |
| **Binomial** | $a + b$, $c^5 - c^2$, $x - 1$, $a^2b - ab$ |
| **Trinomial** | $a + b + c$, $m^5n^2 + m^4 - 2$ |

► The degree of a monomial is the sum of the exponents of the variables in the term. A monomial that is a nonzero constant has a degree of 0.

$8x$ has degree 1. $x^3y$ has degree $3 + 1 = 4$.
$341p^4$ has degree 4. $a^2b^2$ has degree $2 + 2 = 4$.

**Remember:** If a variable is written without an exponent, its exponent is considered to be 1.

The degree of a polynomial is the same as that of the term (monomial) with the greatest degree.

| Polynomial | Degrees of its Monomials | Degree of Polynomial |
|---|---|---|
| $-y + 10$ | 1, 0 | 1 |
| $3z^4 - 2z^3 + z - 99$ | 4, 3, 1, 0 | 4 |
| $a^2b - ab$ | 3, 2 | 3 |
| $m^5n^2 + m^4 - 2$ | 7, 4, 0 | 7 |

▶ A polynomial with one variable is in standard form when its terms are in order from greatest degree to least degree. Because the degrees descend from greatest to least, this is also called descending order.

Polynomials with more than one variable are often written in descending order for *one* of the variables. You can choose one variable that will be written in descending order.

- $3z^3 - 2z^2 + z - 99$ is in descending order. (There is only one variable, $z$.)
- $x^3y^2 + 2x^2y^3 - 3x$ is in descending order for $x$. (There are 2 variables, $x$ and $y$.)

▶ To evaluate a polynomial, substitute the assigned value for each variable, and then simplify. Be sure to follow the order of operations when performing the computations.

## Example

**1** Classify the polynomial $a^3b^2 + a^2b - ab$ as a monomial, binomial, or trinomial, and give its degree. Then evaluate it for $a = -2$ and $b = 4$.

$a^3b^2 + a^2b - ab$ ← trinomial

degree 5

$a^3b^2 + a^2b - ab = (-2)^3(4)^2 + (-2)^2(4) - (-2)(4)$ ← Substitute $-2$ for $a$ and 4 for $b$.

$= (-8)(16) + (4)(4) - (-2)(4)$ ← Evaluate exponents.

$= -128 + 16 + 8$ ← Multiply from left to right, and simplify.

$= -104$ ← Add from left to right.

So $a^3b^2 + a^2b - ab$ is a trinomial, and its degree is 5.
The value of the polynomial is $-104$ for $a = -2$ and $b = 4$.

## Try These

**Classify each expression as *monomial*, *binomial*, or *trinomial*. Then give its degree.**

**1.** $c^3 - 17$ **2.** $m^2n^3p^4$ **3.** $a + b + c$

**Evaluate each expression for $x = 4$ and $y = 2$.**

**4.** $(xy)^2$ **5.** $xy^2$ **6.** $x^3y - xy - y$

**7. Discuss and Write** Find the meanings of the prefixes *poly-*, *mono-*, *bi-*, and *tri-* in a dictionary. Explain how these meanings help you understand the meanings of the words *polynomial*, *monomial*, *binomial*, and *trinomial*. Then find other math words that contain these prefixes.

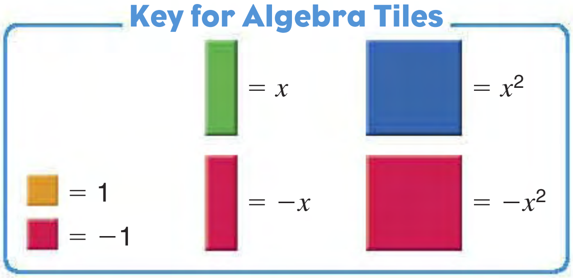
Go to PRACTICE BOOK Lesson 14-1 for exercise sets.

# Model Polynomials

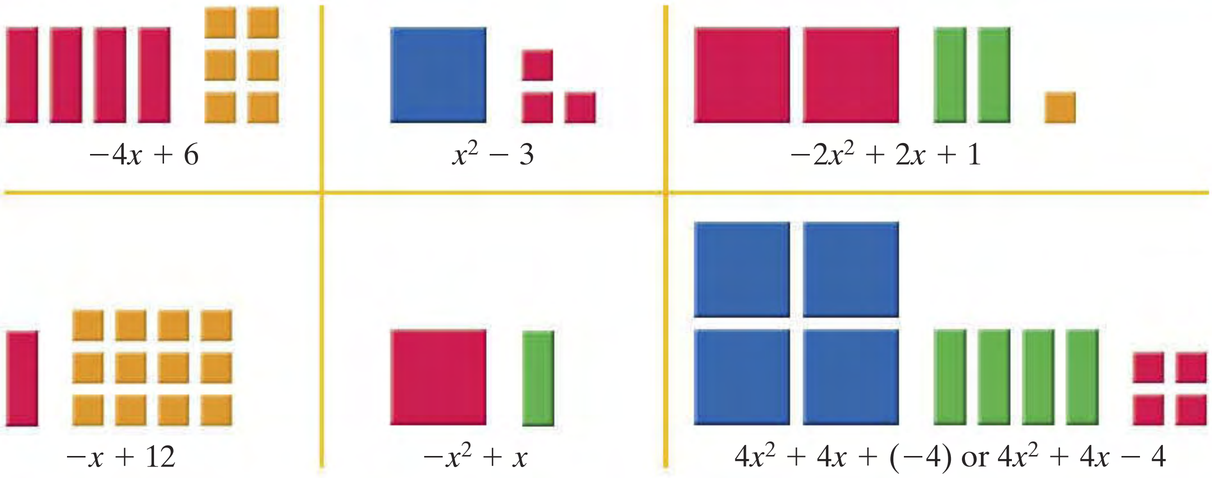
**Objective** To model polynomials using algebra tiles • To simplify polynomials

► Algebra tiles can be used to model polynomials.

The key shows what each tile represents.




Here are some examples of polynomials and their models:

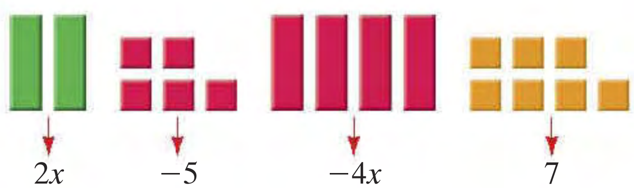



► To write a polynomial in simplest form, it is often helpful to model it first with algebra tiles. Then combine like terms by combining "like tiles."

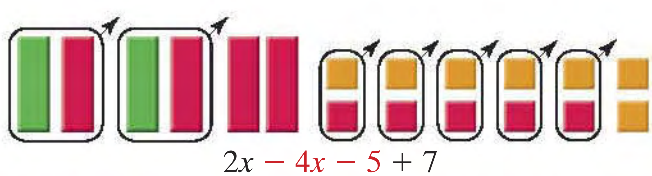
A polynomial is in simplest form when there are no like terms that can be combined.

Use algebra tiles to simplify the polynomial $2x - 5 - 4x + 7$.

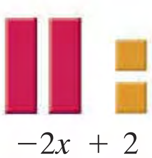
**1** Model the expression with algebra tiles.




**2** Rearrange tiles so that like terms (tiles with the same shape) are next to each other. Then remove the zero pairs.




**3** Write the polynomial for the resulting model. The resulting polynomial is in simplest form.




A number plus its opposite equals 0, so tiles that represent any number and its opposite form a zero pair. Adding 0 to or subtracting 0 from a number does not change the number's value.

So the polynomial $2x - 5 - 4x + 7$ in simplest form is $-2x + 2$.

## Example

**1** Write the polynomial $2x^2 + x - x^2 - 5 + x + 7$ in simplest form.

**1** Model the polynomial with algebra tiles.

$2x^2 + x - x^2 - 5 + x + 7$

**2** Rearrange the tiles so that like terms are next to each other. Then remove the zero pairs.

$2x^2 - x^2 + 2x - 5 + 7$

**3** Write the polynomial for the resulting model.

$x^2 + 2x + 2$

So $2x^2 + x - x^2 - 5 + x + 7$ in simplest form is $x^2 + 2x + 2$.

## Try These

**Model each polynomial using algebra tiles.**

**1.** $3x + 11$

**2.** $-3x^2 + 4x + 2$

**3.** $4x^2 - 2$

**Name the polynomial modeled by the algebra tiles.**

**4.**

**5.**

**Use algebra tiles to simplify each polynomial.**

**6.** $3 - 5x - 2 + x^2$

**7.** $2x^2 - 2x + x - x^2$

**8.** $-5x^2 + x^2 - 4 + x - 3x + 2$

**9.** Write and simplify the polynomial represented by the model.

**10.** **Write and Discuss** Explain how you would simplify the polynomial $-2x^2 + 3x + 1 - 4x$ without using algebra tiles.

Go to PRACTICE BOOK Lesson 14-2 for exercise sets.

Algebra
14-3

# Add Polynomials

**Objective** To model addition of polynomials with algebra tiles • To add polynomials algebraically

Claire's Candles is having a sale: You can buy 2 long-lasting candles for $2 less than the regular price and 4 long-lasting candles for $5 less than the regular price. Suppose you buy 6 candles, and you receive both discounts. What polynomial could you use to represent the total cost of the candles?

To find the polynomial, first write the polynomial that represents the cost when each discount is taken.

**1** Write each addend as a polynomial.
Let $x$ = the price of a single long-lasting candle.

$2x - 2$ ← This polynomial represents the first discount: Buy 2 long-lasting candles for $2 less than the regular price.

$4x - 5$ ← This polynomial represents the second discount: Buy 4 long-lasting candles for $5 less than the regular price.

**2** Model each addend with algebra tiles.

$2x - 2$ $\quad$ $4x - 5$

**3** Rearrange tiles to combine like terms.

$2x + 4x - 5 - 2$

**4** Write the polynomial for the resulting model.

$6x - 7$

So the polynomial $6x - 7$ represents the cost of the 6 candles.

► The properties of operations on whole numbers, integers, and rational numbers are also true for operations on algebraic expressions. In accordance with the Commutative and Associative Properties of Addition, the tiles can be arranged so that like tiles can be grouped together. Steps 3 and 4 above, in which the like terms $2x$ and $4x$ are being combined, involve the Distributive Property of Multiplication over Addition.

$$2x + 4x = x(2 + 4)$$
$$= x \bullet 6 \text{ or } 6x$$

**Remember:**

**Commutative Property of Addition**
$a + b = b + a$

**Associative Property of Addition**
$a + (b + c) = (a + b) + c$

**Distributive Property of Multiplication over Addition**
$a(b + c) = ab + ac$

**Distributive Property of Multiplication over Subtraction**
$a(b - c) = ab - ac$

► You can also add polynomials algebraically, either vertically or horizontally. When finding the sum of two polynomials, write each polynomial in standard form before combining like terms.

What is the sum of $x^2 - 2x$ and $4x - 3x^2$?

**1** Write each polynomial in standard form.

$x^2 - 2x$ ← already in standard form
$4x - 3x^2 \rightarrow -3x^2 + 4x$ ← in standard form

**2** Find the sum of both polynomials using either the horizontal or vertical method.

**Method 1** Compute Horizontally

- Use the Commutative and Associative Properties to group like terms.
- Use the Distributive Property to combine like terms.
- Simplify.

$x^2 - 2x - 3x^2 + 4x$
$(x^2 - 3x^2) + (-2x + 4x)$
$[(1 - 3)x^2] + [(-2 + 4)x]$
$[-2]x^2 + (2)x$
$-2x^2 + 2x$

**Method 2** Compute Vertically

- Align like terms in columns.
- Add the terms in each column separately by adding their coefficients.

$$\begin{array}{rcr} x^2 & + & (-2x) \\ +\ (-3x^2) & + & 4x \\ \hline -2x^2 & + & 2x \end{array}$$

So the sum of $x^2 - 2x$ and $4x - 3x^2$ is $-2x^2 + 2x$.

## Try These

**Find the sum.**

**1.**  and 

**2.**  and 

**Find the sum of the polynomials. Use the horizontal method or the vertical method.**

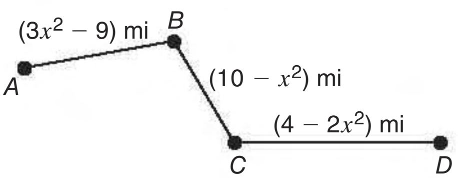
**3.** $3x - 4x^2$ and $2 - 4x$

**4.** $x + x^2$ and $-3x - 5x^2 + 2$

**5.** $2x - 3$ and $4 - 5x^2$ and $x - 4x^2$

**6.** $2x^2 + 3x - 10$ and $-4x + 13 - 3x^2$

**7. Discuss and Write** Jackie is driving from point $A$ to point $D$. What expression represents the distance from point $A$ to point $D$? Write the expression in simplest form. Explain your steps.



Algebra

# 14-4 Subtract Polynomials

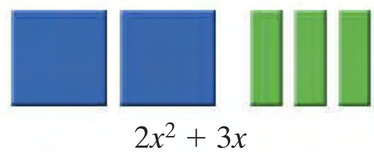
**Objective** To model subtraction of polynomials with algebra tiles • To subtract polynomials algebraically

Duncan is wallpapering one wall of his child's room. He wants to find the area of the paper that will cover the wall. The area of the wall, including the window, is $2x^2 + 3x$. The area of the window is $x^2 + x$. What is the area to be wallpapered written as a polynomial in standard form?

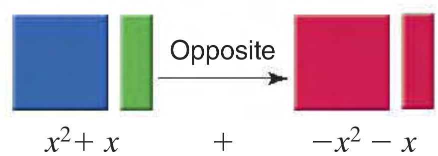
To find the area to be wallpapered, subtract the area of the window from the area of the wall: $(2x^2 + 3x) - (x^2 + x)$

► You can use algebra tiles to model the subtraction of polynomials.

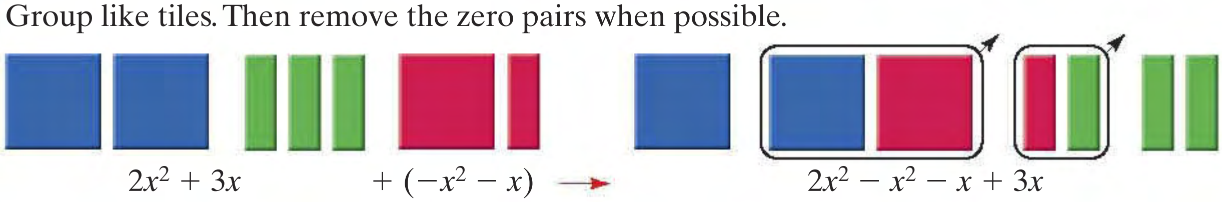
1. Model $2x^2 + 3x$ with algebra tiles.



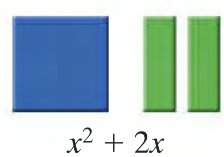

2. Subtract $(x^2 + x)$. To subtract, *add the opposite of* $(x^2 + x)$. Model $x^2 + x$. Then replace each tile with its opposite to model the opposite of $x^2 + x$.




3. Group like tiles. Then remove the zero pairs when possible.




4. Write the polynomial for the resulting model.




So the area to be wallpapered is represented by the polynomial $x^2 + 2x$.

► Notice that in using opposite tiles for $x^2 + x$ in the problem above, you saw that the opposite of $x^2 + x$ is $-x^2 - x$. When a number or expression is multiplied by $-1$, the product is the opposite (additive inverse) of the number or expression. When this property is applied to an expression within parentheses, all the terms within the parentheses are multiplied by $-1$.

**Remember:**
**Multiplicative Property of $-1$**
$(-1) \bullet a = -a$ and $a \bullet (-1) = -a$

**The Opposite of a Sum**
$-(a + b) = -a - b$

**The Opposite of a Difference**
$-(a - b) = -a + b = b - a$

▶ To subtract polynomials algebraically, add the opposite of the polynomial being subtracted.

What is the difference of $19x^2 + 25x$ and $7x^2 - 6x$?

To find the difference, subtract: $(19x^2 + 25x) - (7x^2 - 6x)$
Use either the horizontal or vertical method.

**Remember:**
$a - b = a + (-b)$
$-a = -1 \bullet a$
$a(b - c) = ab - ac$

**Method 1** Compute Horizontally

- To subtract, add the opposite.

$$(19x^2 + 25x) - (7x^2 - 6x)$$
$$(19x^2 + 25x) + [-(7x^2 - 6x)]$$

- The opposite (or additive inverse) of a polynomial is the product of the polynomial and $-1$.

$$(19x^2 + 25x) + (-1)(7x^2 - 6x)$$

- Apply the Distributive Property of Multiplication over Subtraction. Then simplify.

$$(19x^2 + 25x) + [-1 \bullet 7x^2 - (-1 \bullet 6x)]$$
$$(19x^2 + 25x) + [-7x^2 - (-6x)]$$
$$(19x^2 + 25x) + (-7x^2 + 6x)$$

- Apply the Commutative and Associative Properties to group like terms.

$$[19x^2 + (-7x^2)] + [25x + 6x]$$

- Combine like terms.

$$12x^2 + 31x$$

**Method 2** Compute Vertically

- Align like terms in columns.

$$\begin{array}{r} 19x^2 + 25x \\ -\ (7x^2 - \ \ 6x) \\ \hline \end{array}$$

- Add the opposite of each term.

$$\begin{array}{r} 19x^2 + 25x \\ +\ (-7x^2 + \ \ 6x) \\ \hline 12x^2 + 31x \end{array}$$

So the difference of $19x^2 + 25x$ and $7x^2 - 6x$ is $12x^2 + 31x$.

## Try These

**Use algebra tiles to find the difference. Write your answer in standard form.**

**1.** $(x - 5 - x^2) - (2x^2 - 2x)$

**2.** $(3 + 2x^2) - (-x + x^2 - 1)$

**Subtract algebraically. Use the horizontal or vertical method.**

**3.** $(17x + 18) - (22x + 7)$

**4.** $(4x^2 - 4x) - (5x^2 - 5x)$

**5. Discuss and Write** A rectangle has a length of $(7c^2 + 7c + 20)$ units. Its width is $(5c - 25)$ units shorter than its length. What polynomial can express the rectangle's width? Explain your answer.

Go to PRACTICE BOOK Lesson 14-4 for exercise sets.

Algebra 14-5

# Multiply and Divide Monomials

**Objective** To model multiplication and division of monomials with algebra tiles • To apply the Laws of Exponents to multiply and divide monomials algebraically

Angela plans to print a poster in different sizes. She expresses each of the poster's dimensions in the form of a monomial. She expresses the length of the poster as $4x$ and the width as $3x$. What monomial can she write to express the area of the poster?

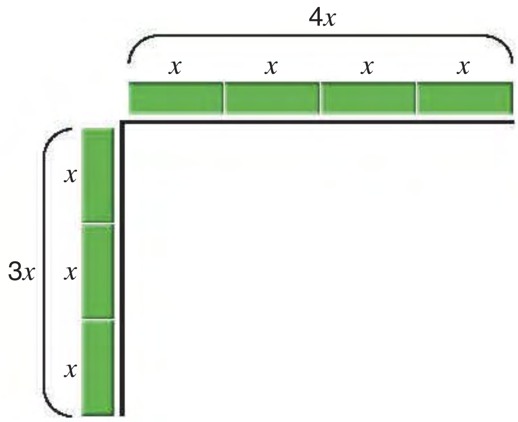
To express the area as a monomial, first substitute $4x$ for $\ell$ and $3x$ for $w$ in the formula for the area of a rectangle.

Then multiply: $A = \ell w = 4x(3x)$

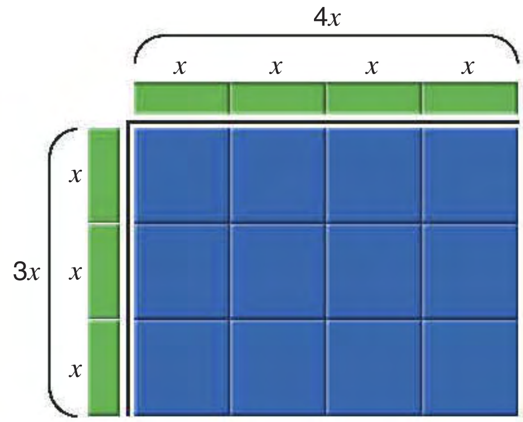
> **Remember:**
> **Area of a Rectangle**
> $A = \ell w$, where $A$ = area, $\ell$ = length, and $w$ = width

► You can use algebra tiles to model the multiplication of monomials.

**1** Place the three $x$ tiles that represent the second monomial factor vertically, and place the four $x$ tiles that represent the first monomial factor horizontally.




**2** Build a rectangle with dimensions $3x$ and $4x$. The area of the rectangle, $12x^2$, represents the product of $3x$ and $4x$.




Since the rectangle contains $12x^2$ tiles, the monomial $12x^2$ expresses the area of the poster.

► You can also multiply two or more monomials algebraically by following these steps:

**1** First, multiply the coefficients.

**2** Then multiply the variables. Be sure to use the appropriate Laws of Exponents.

> **Remember:**
> **Laws of Exponents**
> $a^1 = a$
> $a^m \bullet a^n = a^{(m+n)}$
> $(a^m)^n = a^{mn}$

$4x(3x) = [(4)(3)][(x)(x)]$ ← Apply the Commutative and Associative Properties.

$= 12x^{(1+1)}$ ← Apply the Law of Exponents for Multiplication.

$= 12x^2$ ← Simplify.

So the monomial $12x^2$ expresses the area of Angela's poster.

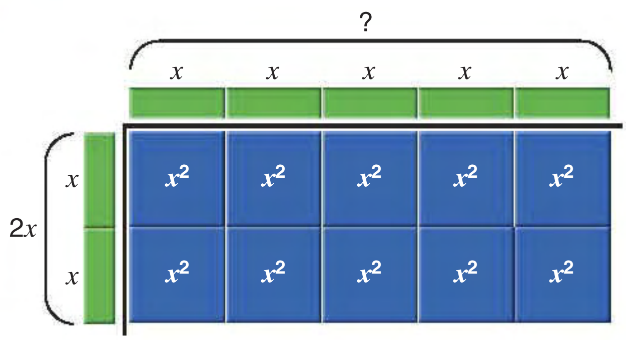
► You can use division of a monomial to find missing dimensions.

Suppose that Angela is working on a different poster. This poster will have an area of $10x^2$ square meters and a width of $2x$ meters. What will the length of this poster be?

To find the length of the poster, divide: $\frac{10x^2}{2x}$

► You can use algebra tiles to model the division of monomials such as $\frac{10x^2}{2x}$.

**1** Form a rectangle with a width of $2x$ using ten $x^2$ tiles.




**2** Identify the length of the rectangle.
$5x$ represents the length.

So $\frac{10x^2}{2x} = 5x$.

**Remember:**
**Law of Exponents for Division**
$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

► To solve the same problem algebraically, you can also divide the two monomials by dividing the coefficients and dividing the variables. Be sure to use the appropriate Laws of Exponents.

$\frac{10x^2}{2x} = \frac{10}{2} \bullet \frac{x^2}{x}$ ← Divide coefficients; divide variables.

$= 5x^{(2-1)}$ ← Apply the Law of Exponents for Division.

$= 5x^1 = 5x$ ← Simplify.

So the length of Angela's poster is $5x$ meters.

## Try These

**Use algebra tiles to multiply or divide.**

**1.** $2x(5x)$   **2.** $6x \bullet 3x$   **3.** $\frac{12x^2}{4x}$   **4.** $\frac{8x^2}{4x}$

**Multiply or divide algebraically.**

**5.** $(-2y)(3y^4)$   **6.** $-m \bullet m^5$   **7.** $\frac{59r^3}{59r}$   **8.** $\frac{100p^4}{10p}$

**9.** **Discuss and Write** Find the monomial to complete the statement $12x^3y^6 = \underline{\ ?\ } \bullet 3x^2y^4$. Explain how you found your answer.

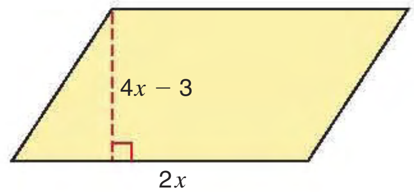
Go to PRACTICE BOOK Lesson 14-5 for exercise sets.

Algebra

# 14-6 Multiply Polynomials by Monomials

**Objective** To model the product of polynomials and monomials with algebra tiles
• To apply the Distributive Property to multiply a polynomial by a monomial

Robert wants to find the area of the parallelogram at the right. It has a base of $2x$ centimeters and a height of $(4x - 3)$ centimeters. What is the area of the parallelogram?

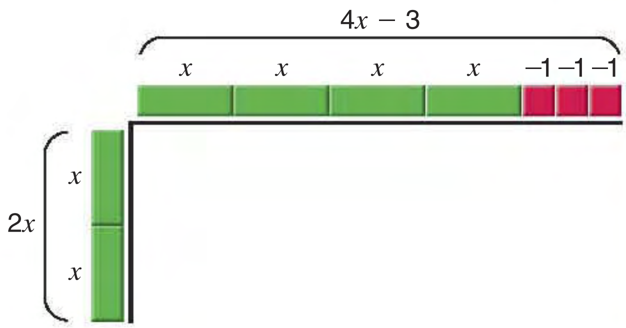



To find the area of the parallelogram, substitute $2x$ for $b$ and $4x - 3$ for $h$ in the formula for the area of a parallelogram. Then multiply: $A = bh = 2x(4x - 3)$

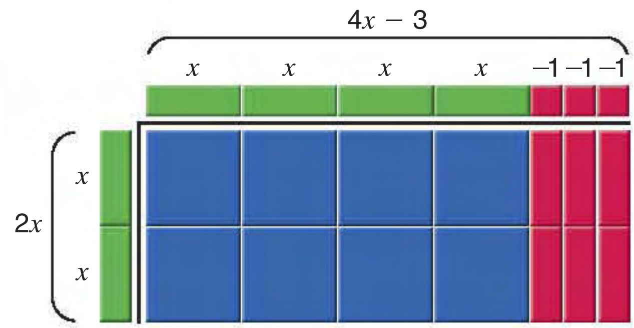
► You can use algebra tiles to model the product of a polynomial and a monomial.

**Remember:**
**Area of a Parallelogram**
$A = bh$, where $A$ = area, $b$ = base, and $h$ = height

**1** Place the tiles for the monomial $2x$ vertically and the tiles for the polynomial $4x - 3$ horizontally.




**2** Build a rectangle using algebra tiles with the dimensions $2x$ and $4x - 3$. The area of the rectangle represents the product of $2x$ and $4x - 3$.




Since the rectangle contains eight $x^2$ tiles and six $-x$ tiles, the area is $(8x^2 - 6x)$ cm$^2$.

► You can also find the product of a polynomial and a monomial algebraically by following these steps:

**1** Apply the Distributive Property to distribute the monomial across the terms of the polynomial.

**2** Then multiply the coefficients and multiply the variables. Use the appropriate Law of Exponents.

$2x(4x - 3) = 2x(4x) - 2x(3)$ ← Apply the Distributive Property of Multiplication over Subtraction.

$= (2 \bullet 4)(x \bullet x) - (2 \bullet 3)x$ ← Using the Associative and Commutative Properties, group coefficients and variables.

$= 8x^{(1 + 1)} - 6x$ ← Apply Law of Exponents for Multiplication.

$= 8x^2 - 6x$ ← Simplify.

So the parallelogram has an area of $(8x^2 - 6x)$ cm$^2$.

▶ The chart below shows some examples of multiplying a polynomial by a monomial.

| Expression | Apply the Distributive Property | Simplify |
|---|---|---|
| $10(3y + 5)$ | $10 \bullet 3y + 10 \bullet 5$ | $30y + 50$ |
| $y(3y - 5)$ | $y \bullet 3y - y \bullet 5$ | $3y^2 - 5y$ |
| $y^2(3y - 5)$ | $y^2 \bullet 3y - y^2 \bullet 5$ | $3y^3 - 5y^2$ |
| $10y^2(3y + 5)$ | $10y^2 \bullet 3y + 10y^2 \bullet 5$ | $30y^3 + 50y^2$ |
| $10y(-3y + 5)$ | $10y(-3y) + 10y \bullet 5$ | $-30y^2 + 50y$ |

## Examples

**1** Multiply: $-6b^2(4 - 5b)$

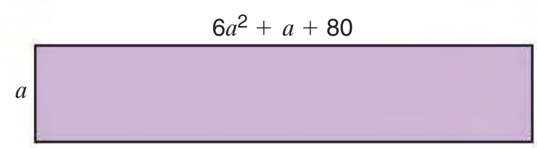
$-6b^2(4 - 5b) = (-6b^2)(4) - (-6b^2)(5b)$ ← Apply the Distributive Property of Multiplication over Subtraction.

$= (-6 \bullet 4)b^2 - (-6 \bullet 5)(b^2 \bullet b)$ ← Using the Associative and Commutative Properties, group coefficients and variables.

$= -24b^2 - (-30) \bullet b^{2+1}$ ← Apply Law of Exponents for Multiplication.

$= -24b^2 + 30b^3$ ← Simplify.

**2** Write a simplified expression for the area of the rectangle.




$A = \ell w$ ← Recall the area formula for a rectangle.

$= (6a^2 + a + 80)a$ ← Substitute the given dimensions into the formula.

$= (6a^2)a + (a)a + (80)a$ ← Apply the Distributive Property of Multiplication over Addition.

$= 6a^{2+1} + a^{1+1} + 80a$ ← Apply the Law of Exponents for Multiplication.

$= 6a^3 + a^2 + 80a$ ← Write in standard form.

## Try These

**Multiply. Write the product in standard form.**

**1.** $7(4 + y)$ **2.** $11a(a^2 - a)$ **3.** $-2c(c + 1)$ **4.** $10m^2(3m - 4)$

**5.** $w^2(4 - w^{10})$ **6.** $j(64 + j^3)$ **7.** $10n(-3 + n^2)$ **8.** $3g(g^2 + 2g + 1)$

**9. Discuss and Write** If you substitute 2 for the variable in exercise 1 and then simplify, what is the value of the given expression? Now substitute 2 for the variable in your answer to exercise 1. Does your answer have the same value? Use this method to verify the answer you found for exercise 2.

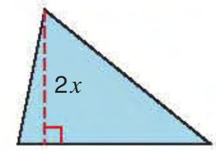
Go to PRACTICE BOOK Lesson 14-6 for exercise sets.

Algebra

# 14-7 Divide Polynomials by Monomials

**Objective** To model division by a monomial with algebra tiles • To apply the Law of Exponents to divide a polynomial by a monomial

The triangle at the right has an area of $(2x^2 + 3x)$ square units and a height of $2x$ units. What is the length of the base?



$A = 2x^2 + 3x$

To find the length of the base, first transform the formula for the area of a triangle as $b = \frac{2A}{h}$, and substitute the given dimensions, $2x^2 + 3x$ for $A$ and $2x$ for $h$:

$$b = \frac{2(2x^2 + 3x)}{2x} = \frac{4x^2 + 6x}{2x}$$

Then to find $b$, divide the polynomial $4x^2 + 6x$ by the monomial $2x$.

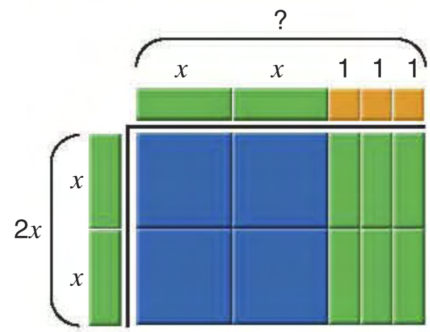
**Remember:**
**Area of a Triangle**
$A = \frac{1}{2}bh$, where $A$ = area, $b$ = base, and $h$ = height

When dividing a polynomial by a monomial, make sure the quotient has the same number of terms as the dividend. In this case, both the dividend and quotient have 2 terms.

► You can use algebra tiles to model the division of polynomials by monomials.

1. To model $(4x^2 + 6x) \div 2x$, start by building a rectangle from four $x^2$ tiles and six $x$ tiles. Be sure the rectangle has a width of $2x$.

2. To find the quotient, find the length of the remaining side. In this case, it is $2x + 3$. So $(4x^2 + 6x) \div 2x = 2x + 3$.



Area of rectangle = $4x^2 + 6x$

► You can also divide a polynomial by a monomial algebraically by following these steps:

**Remember:** $a \div b = \frac{a}{b} = a \bullet \frac{1}{b}$

1. Rewrite the division of a polynomial by a monomial as the *multiplication* of the polynomial by the *reciprocal* of the monomial.
$$\frac{4x^2 + 6x}{2x} = (4x^2 + 6x) \bullet \frac{1}{2x}$$

2. Apply the Distributive Property to distribute the monomial across the terms of the polynomial.
$$= 4x^2 \bullet \frac{1}{2x} + 6x \bullet \frac{1}{2x}$$

3. Simplify if necessary.
$$= \frac{4x^2}{2x} + \frac{6x}{2x}$$

4. Divide the coefficients.
$$= \frac{4}{2}\left(\frac{x^2}{x}\right) + \frac{6}{2}\left(\frac{x}{x}\right)$$

5. Apply the Law of Exponents for Division to divide variables.
$$= 2x^{(2-1)} + 3x^{(1-1)}$$

6. Simplify.
$$= 2x + 3$$

So the length of the base of the triangle is $(2x + 3)$ units.

## Examples

**1** **Divide:** $(4m^3 - 2m^2 + 8m) \div 2m$

$$\frac{4m^3 - 2m^2 + 8m}{2m} = \frac{1}{2m}(4m^3 - 2m^2 + 8m) \leftarrow \text{Rewrite as multiplication.}$$

$$= \frac{4m^3}{2m} - \frac{2m^2}{2m} + \frac{8m}{2m} \leftarrow \text{Apply the Distributive Property.}$$

$$= \frac{4}{2} \bullet \frac{m^3}{m} - \frac{2}{2} \bullet \frac{m^2}{m} + \frac{8}{2} \bullet \frac{m}{m} \leftarrow \text{First divide the coefficients, then divide the variables in each term.}$$

$$= 2m^{3-1} - 1m^{2-1} + 4m^{1-1} \leftarrow \text{Apply the Law of Exponents for Division.}$$

$$= 2m^2 - m + 4 \leftarrow \text{Simplify.}$$

**Remember:**
Divisor • Quotient = Dividend

**Check:** $2m(2m^2 - m + 4) \stackrel{?}{=} 4m^3 - 2m^2 + 8m$

$2m \bullet 2m^2 - 2m \bullet m + 2m \bullet 4 \stackrel{?}{=} 4m^3 - 2m^2 + 8m$

$4m^3 - 2m^2 + 8m = 4m^3 - 2m^2 + 8m$ True

**2** **Divide:** $(9a^4 + 12a^3 - 3a^2) \div 3a^2$

$$\frac{9a^4 + 12a^3 - 3a^2}{3a^2} = \frac{1}{3a^2}(9a^4 + 12a^3 - 3a^2) \leftarrow \text{Rewrite as multiplication.}$$

$$= \frac{9a^4}{3a^2} + \frac{12a^3}{3a^2} - \frac{3a^2}{3a^2} \leftarrow \text{Apply the Distributive Property.}$$

$$= \frac{9}{3} \bullet \frac{a^4}{a^2} + \frac{12}{3} \bullet \frac{a^3}{a^2} - \frac{3}{3} \bullet \frac{a^2}{a^2} \leftarrow \text{First divide the coefficients, then divide the variables in each term.}$$

$$= 3a^{4-2} + 4a^{3-2} - 1a^{2-2} \leftarrow \text{Apply the Law of Exponents for Division.}$$

$$= 3a^2 + 4a - 1 \leftarrow \text{Simplify.}$$

**Check:** $3a^2(3a^2 + 4a - 1) \stackrel{?}{=} 9a^4 + 12a^3 - 3a^2$

$3a^2 \bullet 3a^2 + 3a^2 \bullet 4a - 3a^2 \bullet 1 \stackrel{?}{=} 9a^4 + 12a^3 - 3a^2$

$9a^4 + 12a^3 - 3a^2 = 9a^4 + 12a^3 - 3a^2$ True

## Try These

**Find the quotient, and write it in standard form. Check by multiplication.**

**1.** $(12x - 4) \div 4$

**2.** $(28y^2 + 14y) \div 7y$

**3.** $\frac{4b^2 - 7b}{b}$

**4.** $\frac{18n - 3n + 6n^2}{3n}$

**5.** $(14r + 21r^2) \div 7r$

**6.** $\frac{9a + 18a}{3a}$

**7.** $\frac{14m^2 + 12m + 2m}{-2m}$

**8.** $\frac{-12x^2 + 9x^7 - 3x^5}{-3x}$

**9.** **Discuss and Write** Find the binomial to complete the statement $8a^6 + 64a^3 = \underline{\ ?\ }\,(8a^3)$. Explain how you found your answer.

Go to PRACTICE BOOK Lesson 14-7 for exercise sets.

Algebra

# 14-8 Solve Multistep Equations

**Objective** To solve multistep linear equations • To solve multistep equations with one variable on both sides of the equation

On Monday, Parker Library received 10 boxes, each with the same number of books. Thirteen of the books had been damaged and were sent back. The next day, 2 more boxes with the same number of books were delivered. On Wednesday, 3 more boxes with the same number of books arrived at the library. After the third delivery, the library had received and kept 137 books. How many books were in each box of books?

► To find the number of books in each box, write and solve a multistep equation—an equation that involves two or more operations.

Let $x$ = the number of books in each box.

$$\underbrace{10x}_{\text{number of books in 10 boxes}} - \underbrace{13}_{\text{number of books sent back}} + \underbrace{2x}_{\text{number of books in 2 boxes}} + \underbrace{3x}_{\text{number of books in 3 boxes}} = \underbrace{137}_{\text{number of books delivered and kept}}$$

**Solve:** $10x - 13 + 2x + 3x = 137$

$15x - 13 = 137$ ← Combine like terms.

$15x - 13 + 13 = 137 + 13$ ← Add 13 to both sides.

$15x = 150$ ← Simplify.

$15x \div 15 = 150 \div 15$ ← Divide both sides by 15.

$x = 10$

**Check:** $10x - 13 + 2x + 3x = 137$

$10(10) - 13 + 2(10) + 3(10) \stackrel{?}{=} 137$

$100 - 13 + 20 + 30 \stackrel{?}{=} 137$

$150 - 13 \stackrel{?}{=} 137$

$137 = 137$ True

So there were 10 books in each box.

## Example

**1** Annie has 20 more DVDs than Kent has. Alvin has 20 times as many DVDs as Kent has. The total number of Annie's and Alvin's DVDs divided by 5 is 67. Find $k$, the number of Kent's DVDs.

**Solve:** $\frac{k + 20 + 20k}{5} = 67$ ← Write an equation to represent the situation.

$(5)\left(\frac{k + 20 + 20k}{5}\right) = (67)(5)$ ← Multiply both sides by 5.

$k + 20 + 20k = 335$ ← Simplify.

$(k + 20k) + 20 = 335$ ← Combine like terms.

$21k + 20 - 20 = 335 - 20$ ← Subtract 20 from both sides.

$21k \div 21 = 315 \div 21$ ← Divide both sides by 21.

$k = 15$

**Check:** $\frac{k + 20 + 20k}{5} = 67$

$\frac{15 + 20 + 20(15)}{5} \stackrel{?}{=} 67$

$\frac{335}{5} \stackrel{?}{=} 67$

$67 = 67$ True

So Kent has 15 DVDs.

▶ Some multistep equations require the use of the Distributive Property.

Roxie has 2 fewer bags than her friend Shelly. Callie has 2 more than five times as many bags as Roxie. If Callie has 17 bags, how many bags does Shelly have?

Let $s$ = number of Shelly's bags.

number of Roxie's bags → $(s - 2)$; number of Callie's bags → $17$

$$2 + 5(s - 2) = 17$$

**Solve:** $2 + 5(s - 2) = 17$

$2 + 5(s) - 5(2) = 17$ ← Apply the Distributive Property of Multiplication over Subtraction.

$2 + 5s - 10 = 17$ ← Combine like terms.

$5s - 8 = 17$ ← Simplify.

$5s - 8 + 8 = 17 + 8$ ← Add 8 to both sides.

$5s = 25$ ← Simplify.

$\frac{5s}{5} = \frac{25}{5}$ ← Divide both sides by 5.

$s = 5$ ← Simplify.

**Check:** $2 + 5(s - 2) = 17$

$2 + 5(5 - 2) \stackrel{?}{=} 17$ ← Substitute.

$2 + 5(3) \stackrel{?}{=} 17$ ← Simplify.

$2 + 15 \stackrel{?}{=} 17$ ← Multiply, then add.

$17 = 17$ True

So Shelly has 5 bags.

▶ Sometimes multistep equations have the variable on both sides of the equation.

**Solve:** $50 + 0.1c = 10 + 0.5c$

$50 + 0.1c - 0.1c = 10 + 0.5c - 0.1c$ ← Subtract $0.1c$ from both sides.

$50 = 10 + 0.4c$ ← Simplify.

$50 - 10 = 10 - 10 + 0.4c$ ← Subtract 10 from both sides.

$40 = 0.4c$ ← Simplify.

$\frac{40}{0.4} = \frac{0.4c}{0.4}$ ← Divide both sides by 0.4.

$100 = c$ ← Simplify.

**Check:** $50 + 0.1c = 10 + 0.5c$

$50 + 0.1(100) \stackrel{?}{=} 10 + 0.5(100)$ ← Substitute 100 for $c$.

$50 + 10 \stackrel{?}{=} 10 + 50$ ← Simplify.

$60 = 60$ True

So, in the equation $50 + 0.1c = 10 + 0.5c$, the value of $c$ is 100.

## Try These

**Solve. Check to justify your answer.**

1. $-z + 6z - 10 = 15$
2. $-6(4 + 7x) = -2(3 + 30x)$
3. $\frac{10n + 20}{20} = -12$
4. **Discuss and Write** Your classmate began solving exercise 2 by dividing both sides by $-6$. You began by distributing the $-6$ and the $-2$ over the expressions in parentheses. Are both methods correct? Explain why or why not.

Go to PRACTICE BOOK Lesson 14-8 for exercise sets.

Algebra

14-9

# Addition and Subtraction: Inequalities with Rational Numbers

**Objective** To solve one-step inequalities using the Addition and Subtraction Properties of Inequality • To graph the solution sets of inequalities involving rational numbers

A water ride at the amusement park limits each ride to a maximum mass of 527.25 kg. The "boat" itself and its equipment have a mass of 154.55 kg. What is the maximum combined mass of passengers allowed for each ride?

To determine the maximum combined mass of passengers, write and solve an inequality.

Let $m$ = the passengers' maximum mass.

| Passengers' maximum mass | plus | Mass of boat and equipment | is at most | Boat's maximum mass |
|---|---|---|---|---|
| $m$ | $+$ | 154.55 | $\leq$ | 527.25 |

► Solve inequalities with rational numbers the same way that you solve inequalities with integers. Remember to use the properties of inequalities.

**Remember:**

**Addition Property of Inequality**
For any real numbers $a$, $b$, and $c$, if $a < b$, then $a + c < b + c$.

**Subtraction Property of Inequality**
For any real numbers $a$, $b$, and $c$, if $a > b$, then $a - c > b - c$.

**Solve:** $m + 154.55 \leq 527.25$

$m + 154.55 - 154.55 \leq 527.25 - 154.55$ ← Subtract 154.55 from both sides.

$m \leq 372.7$

To check, choose two numbers from the solution set, and substitute each into the inequality.

**Check:** Try $m = 372.7$.

$372.7 + 154.55 \stackrel{?}{\leq} 527.25$ ← Substitute 372.7 for $m$.

$527.25 \leq 527.25$ True

Try $m = 300$.

$300 + 154.55 \stackrel{?}{\leq} 527.25$ ← Substitute 300 for $m$.

$454.55 \leq 527.25$ True

So the passengers' maximum mass for each ride is at most 372.7 kg.

## Example

**1** **Solve:** $a - 235.6 > 1200.7$

$a - 235.6 + 235.6 > 1200.7 + 235.6$ ← Add 235.6 to both sides.

$a > 1436.3$

**Check:** Try $a = 1436.3$.

$1436.3 - 235.6 \stackrel{?}{>} 1200.7$ ← Substitute 1436.3 for $a$.

$1200.7 > 1200.7$ False

Try $a = 1535.6$.

$1535.6 - 235.6 \stackrel{?}{>} 1200.7$ ← Substitute 1535.6 for $a$.

$1300 > 1200.7$ True

So the solution set is all numbers greater than 1436.3.

As with integers, you can graph the solution set of an inequality with rational numbers on a real-number line.

**Solve:** $x - \frac{1}{2} \geq 4\frac{3}{5}$

$x - \frac{1}{2} + \frac{1}{2} \geq 4\frac{3}{5} + \frac{1}{2}$ ← Add $\frac{1}{2}$ to both sides.

$x \geq 4\frac{6}{10} + \frac{5}{10}$ ← Rename using the LCD.

$x \geq 5\frac{1}{10}$ ← Simplify.

**Graph:**


According to the graph, 5 is *not* a solution, and $5\frac{1}{10}$ *is* a solution.

**Remember:**
When graphing an inequality, use a dot to show that a number *is* in the solution set. Use a circle to show that a number is *not* in the solution set.

**Check:** Try $x = 5$.

$5 - \frac{1}{2} \overset{?}{\geq} 4\frac{3}{5}$ ← Substitute 5 for $x$. Simplify.

$4\frac{1}{2} \geq 4\frac{3}{5}$ False

Try $x = 5\frac{1}{10}$.

$5\frac{1}{10} - \frac{1}{2} \overset{?}{\geq} 4\frac{3}{5}$ ← Substitute $5\frac{1}{10}$ for $x$.

$5\frac{1}{10} - \frac{5}{10} \overset{?}{\geq} 4\frac{6}{10}$

$4\frac{6}{10} \overset{?}{\geq} 4\frac{6}{10}$ ← Simplify.

$4\frac{3}{5} \geq 4\frac{3}{5}$ True

So the solution of $x - \frac{1}{2} \geq 4\frac{3}{5}$ is $x \geq 5\frac{1}{10}$.

Sometimes when solving an inequality, you may need to combine like terms.

**Solve:** $8.1 - c + 2c < 10.2$

$8.1 + (-1 + 2)c < 10.2$ ← Use the Distributive Property to combine like terms.

$8.1 + c < 10.2$ ← Simplify.

$8.1 - 8.1 + c < 10.2 - 8.1$ ← Subtract 8.1 from both sides.

$c < 2.1$

**Graph:**


**Check:** Try $c = 2$.

$8.1 - 2 + 2(2) \overset{?}{<} 10.2$ ← Substitute 2 for $c$.

$8.1 - 2 + 4 \overset{?}{<} 10.2$ ← Simplify.

$10.1 < 10.2$ True

Try $c = 2.1$

$8.1 - 2.1 + 2(2.1) \overset{?}{<} 10.2$ ← Substitute 2.1 for $c$.

$8.1 - 2.1 + 4.2 \overset{?}{<} 10.2$ ← Simplify.

$10.2 < 10.2$ False

So the solution set is all numbers less than 2.1.

## Try These

**Solve the inequality and graph the solution. Check to justify your answer.**

**1.** $r - 17.4 \geq 20$

**2.** $8\frac{1}{4} > 2t - t + \frac{1}{2}$

**3.** $x + 99.9 < 151.2$

**4.** $y + 11 \leq -2\frac{1}{5}$

**5. Discuss and Write** For $e \geq \frac{2}{3}$, is $e = 0.7$ a solution? Is $e = \frac{5}{6}$ a solution? What about $e = 0.67$? In your own words, describe the meaning of $e \geq \frac{2}{3}$.

Go to PRACTICE BOOK Lesson 14-9 for exercise sets.

Algebra
# 14-10 Multiplication and Division: Inequalities with Rational Numbers

**Objective** To solve one- and two-step inequalities involving rational numbers using the Multiplication and Division Properties of Inequality • To graph the solution sets of inequalities involving rational numbers

Problem-Solving Strategy 10

The giraffe at the local zoo is at least twice as tall as the zoo's African elephant. The elephant is 1.85 meters taller than the zoo's wildebeest. If the giraffe is 6 meters tall, how tall is the wildebeest?

To find the wildebeest's height, write and solve an inequality.

Let $h$ = the wildebeest's height.

| Giraffe's height | is at least | two | times | Elephant's height |
|---|---|---|---|---|
| $6$ | $\geq$ | $2$ | $\bullet$ | $(h + 1.85)$ |

► Multiplication inequalities with rational numbers can be solved the same way as inequalities with integers.

**Solve:** $6 \geq 2(h + 1.85)$

$6 \geq 2h + 3.7$ ← Apply the Distributive Property.

$6 - 3.7 \geq 2h + 3.7 - 3.7$ ← Subtract 3.7 from both sides.

$2.3 \geq 2h$ ← Simplify.

$\frac{2.3}{2} \geq \frac{2h}{2}$ ← Divide both sides by 2 to isolate $h$.

$1.15 \geq h$ ← Simplify.

**Graph:**


**Remember:**

**Multiplication Property of Inequality**

For any real numbers $a$, $b$, and $c$, where $c$ is positive, if $a < b$, then $ac < bc$.

For any real numbers $a$, $b$, and $c$, where $c$ is negative, if $a < b$, then $ac > bc$.

**Division Property of Inequality**

For any real numbers $a$, $b$, and $c$, where $c$ is positive, if $a < b$, then $a \div c < b \div c$.

For any real numbers $a$, $b$, and $c$, where $c$ is negative, if $a < b$, then $a \div c > b \div c$.

Similar statements can be written for $a > b$, $a \leq b$, and $a \geq b$ for both properties.

**Check:** Try $h = 1.15$.

$6 \overset{?}{\geq} 2(1.15 + 1.85)$ ← Substitute 1.15 for $h$.

$6 \overset{?}{\geq} 2(3)$ ← Simplify.

$6 \geq 6$ True

Try $h = 0.85$.

$6 \overset{?}{\geq} 2(0.85 + 1.85)$ ← Substitute 0.85 for $h$.

$6 \overset{?}{\geq} 2(2.7)$ ← Simplify.

$6 \geq 5.4$ True

The zoo's wildebeest is no more than 1.15 meters tall.

## Example

**1** Solve, graph, and check your solution to the inequality $-\frac{5}{12}t \le 8\frac{1}{3}$.

**Solve:** $-\frac{5}{12}t \le 8\frac{1}{3}$

$-\frac{5}{12}t \div \left(-\frac{5}{12}\right) \ge 8\frac{1}{3} \div \left(-\frac{5}{12}\right)$ ← Divide both sides by $-\frac{5}{12}$, and reverse the inequality symbol.

$t \ge -20$

**Graph:**


According to the graph, −24 is *not* a solution, and −20 *is* a solution.

**Check:** Try $t = -24$.

$-\frac{5}{12}t \le 8\frac{1}{3}$

$-\frac{5}{12}(-24) \overset{?}{\le} 8\frac{1}{3}$ ← Substitute −24 for *t*.

$10 \le 8\frac{1}{3}$ False

Try $t = -20$.

$-\frac{5}{12}t \le 8\frac{1}{3}$

$-\frac{5}{12}(-20) \overset{?}{\le} 8\frac{1}{3}$ ← Substitute −20 for *t*.

$8\frac{1}{3} \le 8\frac{1}{3}$ True

So the solution set is all numbers greater than or equal to −20.

► When solving a multiplication or division inequality involving rational numbers, you may need to combine like terms.

**Solve**: $130 > -2.5x - 6.5 + 0.75x$

$130 > (-2.5 + 0.75)x - 6.5$ ← Apply the Commutative and Distributive Properties.

$130 + 6.5 > -1.75x - 6.5 + 6.5$ ← Simplify. Add 6.5 to both sides.

$\frac{136.5}{-1.75} < \frac{-1.75x}{-1.75}$ ← Divide both sides by −1.75 to isolate *x*, and reverse the inequality symbol.

$-78 < x$

**Check:** Try $t = -78$.

$130 \overset{?}{>} -2.5(-78) - 6.5 + 0.75(-78)$ ← Substitute −78 for *x*.

$130 \overset{?}{>} 195 - 6.5 + (-58.5)$ ← Simplify.

$130 > 130$ False

Try $t = -70$.

$130 \overset{?}{>} -2.5(-70) - 6.5 + 0.75(-70)$ ← Substitute −70 for *x*.

$130 \overset{?}{>} 175 - 6.5 + (-52.5)$ ← Simplify.

$130 > 116$ True

So the solution set is all numbers greater than −78.

## Try These

**Solve the inequality and graph its solution. Check to justify your answer.**

**1.** $\frac{a}{3.8} \le 45$ **2.** $9\frac{1}{5} < 2\frac{3}{4}z + \left(-1\frac{2}{3}z\right) + 4$ **3.** $-0.77k < -23.254$ **4.** $\frac{7}{8} \le \frac{y}{\frac{1}{-2}}$

**5. Discuss and Write** Describe how solving a multistep equation and solving a multistep inequality are similar. How are the processes different? Explain the reason for the differences.

Go to PRACTICE BOOK Lesson 14-10 for exercise sets.

# 14-11 Problem Solving: Review of Strategies

Read Plan Solve Check

**Objective** To solve problems using a variety of strategies

**Problem:** Fred and Phil wish to open a business together, so they pool their resources. When they combine their money, they have a total of \$4000. If the amount Phil contributed is subtracted from the amount Fred contributed, the result is \$680. How much money did each contribute to their \$4000 pool of funds?

**Read to understand what is being asked.**
List the facts and restate the question.

**Facts:** Fred and Phil contributed a total of \$4000.
Fred contributed \$680 more than Phil.

**Question:** How much money did Fred contribute?
How much did Phil contribute?

**Select a strategy.**
There are many ways to solve this problem. You can use the *Make a Drawing* strategy to help you understand the relationships. You could also use the *Guess and Test* strategy.

**Problem-Solving Strategies**

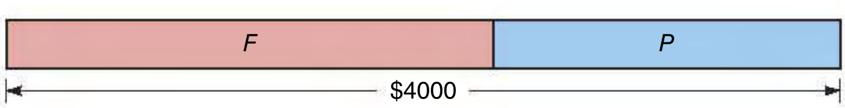
1. Guess and Test
2. Organize Data
3. Find a Pattern
4. Make a Drawing
5. Solve a Simpler Problem
6. Reason Logically
7. Adopt a Different Point of View
8. Account for All Possibilities
9. Work Backward
10. Consider Extreme Cases

**Apply the strategy.**

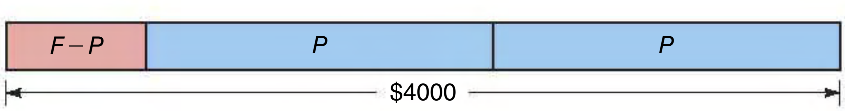
### ▶ Method 1: Make a Drawing

Let $F$ represent the amount Fred contributed, and let $P$ represent the amount Phil contributed.

- Draw rectangles $F$ and $P$, as shown in the figure below, with $F$ longer than $P$ because $F$ is greater than $P$. The total is \$4000.




- Make a copy of rectangle $P$, and place it so its right end is exactly on top of the right end of length $F$. The uncovered portion of $F$ has length $F - P$.



- Since $F - P$ represents \$680, relabel the diagram as shown below.


From this diagram, you can see that $2P + \$680 = \$4000$. Now solve for $P$.

$$2P + \$680 = \$4000$$

$$2P + \$680 - \$680 = \$4000 - \$680$$ Subtract \$680 from both sides.

$$2P = \$3320$$ Simplify.

$$\frac{2P}{2} = \frac{\$3320}{2}$$ Divide both sides by 2.

$$P = \$1660$$ Simplify.

So Phil contributed \$1660, and Fred contributed the remaining \$4000 − \$1660, or \$2340.

### ▶ Method 2: Guess and Test

The problem tells us that Fred contributed \$680 more than Phil.

You can guess how much Phil contributed and then add \$680 to your guess to get the corresponding amount for Fred.

Add the two amounts to see if the total is \$4000.
If not, adjust your guess and try again.
Keep track of your guesses in a table.

The amount Phil contributed must be more than \$1000 because \$1000 + \$1680 is clearly less than \$4000.
Also, it must be less than \$2000 because \$2000 + \$2680 is clearly more than \$4000. You might try \$1500 as a first guess and then go on from there.

| Phil's Amount | Fred's Amount | Total | Comment |
|---|---|---|---|
| \$1500 | \$2180 | \$3680 | Not enough |
| \$1700 | \$2380 | \$4080 | Too much |
| \$1650 | \$2330 | \$3980 | Not enough, but close |
| \$1660 | \$2340 | \$4000 | Correct! |

Phil contributed \$1660, and Fred contributed \$2340.

**Check to make sure your answer makes sense.**
Do the two amounts total \$4000?
Yes, \$1660 + \$2340 = \$4000 ✓
Is Fred's amount \$680 more than Phil's?
Yes, \$2340 − \$1660 = \$680 ✓

Go to PRACTICE BOOK Lesson 14-11 for exercise sets.

# Enrichment:
## Graphing with Absolute Values

**Objective** **To make a table of integer values of $x$ for graphing absolute-value functions and to describe how the graphs change as the functions vary**

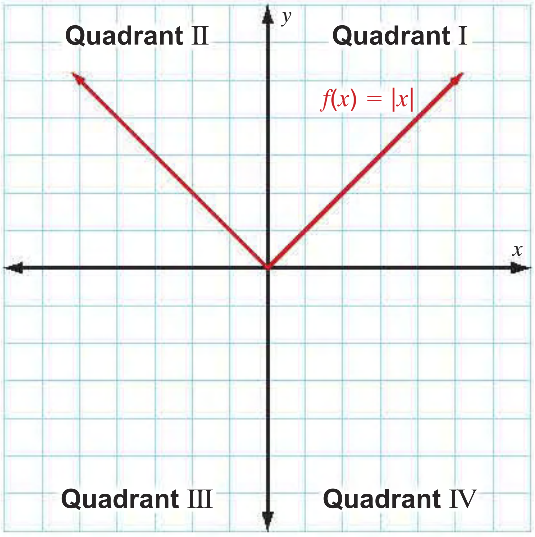
You have found absolute values of numbers and simplified absolute value expressions. Now you will extend your understanding of absolute value to graph absolute-value functions.

► The graphs of absolute-value functions are all related to the graph of the absolute-value function: $f(x) = |x|$. The $f(x)$ notation is another way to represent the $y$-values of $x$.

1 To graph $f(x) = |x|$, first make a table of values. You may use the values in the table below or choose others.

| $f(x) = \lvert x \rvert$ | |
|---|---|
| Input $x$ | Output $f(x)$ |
| 4 | $\lvert 4 \rvert = 4$ |
| 2 | $\lvert 2 \rvert = 2$ |
| 0 | $\lvert 0 \rvert = 0$ |
| −2 | $\lvert -2 \rvert = 2$ |
| −4 | $\lvert -4 \rvert = 4$ |

2 Plot the values from your table. Use the $f(x)$ values as the $y$-coordinates. Then connect the points. The graph below uses the table of values above.

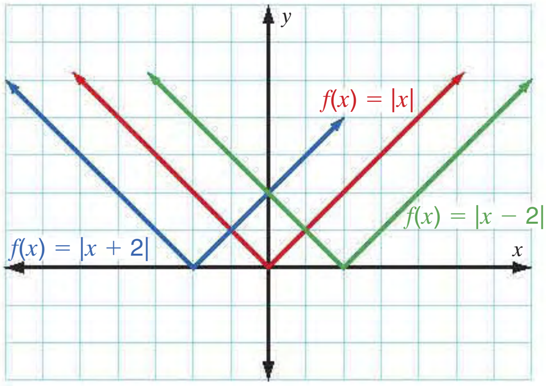



Because $|x|$ is always greater than or equal to zero for all values of $x$, there are no points in Quadrant III or IV. Notice that the part of the graph in Quadrant II is the reflection image of the part of the graph in Quadrant I. This is because $|x| = |-x|$ for any number $x$. That is, the absolute value of a number is the same as the absolute value of its opposite.

- Compare the graphs of $f(x) = |x + 2|$ and $f(x) = |x - 2|$ with the graph of $f(x) = |x|$. Make a table of values for each and then plot the points on the same axes as your graph of $f(x) = |x|$. Connect the points for each function to get the graphs shown below.

| $f(x) = \|x + 2\|$ | |
|---|---|
| Input $x$ | Output $f(x)$ |
| 4 | 6 |
| 2 | 4 |
| 0 | 2 |
| −2 | 0 |
| −4 | 2 |

| $f(x) = \|x - 2\|$ | |
|---|---|
| Input $x$ | Output $f(x)$ |
| 4 | 2 |
| 2 | 0 |
| 0 | 2 |
| −2 | 4 |

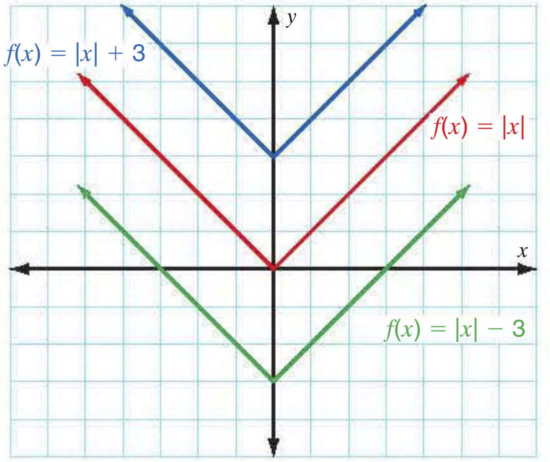



All three graphs have the same shape, but they have different vertices.
The graph of $f(x) = |x + 2|$ is the graph of $f(x) = |x|$ translated left 2 units.
The graph of $f(x) = |x - 2|$ is the graph of $f(x) = |x|$ translated right 2 units.

- Compare the graphs of $f(x) = |x| + 3$ and $f(x) = |x| - 3$ with the graph of $f(x) = |x|$. Make a table of values for each and then plot the points on the same axes as the graph of $f(x) = |x|$.

| $f(x) = \|x\| + 3$ | |
|---|---|
| Input $x$ | Output $f(x)$ |
| 2 | 5 |
| 1 | 4 |
| 0 | 3 |
| −1 | 4 |
| −2 | 5 |

| $f(x) = \|x\| - 3$ | |
|---|---|
| Input $x$ | Output $f(x)$ |
| 6 | 3 |
| 3 | 0 |
| 0 | −3 |
| −3 | 0 |




As before, the graphs have the same shape but different vertices. The graph of $f(x) = |x| + 3$ is the graph of $f(x) = |x|$ translated up 3 units. The graph of $f(x) = |x| - 3$ is the graph of $f(x) = |x|$ translated down 3 units.

## Try These

1. Graph the absolute-value functions $f(x) = |x| + 2$ and $f(x) = |x| - 2$. Explain how their graphs are related to the graph of $f(x) = |x|$.

2. Graph the function $f(x) = -|x|$. Compare this graph to the graph of $f(x) = |x|$.

3. **Discuss and Write** If $c > 0$, describe how the graphs of $f(x) = |x + c|$ and $f(x) = |x| + c$ compare to the graph of $f(x) = |x|$.

Go to PRACTICE BOOK pages 455–456 for exercise sets.

# Test Prep: Short-Answer Questions

## Strategy: Show All Your Work

One way to show all your work when answering short-answer questions is to *justify your steps*. You should state the rules and properties you used to solve the problem.

**Sample Test Item**

Simplify the expression shown below.

$-2x^3(8x^2 + x + 5)$

***Show all your work.*** Explain how you used properties and rules to help you simplify the expression.

**Read the whole test item carefully.**

- Reread the test item carefully.
- Show each step in the solution.
  1. Use the Distributive Property.
  2. Use properties of multiplication and the Laws of Exponents to simplify the terms.

**Test-Taking Tips**

- Reread the item.
- Use the Test-Prep strategy.
- Apply appropriate rules, definitions, properties, or strategies.
- Analyze your answers.

**Solve the problem.**

- Apply appropriate rules, properties, and definitions.

$-2x^3(8x^2 + x + 5) = (-2x^3 \bullet 8x^2) + (-2x^3 \bullet x) + (-2x^3 \bullet 5)$ ← Distributive Property

$= (-2 \bullet 8 \bullet x^3 \bullet x^2) + (-2 \bullet x^3 \bullet x) + (-2 \bullet 5 \bullet x^3)$ ← Commutative Property of Multiplication

$= -16x^5 - 2x^4 - 10x^3$ ← Multiply using rules for signed numbers and the Laws of Exponents.

**Answer:** $-16x^5 - 2x^4 - 10x^3$

To simplify the expression, first use the Distributive Property by multiplying each term within the parentheses by the monomial $2x^3$. To multiply, use the Commutative Property of Multiplication, then apply the rules for signed numbers and the Laws of Exponents.

**Item Analysis**

**Check your work. Review your notes.**
**Make sure you have completed all parts of the item.**

- Analyze your answers. Do they make sense?
  Check by working backwards.
  Find the GCF of the terms and factor the polynomial.

$16x^5 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet x \bullet x \bullet x \bullet x \bullet x$ $\quad 2x^4 = 2 \bullet x \bullet x \bullet x \bullet x$ $\quad 10x^3 = 2 \bullet 5 \bullet x \bullet x \bullet x$

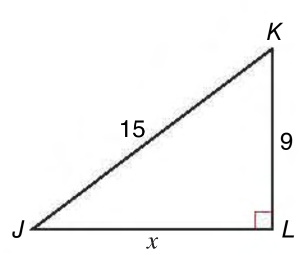
$GCF = 2 \bullet x \bullet x \bullet x = 2x^3$

So $-16x^5 - 2x^4 - 10x^3 = -2x^3(8x^2 + x + 5)$ ✓

**Try These**

**Solve. Justify your steps.**

1. Triangle $JKL$ is a right triangle.
   What is the length of side $x$?
   Explain how you solved the problem.




not drawn to scale

Go to PRACTICE BOOK page 457 for exercise sets.

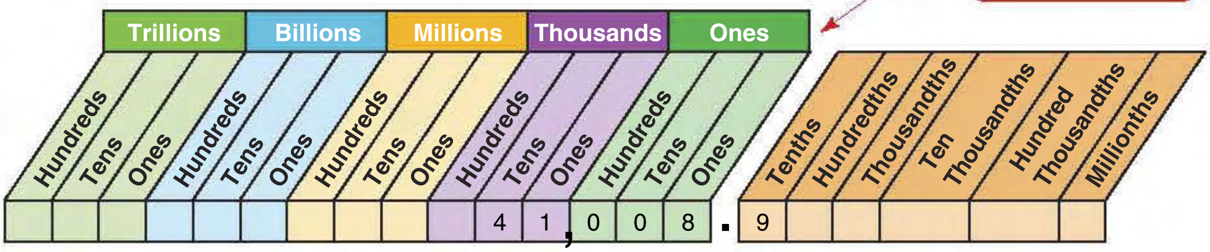
# Skills Update

**A review of prerequisite skills necessary to understand the skills and concepts of *Fundamentals of Algebra***

## I. Place Value

► The digits and the position of each digit in a number determine the value of a number. A place-value chart shows the place value of each digit and helps you read and write numbers. The place value of each place is 10 times greater than the place immediately to its right.

Groups of three digits, separated by a comma, are called *periods*.




The 4 is in the ten-thousands place and has a value of 4 × 10,000, or 40,000. The number 41,008.9 is read "forty-one thousand eight and nine tenths." To read a decimal, say the name of the last place to the right.

## II. Compare and Order Whole Numbers

► You can **compare whole numbers** by comparing the digits in each place-value position. Write one number under another so the digits are aligned by their place values. Start at the left and check each place until the digits are different.

8,532,314
8,539,417

**Think**
8 = 8
5 = 5
3 = 3
9 > 2
So 8,539,417 is greater than 8,532,314.

► You can **order numbers** from least to greatest or from greatest to least by aligning and comparing them in the same way.

**Remember:**
< means "is less than."
> means "is greater than."

Order from greatest to least:
1,657,945; 1,657,948; 658,299; 1,659,816.

| | | |
|---|---|---|
| 1,657,945 | 1,657,945 | 1,657,945 |
| 1,657,948 | 1,657,948 | 1,657,948 |
| 658,299 | 658,299 | 658,299 |
| 1,659,816 | 1,659,816 | 1,659,816 |
| No millions. 658,299 is least. | 6 = 6 and 5 = 5<br>9 > 7<br>So 1,659,816 is greatest. | 9 = 9 and 4 = 4<br>8 > 5<br>So 1,657,948 > 1,657,945. |

So the order from greatest to least is 1,659,816; 1,657,948; 1,657,945; 658,299.

## III. Round Whole Numbers and Decimals

► To **round a number:**

- Find the digit in the place to which you are rounding.
- Look at the next digit on the right.
  If that digit is *less than* 5, then the first digit does not change.
  If that digit is 5 *or greater*, then round the first digit up.
- When rounding whole numbers, replace all digits to the right of the place to which you are rounding with zeros.

Round 8,437 to the nearest hundred.

**Think** 3 < 5, so the 4 in the hundreds place does not change.

8,437 ↓ 8,400

Round 129,502 to the nearest thousand.

**Think** 5 ≥ 5, so the 29 thousands rounds up to 30 thousands.

129,502 ↓ 130,000

When rounding decimals, drop all digits to the right of the place to which you are rounding.

Round 0.3628 to the nearest thousandth.

**Think** 8 > 5, so the 2 in the thousandths place rounds to 3 thousandths.

0.3628 ↓ 0.363

Round 0.96779 to its greatest nonzero place.

**Think** 6 > 5, so 9 tenths rounds up to 10 tenths. Regroup as 1.

0.96779 ↓ 1.0

## IV. Compare and Order Decimals

► **Compare decimals** the same way you compare whole numbers. Align digits by place value. Start at the left. Check each place until the digits are different.

Compare 6.2 and 6.17.

6.20
6.17

6.2 = 6.20

6.20
6.17

**Think** 6 = 6 and 0.2 > 0.1

So 6.2 is greater than 6.17.

► You can order decimals from least to greatest or from greatest to least the same way you order whole numbers. Align digits by their place values. Start at the left and check each place until the digits are different.
Order from least to greatest: 9.631; 9.615; 8.92.

| | | | |
|---|---|---|---|
| 9.631 | 9.631 | 9.631 | 9.631 |
| 9.615 | 9.615 | 9.615 | 9.615 |
| 8.920 | 8.920 | 8.920 | 8.920 |
| 8.92 = 8.920 | 8 < 9<br>8.92 is the least. | 6 = 6<br>Look at the next digit. | 1 < 3<br>9.615 < 9.631 |

So the order from least to greatest is 8.92; 9.615; 9.631.

## V. Estimate Sums and Differences

► You can use rounding or front-end estimation to estimate sums and differences.

- You can **estimate a sum or difference by rounding**:
  Round each number to the greatest nonzero place of the least number.
  Add or subtract the rounded numbers.

Add:

$$\begin{array}{rcr} 6917 & \rightarrow & 6920 \\ 78 & \rightarrow & 80 \\ +\ 434 & \rightarrow & +\ 430 \\ \hline & \text{about} & 7430 \end{array} \qquad \begin{array}{rcr} 1.82 & \rightarrow & 1.8 \\ 0.29 & \rightarrow & 0.3 \\ +\ 0.36 & \rightarrow & +\ 0.4 \\ \hline & \text{about} & 2.5 \end{array}$$

Subtract:

$$\begin{array}{rcr} 5931 & \rightarrow & 5900 \\ -\ 723 & \rightarrow & -\ 700 \\ \hline & \text{about} & 5200 \end{array} \qquad \begin{array}{rcr} 0.087 & \rightarrow & 0.09 \\ -\ 0.058 & \rightarrow & -\ 0.06 \\ \hline & \text{about} & 0.03 \end{array}$$

- You can **estimate a sum or difference using front-end estimation**.
  Add or subtract the front digits of the numbers with the greatest place value. Write zeros for the other digits. Adjust the addition estimate with the back digits.

$$\begin{array}{rr} & 4164 \\ & 987 \\ & +\ 3895 \\ \hline \text{Front-digit sum} \rightarrow & 7000 \\ & +\ 2000 \\ \hline \text{Adjusted estimate} \rightarrow & 9000 \end{array} \qquad \begin{array}{r} 1.82 \\ 0.29 \\ +\ 0.36 \\ \hline 1 \\ +\ 1 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 9561 \\ -\ 742 \\ \hline \text{about } 9000 \end{array} \qquad \begin{array}{r} 0.47 \\ -\ 0.13 \\ \hline \text{about } 0.3 \end{array}$$

## VI. Add and Subtract Whole Numbers and Decimals

► To **add and subtract whole numbers and decimals**, estimate or check to make sure that your answer is reasonable.

- Add: 3,165,892 + 2,096,089
  Use front digits to estimate the sum as 5,000,000. Align digits by place value.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ 1\ 1\ \ } \\ 3{,}165{,}892 \\ +\ 2{,}096{,}089 \\ \hline 5{,}261{,}981 \end{array}$$

  The answer is close to the estimate.

- Add: 5.0953 + 3.0107
  Use front digits to estimate the sum as 8. Align the digits by place value.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 5.0953 \\ +\ 3.0107 \\ \hline 8.106 \end{array} \leftarrow \boxed{8.1060 = 8.106}$$

  The answer is close to the estimate.

- Subtract: 8,309,000 − 777,625
  Align digits by place value. Subtract.

$$\begin{array}{r} 8{,}309{,}000 \\ -\ 777{,}625 \\ \hline 7{,}531{,}375 \end{array}$$

  Add to check:
  7,531,375 + 777,625 = 8,309,000 ✓

- Subtract: 3 − 0.7185
  Align digits by place value. Subtract.

$$\begin{array}{r} 3.0000 \\ -\ 0.7185 \\ \hline 2.2815 \end{array} \leftarrow \boxed{3 = 3.0000}$$

  Add to check:
  2.2815 + 0.7185 = 3.0 ✓

## VII. Multiplication Patterns

You can use patterns to **multiply by powers and multiples of 10**.

► To multiply a whole number by a power or multiple of 10:

- Multiply the nonzero digits in the factors
- Write one zero to the right of the product for each zero in the factors.

**Multiple of 10:**
$10 \times 1 = 10$
$10 \times 2 = 20$
$10 \times 3 = 30$, and so on.

nonzero digits

| | | |
|---|---|---|
| $1 \times 34 = 34$ | $35 \times 2 = 70$ | $6 \times 5 = 30$ |
| $10 \times 34 = 340$ | $35 \times 20 = 700$ | $60 \times 50 = 3000$ |
| $100 \times 34 = 3400$ | $35 \times 200 = 7000$ | $600 \times 500 = 300{,}000$ |
| $1000 \times 34 = 34{,}000$ | $35 \times 2000 = 70{,}000$ | $6000 \times 5000 = 30{,}000{,}000$ |

► To multiply a decimal by a power of 10:

- Count the number of zeros in the power of 10.
- Move the decimal point to the *right* one place for each zero.
- Write as many zeros in the product as needed to place the decimal point correctly. $1000 \times 0.07 = 0.070.$

**Power of 10:**
$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$, and so on.

## VIII. Division Patterns

You can use patterns to **divide by powers and multiples of 10**.

► To divide a whole number by a power or multiple of 10:

- Divide the nonzero digits.
- To determine the number of zeros in the quotient, subtract the number of zeros in the divisor from the number of zeros in the dividend.

| | | |
|---|---|---|
| $34{,}000 \div 1 = 34{,}000$ | $40{,}000 \div 1 = 40{,}000$ | $30{,}000 \div 6 = 5000$ |
| $34{,}000 \div 10 = 3400$ | $40{,}000 \div 10 = 4000$ | $30{,}000 \div 60 = 500$ |
| $34{,}000 \div 100 = 340$ | $40{,}000 \div 100 = 400$ | $30{,}000 \div 600 = 50$ |
| $34{,}000 \div 1000 = 34$ | $40{,}000 \div 1000 = 40$ | $30{,}000 \div 6000 = 5$ |

► To divide a decimal by a power of 10:

- Count the number of zeros in the divisor.
- Move the decimal point to the *left* one place in the dividend for each zero in the divisor.
- Write zeros in the quotient as needed.

$79.4 \div 10 = 7.9.4 = 7.94$

$79.4 \div 100 = .79.4 = 0.794$

## IX. Estimate Products

► To **estimate a product by rounding**:
- Round each factor to its greatest place.
- Multiply the rounded factors.

Estimate: $365 \times 4840$

$400 \times 5000 = 200{,}000$

**Think**
6 > 5, so 3 hundreds rounds up to 4 hundreds.
8 > 5, so 4 thousands rounds up to 5 thousands.

Both factors were rounded up, so $365 \times 4840$ is less than 200,000.

Estimate: $10.25 \times 0.87$

$10 \times 0.9 = 9$

**Think**
0 < 5, so 1 ten stays as 1 ten and 10.25 rounds down to 10.
7 > 5, so 8 tenths rounds up to 9 tenths.

One factor is rounded down and the other is rounded up. So $10.25 \times 0.87$ is close to 9.

## X. Estimate Quotients

► You can **estimate quotients using compatible numbers**, numbers that are easy to compute with. Two whole numbers are compatible if one divides the other evenly.

**Remember:**
The symbol ≈ means "is approximately equal to."

Estimate: $1895 \div 48$

about 2000, about 50

So $1895 \div 48 \approx 2000 \div 50 = 40$.

Estimate: $23.50 \div 5.65$

about 24, about 6

So $23.50 \div 5.65 \approx 24 \div 6 = 4$.

## XI. Multiply Whole Numbers

► To **multiply by a two-digit number**, multiply by ones, then by tens. Add the partial products.

To **multiply by a three-digit number**, multiply by ones, then by tens, then by hundreds. Add the partial products.

Multiply: $17 \times 24$

Estimate

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline \end{array} \rightarrow \begin{array}{r} 20 \\ \times\ 20 \\ \hline 400 \end{array}$$

Compute

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 168 \\ +\ 24\phantom{0} \\ \hline 408 \end{array}$$

Multiply: $126 \times 78$

Estimate

$$\begin{array}{r} 78 \\ \times\ 126 \\ \hline \end{array} \rightarrow \begin{array}{r} 80 \\ \times\ 100 \\ \hline 8000 \end{array}$$

Compute

$$\begin{array}{r} 78 \\ \times\ 126 \\ \hline 468 \\ 156\phantom{0} \\ +\ 78\phantom{00} \\ \hline 9828 \end{array}$$

## XII. Divide Whole Numbers

► To **divide by a 1-digit number**, you can use short division.

- Divide to find the first digit of the quotient.
- Multiply and subtract mentally.
- Write each remainder in front of the next digit in the dividend.
- Repeat the steps until the division is completed.

Divide: $64{,}448 \div 8$

Estimate: $64{,}000 \div 8 = 8000$

Compute: $8\overline{)6\,4{,}4^{4}4^{4}8}$ with quotient $8\,0\,5\,6$

► To **divide by a 2- or 3-digit number**:

- Decide where to begin the quotient. If there are not enough hundreds, the quotient begins in the tens place. Divide the tens and ones.

Find the quotient: $33{,}904 \div 618$

$600\overline{)36{,}000}$ → $60$ ← estimated quotient

$$
\begin{array}{r}
54 \text{ R532} \\
618\overline{)33{,}904} \\
-\,3090\phantom{0} \\
\hline
3004 \\
-\,2472 \\
\hline
532
\end{array}
$$

## XIII. Multiply Decimals

► To **multiply a decimal** by a whole number or another decimal:

- Multiply as you would with whole numbers.
- Count the number of decimal places in both factors.
- Mark off the *same* number of decimal places in the product.

Multiply: $11 \times 0.31$

Estimate: $10 \times 0.3 = 3$

Compute:

$$
\begin{array}{r}
0.31 \\
\times\ \ 11 \\
\hline
31 \\
+\,31\phantom{0} \\
\hline
3.41
\end{array}
$$

2 decimal places

Find the product: $0.42 \times 0.329$

Estimate: $0.4 \times 0.3 = 0.12$

Compute:

$$
\begin{array}{rl}
0.329 & \leftarrow \text{3 decimal places} \\
\times\ \ 0.42 & \leftarrow \text{2 decimal places} \\
\hline
658 & \\
+\ 1316\phantom{0} & \\
\hline
0.13818 & \leftarrow \text{5 decimal places}
\end{array}
$$

Sometimes you will need to write one or more zeros in the product.

$$
\begin{array}{rl}
0.425 & \leftarrow \text{3 decimal places} \\
\times\ \ 0.02 & \leftarrow \text{2 decimal places} \\
\hline
0.00850 & \leftarrow \text{5 decimal places}
\end{array}
$$

REVIEW OF PREREQUISITE SKILLS

## XIV. Divide Decimals

► To **divide a decimal by a decimal**:

- Move the decimal point in the *divisor* to form a whole-number divisor.
- Move the decimal point in the *dividend* to the right *the same number* of places.
- Write the decimal point in the quotient directly above the decimal point in the dividend.
- Divide as you would with whole numbers.

Divide: $8.46 \div 0.2$

Estimate: $8 \div 0.2 = 40$

$$0.2\overline{)8.4\,6}$$

Move the decimal points one place to the right.

$$\begin{array}{r} 4\,2.3 \\ 2\overline{)8\,4.6} \\ -\,8\phantom{4.6} \\ \hline 4\phantom{.6} \\ -\,4\phantom{.6} \\ \hline 6 \\ -\,6 \\ \hline 0 \end{array}$$

So $8.46 \div 0.2 = 42.3$.

## XV. Fractions Greater Than or Equal to 1

A fraction that is greater than or equal to 1 has its numerator greater than or equal to the denominator. This type of number is sometimes called an improper fraction.

► You can express an improper fraction as a whole number or a mixed number.

$\frac{2}{2} = \frac{2 \div 2}{2 \div 2} = \frac{1}{1} = 1 \leftarrow$ whole number $\qquad \frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2} = 1\frac{1}{2} \leftarrow$ mixed number

► To **rename a mixed number** as a fraction:

- Multiply the denominator by the whole number.
- Add the product to the numerator.
- Write the sum as the numerator and the given denominator as the denominator.

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2}$$
$$= \frac{6 + 1}{2} = \frac{7}{2} \leftarrow \text{fraction}$$

Rename $\frac{22}{6}$ as a mixed number.

► To **rename a fraction greater than 1** as a mixed number:

- Divide the numerator by the denominator. Write the quotient as the whole number part.
- If there is a remainder, write it over the denominator. Express the fraction in simplest form.

$$\frac{22}{6} \rightarrow \begin{array}{r} 3\ \text{R}4 \\ 6\overline{)22} \end{array}$$

$$\frac{22}{6} = 3\frac{4}{6} \rightarrow 3\frac{4 \div 2}{6 \div 2}$$
$$= 3\frac{2}{3} \leftarrow \text{mixed number}$$

## XVI. Add and Subtract Fractions

► To **add fractions**:

Add: $\frac{3}{4} + \frac{2}{3}$

- Find the least common denominator (LCD) of the fractions.

Multiples of 4: 4, 8, 12, 16, . . .
Multiples of 3: 3, 6, 9, 12, 15, . . .
The LCD is 12.

**Think**
The LCD is the least common multiple (LCM) of the denominators.

- Rename each fraction as an equivalent fraction with the LCD as the denominator.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$
$$+\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

- Add. Express the sum in simplest form.

$$\frac{17}{12} = 1\frac{5}{12}$$

► To **subtract fractions**:

Subtract: $\frac{1}{2} - \frac{1}{10}$

- Find the least common denominator (LCD) of the fractions.

LCD of $\frac{1}{2} - \frac{1}{10}$: 10

- Rename each fraction as an equivalent fraction with the LCD as the denominator.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$
$$-\frac{1}{10} = \frac{1}{10}$$

- Subtract. Express the difference in simplest form.

$$\frac{4}{10} = \frac{2}{5}$$ ← simplest form

## XVII. Multiply Fractions

► To **multiply fractions**:

Multiply: $\frac{2}{3} \times \frac{3}{8}$

- Multiply the numerators. Then multiply the denominators.

$$\frac{2}{3} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 8} = \frac{6}{24}$$

- Write the product in simplest form.

$$\frac{6}{24} = \frac{6 \div 6}{24 \div 6} = \frac{1}{4}$$ ← simplest form

► To **multiply fractions using the greatest common factor (GCF)**:

- Divide *any* numerator and denominator by their GCF.

$$\frac{2}{3} \times \frac{3}{8} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}}$$ ← Divide by the GCF.

- Multiply the numerators. Then multiply the denominators. The product will be in simplest form.

$$= \frac{1 \times 1}{1 \times 4}$$
$$= \frac{1}{4}$$ ← simplest form



REVIEW OF PREREQUISITE SKILLS

## XVIII. Divide Fractions

► To **divide fractions**:

- Multiply by the reciprocal of the divisor.
- Simplify using the GCF, where possible. Then multiply the numerators and the denominators.
- Rename the product as a whole or mixed number when needed.

The product of reciprocals = 1.
$\frac{1}{12} \times \frac{12}{1} = 1$

Divide: $\frac{5}{6} \div \frac{1}{12}$

reciprocals

$$\frac{5}{6} \div \frac{1}{12} = \frac{5}{6} \times \frac{12}{1}$$

$$= \frac{5}{\cancel{6}_{1}} \times \frac{\cancel{12}^{2}}{1} = \frac{5 \times 2}{1 \times 1}$$

$$= \frac{10}{1} = 10$$

Find the quotient: $\frac{4}{25} \div \frac{3}{7}$

reciprocals

$$\frac{4}{25} \div \frac{3}{7} = \frac{4}{25} \times \frac{7}{3}$$

$$= \frac{28}{75}$$

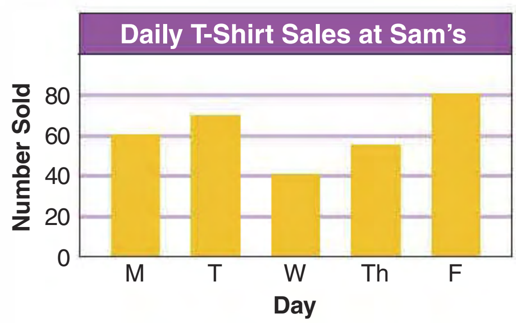
## XIX. Bar Graphs

The owner of Sam's Shirt Shop recorded daily sales of T-shirts for a week. He then displayed the results in a bar graph. A bar graph is especially useful for comparing numerical data.

| Daily T-Shirt Sales at Sam's | | | | | |
|---|---|---|---|---|---|
| **Day** | Mon. | Tues. | Wed. | Thurs. | Fri. |
| **Number of Shirts** | 60 | 70 | 40 | 55 | 80 |

► A bar graph uses vertical bars or horizontal bars of different lengths.

The length of each bar is proportional to the number the bar represents. The scale on the bar graph is divided into equal intervals.



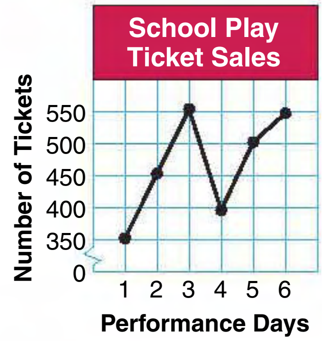

► The bar graph shows the same information as the table. On which day was the least number of T-shirts sold? On which day was the greatest number of T-shirts sold? How does the bar graph help you answer these questions?

The least number of T-shirts was sold on Wednesday.
The greatest number of T-shirts was sold on Friday.
The bar for Wednesday is the shortest bar.
The bar for Friday is the tallest bar.

## XX. Line Graphs

Mr. Jazwinski organized recent school play ticket sales data in a table. Then he displayed the data in a line graph.

| School Play Ticket Sales | | | | | | |
|---|---|---|---|---|---|---|
| **Day** | 1 | 2 | 3 | 4 | 5 | 6 |
| **Tickets Sold** | 352 | 453 | 554 | 396 | 503 | 548 |




► Line graphs are useful for showing changes in data over time.

What trend does the graph show about ticket sales? To determine the trend, look for a rise (shows the data is increasing) or a fall (shows the data is decreasing) in the line between two points.

Ticket sales increased from day 1 to day 3 and from day 4 to day 6; ticket sales decreased from day 3 to day 4.

## XXI. Compute with Units of Measure

► You can **add, subtract, multiply, and divide measures**.

$$\begin{array}{rl} & 5 \text{ ft } 10 \text{ in.} \\ + & 8 \text{ ft } \ \ 8 \text{ in.} \\ \hline & 13 \text{ ft } 18 \text{ in.} = 14 \text{ ft } 6 \text{ in.}, \\ \text{or} & \qquad\qquad\quad 14\tfrac{1}{2} \text{ ft} \end{array}$$

**Think**
13 ft 18 in.
= 13 ft + 1 ft 6 in.
= 14 ft 6 in.

$$\begin{array}{rl} & \overset{3}{\cancel{4}} \text{ gal } \overset{6}{\cancel{2}} \text{ qt} \\ - & 2 \text{ gal } 3 \text{ qt} \\ \hline & 1 \text{ gal } 3 \text{ qt, or } 1\tfrac{3}{4} \text{ gal} \end{array}$$

**Think**
4 gal 2 qt
= 3 gal 4 qt + 2 qt
= 3 gal 6 qt

$$\begin{array}{rl} & \overset{9}{\cancel{10}} \text{ lb } \ \overset{24}{\cancel{8}} \text{ oz} \\ - & 6 \text{ lb } \ 9 \text{ oz} \\ \hline & 3 \text{ lb } 15 \text{ oz} \end{array}$$

**Think**
10 lb 8 oz = 9 lb 24 oz

$$\begin{array}{rl} & 8 \text{ qt } \quad 3\text{c} \\ \times & \qquad\quad 4 \\ \hline & 32 \text{ qt } 12 \text{ c} = 35 \text{ qt} \end{array}$$

**Think**
4 c = 1 qt
so 12 c = 3 qt

3 lb 8 oz ÷ 2 = __?__

$$\begin{aligned} 3 \text{ lb } 8 \text{ oz} &= (3 \times 16) \text{ oz} + 8 \text{ oz} \\ &= 48 \text{ oz} + 8 \text{ oz} \\ &= 56 \text{ oz} \end{aligned}$$

$$\begin{aligned} 56 \text{ oz} \div 2 &= 28 \text{ oz} \\ &= (28 \div 16) \text{ lb} \\ &= 1 \text{ lb } 12 \text{ oz} \end{aligned}$$

**Listen to the directions.**
**You do not need paper and pencil.**

## SET 1

1. What must be added to each in order to reach zero? $3, -3, -16, -9, 8, 15, -5, 1, -11$
2. Divide. $-48 \div -6; -14 \div 7; 0 \div -12; 18 \div -6$
3. Compute and compare. (Use $<, =, >$.) $7 + 9 \underline{\ ?\ } 9 + 7; -3 + 6 \underline{\ ?\ } -6 + 9;$ $0 + (-4) \underline{\ ?\ } 4 + (-4)$
4. Find the missing quotient. $16 \div (-2) \div 2;$ $-125 \div 25 \div (-1); 36 \div (-2) \div (-3)$
5. Read the numeral, then rename as factors: $10^3, 4^2,$ $5^4, 10^6, 3^5, 4^4, 9^2, 8^3, 6^4, 7^3$
6. Multiply or divide. $2^2 \bullet 3^2, 10^2 \div 5^2, 5^2(-2)^2,$ $-3^2(-2)^2, 5^3 \div 5^1, \frac{10^4}{2^2}, -3^3(-3)^2$
7. Give the simplest number for each. $10 \times 10 \times 10 \times 10; 10^3; 10 \times 10^2; -10 \times 10^2;$ $-10^2 \times (-10)$
8. Write the numeral: six billion, twelve million, eighteen; five million, four hundred thousand; ninety thousand
9. Say each in expanded notation. 16,000; 109,400; 50,005; 10,110; 6,000,500; 25,000,250
10. Round to the nearest thousand: 24,123; 986; 7645
    Round to the nearest thousandth: 0.9665; 1.0743
11. Round to the nearest whole number: $4\frac{3}{4}, 1\frac{1}{5}, 6\frac{7}{8}, 24\frac{11}{15}, 16\frac{2}{3}, 9\frac{2}{3}, 14\frac{3}{8}, 5\frac{4}{9}$
12. Round to the nearest whole number: 7.5, 8.25, 6.19, 5.763, 4.199, 0.56, 9.67, 3.402
13. Solve. $5x = 35, a \bullet 8 = 88, -7c = 42, -45 = a \bullet 9,$ $0 = x \bullet 6, -y \bullet 6 = 36, x \bullet 10 = 0$
14. When $n = 5.5$ what number is named by $-4n$?

## SET 2

1. What operation is suggested by moving on a number line from 2 to $-1$? 4 to $-4$? $-3$ to 6? 5 to $-5$?
2. Say the word phrase for each algebraic expression. $6^3 \bullet 4; 7x + 9; 5 + b; 8 = \frac{y}{4}; x(8 - 2); r \div 15;$ $4(12 - n); d + 7; -5 \bullet t; 96 \div x$
3. Name an algebraic expression for: the quotient when the sum of three and a number is divided by two.
4. Name an algebraic expression for: the absolute value of negative three minus a number cubed.
5. On a horizontal number line, what integer is: 2 units left of zero, 4 units right of zero, 5 units left of zero?
6. On a horizontal number line, how many units to the right or left of zero is each integer: $-1, -6, 5, -3, 8$
7. What integer is suggested by a temperature of 15 degrees below zero? 6 degrees above zero?
8. Name the opposite integer: $5, 8, 11, -9, -4, -7$
   Tell what integer is one less than: $0, -3, 14, -4, 9$
9. Evaluate $a + b$ when $a = 2$ and $b$ is: $0, 3, -3, -2,$ $-5, 9, 17, -6, 4, 10, 10^2, 100^2$
10. On a horizontal number line, what integer is 7 units to the right of $-7$? 1 unit to the right of $-2$?
11. Write a number sentence suggested by a number line move from $-6$ to zero. from $-6$ to $-9$
12. To show each of the following, show where you would start on a number line and the number of units left or right you would move: $2 + 4; 4 - 5; -3 - 2$

## SET 3

1. When $\frac{2}{3}$ of a number is 12, what is the number?
2. If $\frac{5}{8}$ of a number is 15, what is $\frac{2}{3}$ of the same number?
3. Find $\frac{1}{5}$ of: 60, 30, 40, 55, 35, 45, 25, 20
   Find $\frac{1}{8}$ of: 72, 40, 56, 48, 24, 64, 88, 32
4. Complete: $-4 - (-2) = -4 + \underline{\ ?\ };$ $-6 - (-7) = -6 + \underline{\ ?\ }; -5 - (-5) = -5 + \underline{\ ?\ }$
5. Find the missing factor. $6(\underline{\ ?\ }) = -36; -4(\underline{\ ?\ }) = 16;$ $-18 = -6(\underline{\ ?\ }); \underline{\ ?\ } \bullet 9 = 72$
6. Find $\frac{1}{9}$ of : 72, 81, 54, 45, 27, 63, 99, 36
7. Find the product. $-2 \times (-8); 3 \times (-10); 4 \times (-8);$ $-11 \times 9; -1 \times 1; -6 \times 12; 7 \times (-2); 0 \times 99$
8. Identify the property. $(4 + 7) + 2 = 4 + (7 + 2);$ $18 + 0 = 18; 0 \bullet (-6) = 0; 6 \times (-6) = -6 \times 6$
9. What number makes the sentence true? Name the property used. $(15 \times 3) + (15 \times 7) = 15(3 + y)$
10. Compute and compare. (Use $<, =, >$.) $0 \times 1 \underline{\ ?\ } -6;$ $-1 \times (-7) \underline{\ ?\ } -1 \times 7; -1 \times (-3) \underline{\ ?\ } 1 \times 3$

## SET 4

1. Compare. (Use $<, =, >$.) $-4 \underline{\ ?\ } -2; 4 \underline{\ ?\ } 2,$ $3 \underline{\ ?\ } -3; 7 \underline{\ ?\ } -6; -9 \underline{\ ?\ } -10; -5 \underline{\ ?\ } -1$
2. Order from greatest to least: $3, -1, 0, 4; -5, -6, 0,$ $-1; -6, 10, -7, 3; 2, -2, 4, 0$
3. Subtract $-10$ from: $4, 5, 6, -4, 15, -15, -7, 7$
4. If $n = 9$, what number is named by: $(24 - 6) \div n$? $3(n + 12)$? $(45 - 11) - n$? $6n \div 9$?
5. Compute and compare. (Use $<, =, >$.) $9 \times 8 \underline{\ ?\ } 70;$ $4 \times 12 \underline{\ ?\ } 50; 24 + 26 \underline{\ ?\ } 50$
6. How many times greater than 6 is 72?

MAINTENANCE

## SET 5

1. Write an equation for each. one less than ten is nine; one more than twice three is seven; five times seven is thirty-five
2. Evaluate the expression $7a$ when $a = 11$. Evaluate the expression $5b$ when $b = 8$.
3. Solve. $\frac{k}{3} = -3, 7 = x \div 42, \frac{n}{-6} = -36, 0 = \frac{x}{1000}$
4. Divide by $-3$: $12, -9, -3, 6, -6, -15, 15, 0, 18$
5. Write an inequality for each. double a value is less than or equal to 12; the sum of $y$ and 7 is less than or equal to 11
6. Write each as a word sentence. $4 > x \div 15; 21 < 4x; x(-4) < 9; -9 + 6 = m; 4 > a \bullet 2.2; 12 < y$
7. Which is least: $\frac{1}{2}, \frac{50}{100}, \frac{5}{10}$, or $\frac{2}{6}$?
8. Simplify each expression. $5a + 11 + 3a; 6b + 2ab - 2(a + b); 11xy - 9y + 17xy$
9. Write each as an expression. the sum of 7 and 8.03, a number decreased by 19.9, 24 divided by twice a number, 2 increased by $-9$
10. Tell the correct relation. (Use $<, =, >$.) $6 \underline{\ ?\ } -7; -3 \underline{\ ?\ } -2; -5 \underline{\ ?\ } -9; 4 \underline{\ ?\ } -4; -9 \underline{\ ?\ } 9$
11. Tell which fraction is less: $\frac{2}{3}$ or $\frac{3}{4}$; $\frac{2}{5}$ or $\frac{1}{6}$; $\frac{7}{3}$ or $\frac{3}{7}$; $\frac{4}{9}$ or $\frac{9}{4}$; $\frac{3}{2}$ or $\frac{2}{3}$; $\frac{5}{6}$ or $\frac{6}{5}$
12. On a horizontal number line there are two fractions: $\frac{5}{8}$ and $\frac{6}{8}$. Name 3 fractions between these two.
13. 90 is $\frac{3}{4}$ of what number? 9 is $\frac{3}{4}$ of what number?
14. $\frac{1}{3} = \frac{?}{12} = \frac{?}{18} = \frac{?}{24} = \frac{?}{30} = \frac{?}{21} = \frac{?}{27} = \frac{?}{15}$

## SET 6

1. Solve. $8 + x = 19; a + 16 + 1 = 20; 7 + r + 5 = -11; -3 = -9 + y; -3 = (-2)^2 + x; 3 + x = 10; n + 6 = -7$
2. Tell an equivalent fraction that has a denominator of 1000: $\frac{1}{250}, \frac{1}{8}, \frac{2}{50}, \frac{1}{25}, \frac{1}{5}$
3. Express the following as decimals: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{10}, \frac{9}{10}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{10}, \frac{1}{3}$
4. Rename each fraction or mixed number as an equivalent decimal. $\frac{4}{5}; \frac{3}{20}; \frac{1}{8}; \frac{49}{50}; 3\frac{3}{4}; 9\frac{1}{2}; 2\frac{3}{10}; 2\frac{11}{20}; 5\frac{2}{5}; 1\frac{3}{8}$
5. Add. $25.9 + 15.6; 21.09 + 17.9; -15.29 + 11.01; -4.3 + (-2.7); 9.08 + (-1.3); -100.5 + (-90.5)$
6. Find the difference. $1 - (-9) = n, -14 - (-2) = n, 10 - (-6) = n, 15 - 5 = n, -2 - 5 = n, -8 - (-7) = n$
7. Add or subtract. $14 + (-8) = n, -14 - 8 = n, -14 - (-8) = n, -18 + (-9) = n, -18 - 9 = n$
8. Estimate the sum or difference by rounding. $60.1 + 9.7; 15.9 - 4.1; 0.075 + 0.12; 0.09 - 0.9$
9. Estimate the sum or difference by using front-end estimation. $56.5 + 9.5; 17.1 - 3.01; 10.5 - 8.08$
10. Subtract. $3.05 - 0.95; 22.4 - (-8.5); -7.1 - (-1.1)$
11. Give the simplest number name for each: $2^2, 3^2, 9^2, 4^1, 2^3, 1^3, 5^0, 5^3, 10^0, 10^2$
12. Write the standard numeral for: $(4 \times 10^4) + (2 \times 10^3) + (5 \times 10^2) + (3 \times 10^1)$ $(7 \times 10^5) + (8 \times 10^4) + (3 \times 10^2) + (9 \times 10^1)$
13. Name the expanded form with exponents for 28,404.

## SET 7

1. Estimate each product. $0.6 \bullet 0.7; 8.1 \bullet 1.9; 0.55 \bullet 2.2; 32.5 \bullet 0.511; 45.5 \bullet 0.005$
2. Estimate each quotient. $27.8 \div 9.01; 32.01 \div 4.44; 0.36 \div 0.09; 49.004 \div 7.007; 64.008 \div 8.006$
3. Write the power of 10 in standard form. Multiply the factors. $2.3 \times 10^3; 1.7 \times 10^{-3}; 3.09 \times 10^{-2}; 4.1 \times 10^0$
4. Write in scientific notation: 728,000; 5,100,000; 0.033; 0.0065; $-42{,}500{,}000$; $-0.0003$; $-0.0011$
5. Rename each as a mixed number. $\frac{65}{9}, \frac{76}{9}, \frac{47}{9}, \frac{58}{9}, \frac{40}{9}, \frac{28}{9}, \frac{89}{9}, \frac{95}{9}, \frac{109}{9}$
6. Change to improper fractions. $3\frac{3}{7}, 4\frac{2}{7}, 2\frac{4}{7}, 5\frac{5}{7}, 6\frac{6}{7}, 7\frac{1}{7}, 5\frac{2}{7}, 3\frac{2}{9}, 2\frac{6}{7}, 7\frac{5}{7}$
7. Mt. Whitney is 14,484 ft high. If you climbed to a point 250 ft below the top, how high would you have climbed?
8. Jim's lunches costs \$4.95. If he wants to leave a 15% tip, how much money should he leave?

## SET 8

1. Compare. (Use $<, =, >$.) $0.2 \underline{\ ?\ } 0.3; 0.3 \underline{\ ?\ } 0.03; 0.006 \underline{\ ?\ } 0.02; 0.125 \underline{\ ?\ } 0.12; 0.009 \underline{\ ?\ } 0.001$
2. Order from least to greatest: 0.1, 0.03, 0.001, 0.031
3. Change to a mixed number: $\frac{15}{7}, \frac{29}{7}, \frac{32}{7}, \frac{40}{7}$
4. Express as fractions in simplest form: 0.5, 0.2, 0.3, 0.4, 0.25, 0.75, 0.80, 0.125, 0.375,
5. Simplify: $\frac{6}{24}, \frac{18}{45}, \frac{24}{60}, \frac{12}{36}, \frac{36}{42}, \frac{54}{60}, \frac{18}{72}, \frac{21}{24}, \frac{18}{36}, \frac{36}{108}, \frac{45}{135}$
6. Order from greatest to least: 0.009, 0.9, 0.912, 0.091, 0.91
7. A room is 25 ft long. Express the length in yards and feet.
8. What percent of 1 $yd^2$ is 1 $ft^2$?
9. If 6.5 kg of rice is put into 100 boxes, how much rice will there be in each box?
10. What is the unit of measure represented by: L, kL, mL, daL, cL, m, $cm^2$, $dm^3$, mm, mg, kg, hg?

## SET 9

1. Evaluate each expression. $6^{-2}$; $9^{-3}$; $24^{-1}$; $10^{-1}$
2. Evaluate each expression. $2^{-2} \bullet 2^{-1}$; $10^{-2} \bullet 3^{-2}$; $(6^{-1})(2^{-3})$; $(3^{-1})(5^{-3})$; $(1^{-1})(10^{-3})$; $\frac{7^{-2}}{7^{-1}}$
3. Multiply $3 \times 8$; $3 \times 80$; $3 \times 800$; $3 \times 8{,}000$; $3 \times 80{,}000$; $3 \times 800{,}000$; $3 \times 8{,}000{,}000$
4. Multiply $5 \times 9$; $3 \times 9.01$; $5 \times 90.1$; $3 \times 900.1$; $5 \times 9{,}000.1$; $5 \times 90.001$; $5 \times 9{,}000{,}000$
5. Write the answer in scientific notation. $3(4 \times 10^3)$; $2(5.5 \times 10^1)$; $3(2.2 \times 10^2)$; $(1.5 \times 10^0)(1.2)$
6. Multiply by $10^2$: 45, 59, 0.4, 0.05, 0.24, 0.651
7. Find the GCF of 24 and: 4, 12, 8, 48, 16, 32, 27
8. Tell the GCF for each pair: 12 and 18; 36 and 24; 20 and 32; 18 and 27; 36 and 63; 11 and 22
9. The difference between two numbers is 6. If the greater number is 15, what is the other number?
10. Tell the equivalent fraction that has a denominator of 100: $\frac{3}{10}, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}, \frac{9}{20}, \frac{4}{25}$
11. Tell the equivalent fraction that has a denominator of 1000: $\frac{1}{100}, \frac{1}{10}, \frac{1}{2}, \frac{1}{50}, \frac{1}{8}, \frac{1}{5}$
12. Dividing a number by 9 is equivalent to multiplying it by what number?
13. There are 31classes with 24 students in each class. Estimate to the nearest hundred the total number of students.
14. Dividing a number by $2\frac{1}{2}$ is the same as multiplying it by what number?

## SET 10

1. Multiply by 40: 10, 1,000, 100, 20, 40, 120
2. Multiply. $\frac{2}{3} \times 6$; $\frac{4}{5} \times 5$; $\frac{3}{6} \times 12$; $7 \times \frac{5}{7}$; $\frac{1}{2} \times 8$; $\frac{1}{5} \times 5$; $\frac{1}{7} \times 3$; $9 \times \frac{2}{3}$; $6 \times \frac{3}{4}$
3. Find the sum. $\frac{-2}{7} + \frac{4}{7}$; $\frac{15}{30} + \left(\frac{-7}{15}\right)$; $-\frac{11}{24} + \frac{9}{48}$; $\frac{6}{7} + \frac{7}{49}$; $\frac{4}{5} + \left(\frac{-9}{10}\right)$
4. Find the difference. $\frac{1}{2} - \frac{3}{7}$; $\frac{4}{5} - \frac{1}{3}$; $\frac{9}{12} - \frac{1}{6}$; $\frac{3}{10} - \left(\frac{-1}{5}\right)$; $\frac{7}{13} - \frac{1}{26}$; $\frac{1}{8} - \frac{7}{8}$; $\frac{-2}{5} - \left(\frac{-1}{5}\right)$
5. Change to mixed numbers: $\frac{24}{7}, \frac{30}{8}, \frac{18}{7}, \frac{40}{9}, \frac{49}{6}, \frac{15}{4}, \frac{27}{7}, \frac{54}{5}, \frac{64}{9}, \frac{70}{8}, \frac{81}{8}, \frac{37}{7}$
6. Tell which are prime numbers: 7, 11, 14, 18, 19 23, 27, 33, 37, 41, 49, 51, 53, 56, 59, 63, 65
7. Name the next five composite numbers: 4, . . .
8. What fraction subtracted from 2 equals $\frac{1}{3}$?
9. What fraction represents $\frac{1}{5}$ more than the original whole? What fraction represents $\frac{1}{3}$ less than the original whole?
10. How much greater than 1 is $\frac{3}{4} + \frac{7}{8}$?
11. Multiply by 6: $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{2}, 1\frac{1}{6}, 2\frac{1}{6}$
12. Find the missing number. Name the property used. $6 \times b = 0$; $7 + x = x + 7$; $9 + n = 9$; $36x = 36$; $(7 + 8) + 3 = 7 + (n + 3)$
13. What is 7 more than $3 \times 30$? 9 more than $88 \div 8$?
14. Tell the number named by the following: $6^2, 1^2, 4^2, 2^2, 7^2, 5^2, 10^2, 8^2, 3^2, 9^2, 10^3, 10^4$
15. Multiply each by 10, 100 and 1000: 0.1, 0.01, 0.001, 0.41, 0.041, 0.05, 0.005, 0.055

## SET 11

1. Dividing a number by 2.5 is the same as multiplying it by what number?
2. Which is less, the sum of 0.5 and 1.2 or the product of 0.5 and 1.2?
3. Multiply 8 by: $\frac{1}{4}, \frac{3}{4}, \frac{1}{2}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}$
4. Tell whether the fraction is closer to $-1$, $-\frac{1}{2}$, or 0. $-\frac{2}{9}, -\frac{6}{7}, -\frac{30}{55}, -\frac{65}{70}, \frac{3}{18}, -\frac{7}{15}$
5. Replace the $x$ with a number so that the fraction is a little less than $\frac{1}{2}$. $\frac{x}{12}$; $\frac{36}{x}$; $\frac{x}{25}$; $\frac{x}{60}$; $\frac{22}{x}$
6. Which is the greatest: $\frac{9}{10}, \frac{9}{9}, \frac{9}{100}, \frac{90}{100}$?
7. Which is the least: $\frac{4}{8}, \frac{50}{100}, \frac{20}{50}$?
8. If 30 items cost \$4563, how much is 1 item? \$1521 or \$152.10 or \$15.21
9. What are the prime factors of 24?
10. The difference between two numbers is 10. If the lesser number is $-10$, what is the greater number?
11. What is the greatest possible remainder when dividing by 3: 5, 6, 8, 11, 15, 20, 100?

## SET 12

1. Evaluate the expressions. Use order of operations. $(1 \times 8) + (6 \div 3)$; $12 - (14 \div 2) - 4$; $9 + (6 \times 6)$; $(9 \times 7) - 12 + (0 \div 5)$; $(10 + 2 \times 8 - 6) \div 3$
2. Evaluate when $a = -4$ and $b = 6.3$. $(a + b)$; $2(a \times b)$
3. Solve: $\frac{2}{7} = \frac{4}{n}$; $\frac{3}{5} = \frac{n}{20}$; $\frac{5}{9} = \frac{n}{45}$; $\frac{n}{8} = \frac{49}{56}$; $\frac{5}{6} = \frac{n}{18}$; $\frac{3}{10} = \frac{n}{100}$; $\frac{7}{49} = \frac{35}{n}$
4. Solve: $\frac{2}{3} = \frac{x}{9} = \frac{x}{27} = \frac{x}{15} = \frac{x}{6} = \frac{x}{21} = \frac{x}{12}$
5. A concert attracted 450 adults. If there were 100 more men than women, how many men were there?
6. A school team won 15 out of 20 games. Express this as a ratio.
7. What is the least number represented by four digits that are all alike and even?
8. Simplify. $(0.04)^2 \bullet 5$; $(2 \bullet 3.5) + (1.6 \div 2)$

## SET 13

1. Which of these ratios are equivalent? $\frac{9}{18}$ and $\frac{1}{2}$; $\frac{8}{0.8}$ and $\frac{4.8}{0.6}$; $\frac{9}{27} = \frac{18}{45}$; $\frac{0.6}{12.2}$ and $\frac{0.3}{6}$
2. Tell the ratio in simplest form: 3 pt to 1 gal; 18 in. to 1 yd; 3 days to 1 wk; 50 min to 5 sec
3. Write as percents: $\frac{5}{50}, \frac{16}{50}, \frac{75}{100}, \frac{2}{5}, \frac{2}{10}, \frac{50}{100}, \frac{8}{25}, \frac{2}{100}, \frac{7}{10}, \frac{4}{50}, \frac{96}{100}$
4. Give two ratios equivalent to: $\frac{2}{7}, \frac{3}{8}, \frac{1}{4}, \frac{5}{6}, \frac{1}{5}$
5. Tell if the expression is equal. $\frac{3}{8} = \frac{24}{64}$; $\frac{1}{2} = \frac{1}{4}$; $\frac{1}{5} = \frac{4}{20}$; $\frac{1}{6} = \frac{5}{20}$; $\frac{4}{8} = \frac{10}{19}$; $\frac{5}{9} = \frac{45}{81}$
6. Name each decimal as a percent. 0.6, 0.9, 0.52, 0.76, 0.05, 0.01, 0.545, 0.675, 0.885, 0.99
7. Solve for $x$. Find the original number. 50% of $x$ is 80, 15% of $x$ is 100, 25% of $x$ is 125, 75% of $x$ is 250
8. What is a prime number that is greater than 10 and less than 20? What is an even prime number that is greater than 1 and less than 10?
9. What number is $\frac{7}{8}$ of 24?
10. Solve each proportion: $\frac{6}{42} = \frac{1}{n}$; $\frac{30}{n} = \frac{6}{5}$; $\frac{35}{7} = \frac{n}{1}$; $\frac{60}{100} = \frac{n}{20}$; $\frac{12}{30} = \frac{n}{100}$
11. Find the missing term. $\frac{9}{10} = \frac{81}{n}$; $\frac{7}{3} = \frac{28}{n}$; $\frac{5}{1} = \frac{15}{n}$; $\frac{12}{21} = \frac{n}{7}$; $\frac{32}{36} = \frac{8}{n}$; $\frac{3}{4} = \frac{n}{8}$
12. If 10% of the amount sold is \$50, what is the amount of sales? If 20% is \$40 what is the amount sold?
13. Find 150% of 10, 20, 60, 80, 18, 24, 90, 200
14. Express as a decimals: 110%, 130%, 150%, 125%, 190%, 200%, 240%, 899%, 375%

## SET 14

1. Make each ratio equivalent to 2:5. 8:__?__, 10:__?__, 12:__?__, 14:__?__, 18:__?__, 20:__?__
2. Make each set of ratios equivalent: 3:4 = 6:__?__; 2:3 = 4:__?__; 5:6 = __?__:30; 7:8 = __?__:24
3. Tell the value of $n$: $\frac{n}{9} = \frac{2}{3}$; $\frac{4}{5} = \frac{n}{45}$; $\frac{12}{5} = \frac{n}{25}$; $\frac{8}{4} = \frac{24}{n}$; $\frac{5}{8} = \frac{20}{n}$
4. Tell the equivalent fraction that has a denominator of 100: $\frac{9}{10}, \frac{2}{5}, \frac{13}{20}, \frac{3}{25}, \frac{3}{4}, \frac{9}{50}$
5. Tell the equivalent fraction that has a denominator of 100: $\frac{7}{50}, \frac{1}{2}, \frac{11}{20}, \frac{4}{25}, \frac{7}{10}, \frac{11}{10}$
6. Jim receives 60% of the profits and Luc receives 40%. How much more does Jim receive? Express Jim and Luc's profits as a ratio.
7. Bea gave correct answers to 82 out of 100 questions. What percent of her answers were correct?
8. A salt solution contains 2 g of salt to every 3 g of water. What is the ratio of water to salt?
9. Select the better buy: 5 oz for 39¢ or 1 lb for \$1.20; 2 L for 79¢ or 3 L for \$1.19; 3 p for \$4 or 5 p for \$5
10. Find the equivalent ratios: $\frac{3}{4} = \frac{6}{c} = \frac{12}{c} = \frac{15}{c} = \frac{21}{c} = \frac{30}{c}$
11. Write each percent as a decimal. 107%, 226%, 553%, 611%, 101%, 732%, 225%, 178%
12. Write each fraction as a percent. $\frac{3}{1}, \frac{9}{5}, \frac{6}{5}, \frac{1}{200}, \frac{3}{2}, \frac{199}{100}, \frac{25}{5}, \frac{276}{100}, \frac{20}{7}$
13. Write each decimal as a percent. 0.8, 0.67, 5.55

## SET 15

1. What percent of 20 is: 2, 4, 5, 10, 15, 18, 20, 25?
2. Emil saved \$15 out of \$20. What percent of his money did he save?
3. What percent of 300 is 30? What percent of 55 is 5? What percent of 120 is 10? What percent of 88 is 4?
4. 20 is what percent of 80? 5 is what percent of 40? 0.8 is what percent of 34? 6 is what percent of 36?
5. Calculate what the total cost is with a 15% tip. \$20, \$45, \$60, \$75, \$110, \$150, \$250
6. Solve. 90 is what percent of 30? 75 is what percent of 200? \$180 is what percent of \$500?
7. $112\frac{1}{2}$% of a number is the same as what fraction times the number?
8. Rose won 3 out of 4 games. What percent of the games did she win?
9. 300 is 150% of what number? What percent of 50 is 75? 250 is 25% of what number?
10. What percent of 10 is: 5, 2, 3, 7, 8, 4, 9, 15, 25?

## SET 16

1. Find the commission. 10% commission on \$250; 15% commission on \$350; 9% commission of \$600
2. Find each percentage. 10% of 50; 10% of 300; 25% of 50; 5% of 25; 150% of 50; 8% of 200
3. A discount of 5% brings a bill down to \$19. Find the amount of the original bill.
4. Find 25% of: 32, 8, 16, 64, 24, 56, 48, 70, 150
5. Find the mode for this data: 12, 8, 7, 4, 4, 4, 3, 3, 2, 2
6. Which number is divisible by 2 and 5: 8, 15, or 30?
7. Irene's watch gains 26.6 seconds in one week. About how many seconds does her watch gain in a day?
8. To travel 15 miles in 20 minutes, how many miles per hour must you be going?
9. Which number is divisible by 4 and 6: 32, 44, or 60?
10. If a leaky faucet loses 9 pt of water in 24 hours, how much water is lost in a week? in a 30-day month?

## SET 17

**1.** Find the percent of decrease. from \$25 to \$20; from \$65 to \$50; from \$80 to \$45; from \$15 to \$12; from \$150 to \$130; from \$12,000 to \$8,500

**2.** Find the discount rate. a \$30 book on sale for \$20; a \$25,000 automobile on sale for \$17,500

**3.** Express as percents: 6, 6.5, 0.65, 0.065, 4, 4.5 0.045, 9, 9.5, 0.95, 0.095, 8.5, 0.85, 0.085

**4.** Over 5 weeks the recorded rainfall was: 1 in., 2 in., $2\frac{1}{4}$ in., 3 in., $1\frac{1}{2}$ in., $2\frac{1}{2}$ in. What is the range?

**5.** Using the weekly rainfall data above, what is the mode?

**6.** Using the same data, what is the average rainfall?

**7.** Using the same data, what is the median rainfall?

**8.** Edna wants a radio that costs \$90. If she has \$75, how much more money does she need?

**9.** The following are spelling scores: 80, 89, 81, 80, 70. What is the mean? What is the median?

**10.** Of the scores above, which is the mode?

**11.** What is the range for this set of data? 90, 89, 89, 80, 70

**12.** Solve each proportion: $\frac{7}{42} = \frac{1}{n}$; $\frac{60}{n} = \frac{6}{5}$; $\frac{35}{7} = \frac{n}{1}$; $\frac{80}{100} = \frac{n}{20}$; $\frac{12}{30} = \frac{n}{48}$

**13.** Find the number: $4 = 12\frac{1}{2}\%$ of $x$; $10 = 5\%$ of $x$; $150 = 20\%$ of $x$; $200 = 15\%$ of $x$; $75 = 20\%$ of $x$

**14.** Multiply by 1000: 23, 2.3, 0.23, 0.023, 125, 12.5, 1.25, 0.125, 255, 25.5, 2.55, 0.255

## SET 18

**1.** Find 40% of: \$10, \$5, \$50, 35, 75, 90, 400, \$1.75, \$1.50, \$600, \$2, 120, 55, 80, 600

**2.** The following are math scores: 90, 81, 81, 80, 75. What is the mean? What is the median?

**3.** Of the scores above, which is the mode?

**4.** What is the range for the same set of data? 90, 81, 81, 80, 75.

**5.** One angle in a right triangle measures 40°. Find the measure of the third angle.

**6.** Identify the complementary or supplementary angle pairs. 65°, 25°; 110°, 70°; 35°, 55°; 95°, 85°

**7.** Name the measure of the supplement of each angle: 40°, 100°, 35°, 60°, 5°, 45°, 120°, 110°

**8.** What fractional part of an hour is: 20 min; 40 min; 10 min; 5 min; 15 min, 12 min?

**9.** 30% of Jon's day is spent in bed. How long is that?

**10.** To travel 15 miles in 20 minutes, how many miles per hour must you be going?

**11.** A tax collector is paid \$60 commission for collecting \$1500. Find the rate of commission.

**12.** Find the third angle of a triangle if two angles are: 90° and 60°; 82° and 64°; 75° and 58°

**13.** The opposite angles of a parallelogram measure 100° each. Name the measure of the other angles?

**14.** Express in scientific notation: 13,000; 2700; 450,000; 6,000,000; 108,000; 9,810,000.

## SET 19

**1.** Find the number: 80% of $n = 100$; 25% of $n = 2000$; $33\frac{1}{3}\%$ of $n = 20$; 60% of $n = \$3600$; 5% of $n = 75$

**2.** To the nearest penny find 5% of: 60¢, 80¢, 70¢, 50¢

**3.** Find: 15% of 30; 75% of 150; 20% of 60

**4.** What is: 80% of 200; 75% of 48; 60% of 60; 90% of 90; 30% of 60; 200% of 80; 60% of 30?

**5.** Name the kind of triangle in which all angles are less than 90°.

**6.** Name the kind of triangle that has three congruent sides.

**7.** Right triangles have two acute angles. What kind of angle is the third angle?

**8.** Into what two shapes does a diagonal divide a rectangle?

**9.** The measure of two of the three angles of a triangle are given. Name the measure of the remaining angle. 90°, 45°; 105°, 20°; 70°, 75°; 60°, 50°; 100°, 30°

**10.** Multiply by 3: 6, $6\frac{1}{3}$, 5, $5\frac{2}{3}$, 2, $2\frac{2}{3}$, 7, $7\frac{1}{3}$, 4, $4\frac{2}{3}$, 9, $9\frac{1}{3}$, 10, $10\frac{3}{4}$

## SET 20

**1.** Find the number: 50% of $n = 500$; 20% of $n = 1000$; 40% of $n = 100$; 75% of $n = \$750$; 10% of $n = 50$

**2.** To the nearest dollar find 15% of: \$50, \$90, \$120

**3.** From 1000 take: $10^1$, $10^2$, $10^3$, $10^0$, $2^2$, $3^2$, $5^2$

**4.** Take $10^3$ from: 2000, 1000, 3500, 64,000, 10,000

**5.** Tell the range of possible measurements for each degree of precision. nearest cm: 3 cm; nearest tenth of a cm: 5.7 cm; nearest in.: 4 in.; nearest tenth of a cm: 8.1 cm; nearest half in.: 6.7 in.

**6.** Arrange in order from least to most precise. 4.01 cm, 4.1 cm, 4.014 cm, 4 cm; 15.4 g, 15.054 g, 15 g, 15.04 g

**7.** What percent of 360° is: 60°, 90°, 120°, 180°, 30°, 40°, 160°, 200°, 225°, 280°, 300°?

**8.** What percent of 360° is: 20°, 50°, 70°, 85°, 95°, 110°, 155°, 200°, 280°, 310°, 320°?

**9.** Find the circumference of a circle when the diameter is: 14 cm, 2.1 m, 28 in., 3.5 m, 70 mm, 1.4 m

## SET 21

1. Two angles of a triangle measure 28° and 62°. What kind of triangle is it?
2. A man's suit is $290. The tax is 5%. How much tax must be paid? What is the total cost of the suit?
3. The diagonal of a square forms two congruent isosceles right triangles. What is the measure of each congruent angle?
4. Solve each proportion: $\frac{5}{6} = \frac{25}{n}$; $\frac{4}{12} = \frac{n}{2}$; $\frac{3}{4} = \frac{n}{16}$; $\frac{8}{n} = \frac{80}{100}$; $\frac{6}{18} = \frac{n}{36}$; $\frac{7}{10} = \frac{5}{n}$
5. Give the square root of: 4, 9, 25, 49, 16, 121, 144, 225, 64, 36, 81, −4
6. Square each number. 7, −2, $\frac{3}{5}$, 0.5, 6, 8, 7, 12, 13, 10, 100, 20, 15
7. Which measures form a right triangle? 3 m, 4 m, 5 m; 2 m, 2 m, 4 m; 5 cm, 5 cm, 7 mm; 9 cm, 12 cm, 16 cm;
8. A triangle has an altitude of 20 m and a base of 13.7 m. Find its area.
9. Find the perimeter of an equilateral triangle whose sides are 30 cm.
10. Name the prime factors of: 4, 6, 8, 9, 10, 15, 18
11. What are the prime factors of 28?
12. Find the hypotenuse of a right triangle whose base is 1.5 m and height is 2 m.
13. How long is the diagonal brace on a gate that measures 4 ft by 3 ft?

## SET 22

1. The perimeter of a square is 12 feet. Find the length of one side in inches.
2. The side of both a square and a regular octagon is 10 cm. How many times greater is the perimeter of the octagon than that of the square?
3. The circumference of a wheel is 44 cm. What is the radius?
4. Find the diameter of a pipe whose circumference is 44 cm.
5. Find the side of a square whose perimeter is: 48 cm, 3.6 cm, 26 ft, 100 in., 12 m, 6.4 m, 2.3 m
6. What is the perimeter of a merry-go-round 28 ft in diameter? (Use $\frac{22}{7}$ as $\pi$.)
7. Find the surface area of a rectangular prism if the height is 9 cm and base is 6 cm × 4 cm.
8. How much did Sally pay for dog food if $\frac{1}{3}$ off list price saved her 63¢?
9. Out of 150 tickets, 20% were not sold. How many tickets were not sold?
10. After a radio had been reduced 10%, the sale price was $65. What was the original price?
11. A tree broke 12 ft above its base. The top hit the ground 16 ft from the base. How long was the fallen part?
12. Find the surface area of a cube whose edge is: 3 in., 5 ft, 10 dm, 4 m, 9 in., 10 ft, 30 m

## SET 23

1. The area of a rectangle is $1\frac{1}{3}$ times the area of the square. If the rectangle's area is 12 $ft^2$, what is the area of the square?
2. Find the area of a square with sides of: 4 in., 12 m, 1 ft, 11 ft, 5 cm, 10 yd, 6 in., 4 m, 20 cm
3. Find the area of a rectangle with sides of: 8 m and 11 m, 9 ft and 6 ft, 15 in. and 4 in.
4. At $24.75 each, about how much will 4 shirts cost?
5. What number is 4 more than 8 × 6?
6. Let $x = 5$ and $w = 4$. Evaluate: $x + w$; $2x + 2w$; $xw$; $2(x + w)$; $3x \bullet 2w$; $2w \div x$
7. Write the number sentence suggested by a number-line move from −6 to zero.
8. Evaluate $a \div b$ when $a = 12$ and $b = -3$; When $a = -9$ and $b = -6$; when $a = 20$ and $b = -14$
9. Use exponents and write in expanded form: 63,507.
10. Sue's watch gains 2.1 minutes in a week. What is the average number of minutes it gains per day?

## SET 24

1. What is the area of a circular tabletop having a diameter of 1.4 m?
2. Find the circumference of a circular flower bed that is 2.1 m in diameter.
3. $\frac{7}{8}$ of a number is 14. What is the number?
4. Is $\frac{3}{5}$ more or less than 0.7? More or less than $\frac{7}{8}$?
5. If $n = 7$, what number is named by $18 + \frac{n}{5}$?
6. A pamphlet of 30 sheets of paper is 0.12 inches thick. About how thick is one sheet?
7. Express each rate in simplest form: 100 miles in 2 hours; 5 pencils for 40¢; 40 gallons in 40 seconds; 150 meters in 3 seconds; 9 apples for 75¢
8. If a car travels at the rate of 65 miles per hour, what is the distance traveled by the car in $9x$ hr?
9. Write 35% as a fraction.
10. The original amount was $25. The new amount is $35. What is the percent of change?
11. Express as percents: 0.25, 0.05, 0.1, 0.15, 0.061

## SET 25

1. What is the measure of an angle on a circle graph that equals 10%? 35%? 75%? 20%?
2. 10% of what amount equals \$500? equals \$150?
3. 12 is 120% of what number?
4. Roll a 1–6 number cube. Find the theoretical probability. $P$(multiple of 2); $P$(even); $P$(not 1 or 6); $P$(factor of 4 or 18); $P$(odd)
5. Is $9^2$ greater or less than $2^9$?
6. How many permutations can be made using the letters in each word? DOG, LINE, DECIMAL, SUBSET, METER, SQUARE, TENS
7. Complete the proportions: $3:7 = \underline{\ ?\ }:21$; $4:7 = 6:\underline{\ ?\ }$; $6:9 = \underline{\ ?\ }:45$; $2:\underline{\ ?\ } = 6:8$
8. What is the probability: of picking a red item from a bag of 8 items with 4 red, 3 brown, and 1 white? What is the probability: of picking a red or tan item from a bag of 10 items with 4 red, 3 tan, and 3 blue?
9. What is the volume of a rectangular prism when the edges measure: 4 in. × 4 in. × 10 in.?
10. The edges of a box measure 7 in. by 9 in. by 4 in. Find the volume.
11. A small jet travels at a speed of 300 miles per hour. How far could it travel in $2\frac{1}{3}$ hr?
12. At a speed of 200 miles per hour, how long will it take to travel 500 miles? How long will it take to travel 750 miles?

## SET 26

1. Find the area of a triangle with a base of 40 ft and a height of 60 ft.
2. An isosceles right triangle has an area of 98 $cm^2$. Find the length of each leg.
3. By selling for \$100 above cost, a dealer makes $12\frac{1}{2}$% profit. Find the cost.
4. If a truck travels at 90 kilometers per hour, how far does it travel in one minute?
5. Edwin walked 0.2 kilometers in 5 minutes. How long will it take him to walk 1 kilometer?
6. Write two numbers greater than 5 and less than 11 that are divisible by 2.
7. Which is divisible by 6? 99, 84, 963, 156, 140
8. Write the decimal equivalent for $\frac{5}{6}$.
9. How much rope would you buy if you needed 8 pieces, each $1\frac{1}{2}$ yd long?
10. What number is named by $1.6 \times 10^3$?
11. Dan packed 1020 cards 50 to a pack. How many packs were there and how many loose cards?
12. If $\frac{3}{4}$ of a number is 18, what is the number?
13. If $\frac{3}{8}$ of Bill's age is 6 years, how old is he?
14. After spending 60% of her money, Rosa had \$20 left. How much money did she have at first?
15. Solve: $n - 2 = 9$; $a - 7 = 6$; $x - 9 = 20$; $m - 7 = 6$; $n - 4 = 13$; $y - 6 = 14$; $x - 7 = 28$
16. Express the ratio in simplest form: 30 cm : 1 m

## SET 27

1. $8\frac{1}{3}$% of 72 is what percent of 36?
2. Express $2\frac{1}{2}$% as a fraction.
3. What percent of 90¢ is 75¢?
4. Out of 800 employees, 480 are women. What is the ratio of women to the total?
5. Write the decimal equivalent of $\frac{8}{20}$.
6. Round 0.2939 to the nearest hundredth.
7. The area of a rectangle is 48 $m^2$. The length is 3 times the width. What is the width?
8. The width of a garden is $\frac{1}{4}$ of its length. The area is 36 $ft^2$. What is the width?
9. What percent of the diameter of a circle is its radius?
10. What is the ratio of a nickel to: a dime, a quarter, a nickel, a penny, a dollar?
11. Find $\frac{3}{4}$ of: 8, 12, 16, 24, 36, 48, 40, 72, 100
12. Solve: $6 + n = 8$; $y + 7 = 9$; $n + 8 = 10$; $4x = 11$; $8 + n = 16$; $m + 5 = 10$; $8 + b = 17$;
13. Use $\pi$ to find the circumference when the radius is 6.

## SET 28

1. Find the surface area of each cube: edge = 4 in., edge = 8 in., edge = 2.8 m, edge = 20 ft
2. Tell the number of faces for: a rectangular prism, a triangular prism, a cube, a sphere, a cylinder, a cone.
3. Solve for $n$ if one third $n$ equals: 5, 10, 4, 1, 3, 20, 12, 6, 11, 15, 22, 40, 100
4. Find the surface area of a cube whose edge is: 5 yd, 10 cm, 20 m, 25 ft, 100 cm
5. If 1 more than twice a number is subtracted from 6 times the number, the result is 21. Name the number.
6. Write the product using an exponent: 40 × 40
7. What number is named? $6 \times 10^{-4}$
8. The volume of a square pyramid is 21 $yd^3$. The pyramid's height is 7 yd. How long is the base?
9. Which has the greater volume: a can 10 cm high with a radius of $3\frac{1}{2}$ cm or a can 10 cm high with a 7 cm diameter?

# Glossary

also online

## A

**absolute error** (AB-suh-loot ER-ur) An error in a measurement that is the actual size of the error. (p. 273)

**absolute value** (AB-suh-loot VAL-yoo) The distance of a number from 0 on a number line. The symbol for absolute value is | |. (p. 3)

**absolute-value function** (AB-suh-loot VAL-yoo FUHNGK-shuhn) When graphed on a coordinate plane, a function that takes a V-shaped form. (p. 367)

**accuracy** (AK-yuh-ruh-see) A term that refers to how close a measurement is to the actual value. (p. 272)

**acute angle** (uh-KYOOT ANG-guhl) An angle having a measure that is greater than 0° and less than 90°. (p. 242)

**acute scalene triangle** (uh-KYOOT skay-LEEN TRYE-ang-guhl) A triangle with three acute angles and no congruent sides. (p. 254)

**acute triangle** (uh-KYOOT TRYE-ang-guhl) A triangle with three acute angles. (p. 254)

**Addition Property of Equality** (uh-DISH-uhn PROP-ur-tee UHV i-KWOL-uh-tee) When you add the same number to both sides of an equation, you get a true statement. If $a = b$, then $a + c = b + c$. (p. 38)

**Addition Property of Inequality** (uh-DISH-uhn PROP-ur-tee UHV *in*-i-KWOL-uh-tee) If $a < b$, then $a + c < b + c$. This statement is also true if $<$ is replaced by $>$, $\leq$, or $\geq$. (p. 60)

**Additive Inverse Property** (AD-i-tiv IN-vurss PROP-ur-tee) The sum of two inverse (opposite) numbers equals zero. $a + (-a) = 0$. (p. 14)

**additive inverses (opposites)** (AD-i-tiv IN-vurss-iz [OP-uh-zitz]) Two numbers whose sum is 0. (p. 9)

**adjacent angles** (uh-JAY-suhnt ANG-guhlz) Two angles that share a common side and vertex but have no common interior points. (p. 244)

**algebraic equation** (*al*-juh-BRAY-ik i-KWAY-zhuhn) An equation that contains one or more variables and at least one mathematical operation. It may also contain numbers. (p. 34)

**algebraic expression** (al-juh-BRAY-ik ek-SPRESH-uhn) An expression that contains one or more variables and at least one mathematical operation. It may also contain numbers. (p. 30)

**algebra tiles** (AL-juh-bruh TYELZ) Manipulatives used to model algebraic expressions or polynomials. (p. 44)

**alternate exterior angles** (AWL-tur-nit ek-STEER-ee-ur ANG-guhlz) For two lines intersected by a transversal, a pair of nonadjacent angles that lie outside the two lines and are on opposite sides of the transversal. (p. 246)

**alternate interior angles** (AWL-tur-nit in-TEER-ee-ur ANG-guhlz) For two lines intersected by a transversal, a pair of nonadjacent angles that lie between the two lines and are on opposite sides of the transversal. (p. 246)

**altitude of a parallelogram** (AL-ti-tood UHV UH *pa*-ruh-LEL-uh-*gram*) A perpendicular line segment from the base to the opposite side. (p. 282)

**angle** (ANG-guhl) A figure formed by two rays with a common endpoint called the *vertex*. (p. 240)

**angle of rotation** (ANG-guhl UHV roh-TAY-shuhn) For a figure that has rotational symmetry, the smallest turn of the figure that creates a match. (p. 290)

**arc** (ARK) A part of a circle with all its points on the circle. (p. 262)

**area** (AIR-ee-uh) The number of square units that cover a figure. (p. 282)

**arithmetic sequence** (*a*-rith-MET-ik SEE-kwuhnss) A sequence of numbers in which each number is found by adding a fixed number to the number before. (p. 352)

**Associative Property of Addition** (uh-SOH-shuh-tiv PROP-ur-tee UHV uh-DISH-uhn) Changing the grouping of the addends does not change the sum. $(a + b) + c = a + (b + c)$. (p. 14)

**Associative Property of Multiplication** (uh-SOH-shuh-tiv PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn) Changing the grouping of the factors does not change the product. $(a \bullet b) \bullet c = a \bullet (b \bullet c)$. (p. 14)

## B

**back-to-back stem-and-leaf plot** (BAK-TOO-BAK STEM-AND-LEEF PLOT) A display that compares two sets of data by showing "leaves" on both sides of the "stem." (p. 221)

**balance** (BAL-uhnss) The sum of the principal plus the interest earned. (p. 200)

**base** (BAYSS) 1. In exponential form, the number or expression being used as a factor. (p. 18)
2. A selected side or face of a geometric figure. (pp. 282, 302)

**base angles** (BAYSS ANG-guhlz) The angles opposite the congruent sides of any isosceles or equilateral triangle. (p. 257)

**base of a parallelogram** (BAYSS UHV UH *pa*-ruh-LEL-uh-*gram*) A term that can refer to the length of any side of a parallelogram. (p. 282)

**base of a triangle** (BAYSS UHV UH TRYE-ang-guhl) A term that can refer to the length of any side of a triangle. (p. 284)

**bases of a polyhedron** (BAY-siz UHV UH *pol*-ee-HEE-druhn) The parallel, congruent faces of a polyhedron. (p. 302)

**bias** (BYE-uhss) Anything that favors a particular outcome in a sampling procedure. (p. 208)

**binomial** (bye-NOH-mee-uhl) A polynomial with two terms. (p. 382)

**bisect** (BYE-*sekt*) To divide a line segment or angle into two congruent parts. (p. 248)

**box-and-whisker plot** (BOKS-AND-WISS-kur PLOT) A graph that shows how data in a set are distributed without showing all the values in the data set. (p. 222)

## C

**center** (SEN-tur) The point inside a circle from which all points on the circle are equidistant. (p. 262)

**center of rotation** (SEN-tur UHV roh-TAY-shuhn) The point around which a figure rotates to create a transformation called a *rotation*. (p. 372)

**central angle** (SEN-truhl ANG-guhl) An angle that has its vertex at the center of a circle. (p. 262)

**certain event** (SUR-tuhn i-VENT) An event that *must* happen. (p. 330)

**chord** (KORD) A line segment with its endpoints on a circle. (p. 262)

**circle** (SUR-kuhl) A set of points in a plane, all of which are the same distance from a given point called the *center*. (p. 262)

**circle graph** (SUR-kuhl GRAF) A graph used to show parts of a whole. (p. 264)

**circumference** (sur-KUHM-fur-uhnss) The distance around a circle. (p. 286)

**circumscribed polygon** (sur-kuhm-SKRIBED POL-ee-gon) A polygon that has all its vertices on a circle. (p. 263)

**closed sentence** (KLOHZD SEN-tuhnss) A numerical equation that does *not* contain variables. Closed sentences are either true or false. (p. 35)

**Closure Property** (KLOH-zhuhr PROP-ur-tee) If performing an operation on any two numbers in a set always results in a number that is in that set, then the set is closed under that operation. (p. 16)

**coefficient** (*koh*-uh-FISH-uhnt) The numerical factor of a term that contains a variable. (p. 33)

**collinear points** (koh-LIN-ee-ur POYNTS) Points that lie on the same line. (p. 240)

**combination** (*kom*-buh-NAY-shuhn) A collection of objects or items in which order does not matter. (p. 344)

**commission** (kuh-MISH-uhn) The amount of money earned for selling goods or services. (p. 196)

**commission rate** (kuh-MISH-uhn RAYT) The percent of the total amount of goods or services sold that is earned by the seller. (p. 196)

**common multiples** (KOM-uhn MUL-tuh-puhlz) Multiples shared by two or more whole numbers. (p. 112)

**Commutative Property of Addition** (kuh-MYOO-tuh-tiv PROP-ur-tee UHV uh-DISH-uhn) Changing the order of the addends does not change the sum. $a + b = b + a$. (p. 14)

**Commutative Property of Multiplication** (kuh-MYOO-tuh-tiv PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn) Changing the order of the factors does not change the product. $ab = ba$. (p. 14)

**compatible numbers** (kuhm-PAT-uh-buhl NUHM-burz) Numbers that are easy to compute mentally. (p. 82)

**complement** (KOMP-luh-muhnt) An angle of a pair of angles having a sum of 90°. (p. 244)

**complementary angles** (komp-luh-MEN-tuh-ree ANG-guhlz) Two angles with a sum of 90°. (p. 244)

**complementary events** (komp-luh-MEN-tuh-ree i-VENTS) Two events that cannot occur at the same time. (p. 335)

**complex fraction** (kuhm-PLEKS FRAK-shuhn) A fraction that has a fraction or mixed number in the numerator, the denominator, or both. (p. 127)

**complex figure** (kuhm-PLEKS FIG-yur) A figure that is made up of two or more shapes. (p. 288)

**composite number** (kuhm-POZ-it NUHM-bur) A whole number greater than 1 that has more than two factors. The factors of 4 are 1, 2, and 4, so 4 is a composite number. (p. 108)

**compound event** (KOM-pownd i-vent) An event that consists of two or more simple events considered as a single event. (p. 340)

**compound inequality** (KOM-pownd *in*-i-KWOL-uh-tee) An inequality that consists of two or more connected inequalities. (p. 55)

**compound interest** (KOM-pownd IN-tur-ist) The interest paid on the principal and the interest accumulated to date. (p. 200)

**concave polygon** (kon-KAYV POL-ee-gon) A polygon that contains one or more interior angles that each have a measure greater than 180°. (p. 252)

**concentric circles** (kuhn-SEN-trik SUR-kuhlz) Circles that lie in the same plane and have the same center. (p. 263)

**cone** (KOHN) A three-dimensional figure that has one circular base and one curved surface that comes to a point called the vertex. (p. 303)

**congruent angles** (kuhn-GROO-ent ANG-guhlz) Angles that have the same degree measure. (p. 248)

**congruent figures** (kuhn-GROO-ent FIG-yurz) Figures that have the same size and shape. (p. 248)

**congruent line segments** (kuhn-GROO-ent LINE SEG-muhnts) Line segments that have the same length. (p. 248)

**congruent triangles** (kuhn-GROO-ent TRYE-ang-guhlz) Triangles that have exactly the same size and shape. Their corresponding sides and angles are congruent. (p. 256)

**conjecture** (kuhn-JEK-chur) A prediction that suggests what you expect to happen. (p. 352)

**constant** (KON-stuhnt) A term that does not contain a variable. (p. 33)

**constant difference** (KON-stuhnt DIF-ur-uhnss) The difference between each pair of consecutive numbers in a sequence. (p. 352)

**constant of variation** (KON-stuhnt UHV *vair*-ee-AY-shuhn) The term $k$ in an equation showing direct variation. $y = kx$ or $\frac{y}{x} = k$, where $x, k \neq 0$. (p. 364)

**constant ratio** (KON-stuhnt RAY-shee-oh) The fixed number that is used to calculate each term in a geometric sequence. Each term in the sequence is found by multiplying the preceding term by this fixed number. (p. 353)

**convenience sample** (kuhn-VEEN-yuhnss SAM-puhl) A sample in which members of a population or total group are chosen because they are readily available. (p. 208)

**conversion factors** (kuhn-VUR-zhuhn FAK-turz) The unit ratios needed to convert the given units in dimensional analysis. (p. 166)

**convex polygon** (kon-VEKS POL-ee-gon) A polygon that has all interior angles less than 180°. (p. 252)

**coordinate plane** (koh-OR-duh-nit PLAYN) A grid divided into four quadrants used to locate points by naming ordered pairs. (p. 22)

**coordinates** (koh-OR-duh-nits) Ordered pairs of numbers used to locate a point on a grid. (p. 22)

**coplanar** (koh-PLAY-nur) Points and lines that lie in the same plane. (p. 241)

**corresponding angles** (kor-uh-SPOND-ing ANG-guhlz) 1. Angles of congruent or similar figures that are in the same relative position. (p. 160) 2. For two lines intersected by a transversal, a pair of nonadjacent angles, one inside the two lines and the other outside the two lines, that are both on the same side of the transversal. (p. 246)

**corresponding sides** (kor-uh-SPOND-ing SYEDZ) Sides of congruent or similar figures that are in the same relative position. (p. 160)

**cost** (KAWST) The original amount spent for an item. (p. 189)

**counterexample** (*kown*-tur-eg-ZAM-puhl) 1. An example that disproves a statement or a condition. (p. 16) 2. A case that shows that a conjecture is false. (pp. 16, 356)

**cube** (KYOOB) A rectangular prism whose faces are all squares. Also called a **square prism**. (p. 302)

**cubic units** (KYOOB-ik YOO-nits) The units used to measure volume. (p. 314)

**cumulative frequency** (KYOOM-yuh-luh-tiv FREE-kwuhn-see) A running total of the number of all the responses in a survey. (p. 209)

**cylinder** (SIL-uhn-dur) A three-dimensional figure that has two circular congruent bases that are parallel. (p. 303)

## D

**degree of a monomial** (di-GREE UHV UH mon-OH-mee-uhl) The sum of the exponents of the variables in a term. (p. 382)

**degree of a polynomial** (di-GREE UHV UH *pol*-ee-NOH-mee-uhl) The sum of the exponents of the monomial with the greatest degree in a polynomial expression. (p. 382)

**degrees** (di-GREEZ) Units in which angles are measured. (p. 242)

**Density Property** (DEN-si-tee PROP-ur-tee) An infinite number of rational numbers can be found between any two rational numbers. (p. 116)

**dependent event** (di-PEN-duhnt i-VENT) An event that depends on the outcome of another event. (p. 340)

**diagonal** (dye-AG-uh-nuhl) A line segment that connects two nonadjacent vertices of a polygon. (p. 253)

**diameter** (dye-AM-uh-tur) A chord that passes through the center of a circle. (p. 262)

**dilation** (dye-LAY-shuhn) A transformation that reduces or enlarges the size of a figure. (p. 374)

**dimensional analysis** (duh-MEN-shuhn-uhl uh-NAL-uh-sis) The conversion from one unit system to another. (p. 166)

**direct proportion** (duh-REKT pruh-POR-shuhn) A relationship in which an increase or decrease in one quantity causes the same kind of change in the other quantify. (p. 154)

**direct variation** (duh-REKT *vair*-ee-AY-shuhn) A linear function in which the output (the $y$-value) is directly proportional to the input (the $x$-value), thus making the quotient the same for any ordered-pair solution.
$y = kx$ or $\frac{y}{x} = k$, where $x, k \neq 0$. (p. 364)

**discount** (DISS-kownt) The amount by which the regular or list price of an item is reduced. (p. 194)

**discount rate** (DISS-kownt RAYT) The ratio that represents the percent decrease in the list price. (p. 194)

**disjoint events** (diss-JOYNT i-VENTS) Two events that have no outcomes in common. Also called **mutually exclusive events**. (p. 335)

**Distributive Property of Multiplication over Addition** (diss-TRIB-yoo-tive PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn OH-vur uh-DISH-uhn) Multiplying a sum by a number is the same as multiplying each addend by that number and then adding the two products.
$a(b + c) = (a \bullet b) + (a \bullet c)$. (p. 15)

**Distributive Property of Multiplication over Subtraction** (diss-TRIB-yoo-tive PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn OH-vur suhb-TRAK-shuhn) Multiplying the difference of two numbers by a third number is the same as multiplying each of the two numbers by the third number and then subtracting the two products.
$a(b - c) = (a \bullet b) - (a \bullet c)$. (p. 15)

**Division Property of Equality** (di-VIZH-uhn PROP-ur-tee UHV i-KWOL-uh-tee) When you divide both sides of an equation by the same nonzero number, the result is a true statement.
If $a = b$, then $\frac{a}{c} = \frac{b}{c}, c \neq 0$. (p. 40)

**Division Property of Inequality** (di-VIZH-uhn PROP-ur-tee UHV *in*-i-KWOL-uh-tee)
If $a < b$ and $c$ is positive, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c$ is negative, then $\frac{a}{c} > \frac{b}{c}$. Similar statements can be written for $a > b$, $a \leq b$, or $a \geq b$. (p. 60)

**domain** (doh-MAYN) The set of input values, or $x$-values, in ordered pairs. (p. 358)

## E

**edge** (EJ) A line segment where two faces of a polyhedron meet. (p. 302)

**empty set** (EMP-tee SET) When no number from the replacement set makes the inequality true, the solution set is the empty set, { } or $\varnothing$. (p. 56)

**enlargement** (en-LARJ-muhnt) A dilation that is greater than the original figure. (p. 374)

**equally likely** (EE-kwuhl-ee LIKE-lee) Events that are just as likely to occur as other events. (p. 330)

**equation** (i-KWAY-zhuhn) A statement that contains an equal sign, showing that two mathematical expressions are equal. (p. 34)

**equilateral triangle** (*ee*-kwuh-LAT-ur-uhl TRYE-ang-guhl) A triangle with three congruent sides and angles. (p. 254)

**equivalent fractions** (i-KWIV-uh-luhnt FRAK-shuhnz) Two fractions that have the same value. (p. 111)

**equivalent ratios** (i-KWIV-uh-luhnt RAY-shee-ohz) Two ratios that have the same value. (p. 148)

**estimate** (ESS-ti-mate) 1. To find an approximate answer; to find an answer that is close to the exact answer. (p. 78) 2. To determine if a fraction is close to $-1$, close to $-\frac{1}{2}$, close to 0, close to $\frac{1}{2}$, or close to 1 by comparing the absolute value of its numerator to the absolute value of its denominator. (p. 114)

**evaluate** (i-VAL-yoo-*ate*) To find the value of a numerical expression or an algebraic expression. (p. 32)

**even odds** (EE-vuhn ODZ) Odds that take the form 1 : 1 in simplest form. (p. 339)

**event** (i-VENT) Any grouping of one or more outcomes from the sample space. (p. 330)

**exchange rate** (eks-CHAYNJ RAYT) The conversion factor in dimensional analysis. (p. 167)

**experiment** (ek-SPER-uh-ment) An event set up to test a hypothesis. (p. 330)

**experimental probability** (ek-*sper*-uh-MEN-tuhl *prob*-uh-BIL-uh-tee) An estimate that an event will happen based on how often the event occurs after collecting data from an experiment. (p. 336)

**exponent** (ek-SPOH-nuhnt) A numeral that tells you how many times a number, called the *base*, is used as a factor. (p. 18)

**exponential form** (*ek*-spoh-NEN-shuhl FORM) A notation used to write an expression in simplified form using exponents and bases. In $10^3$, 10 is the base and 3 is the exponent. (p. 18)

**exterior angle** (ek-STEER-ee-ur ANG-guhl) 1. An angle formed outside the parallel lines that are intersected by a transversal. (p. 246) 2. An angle that is formed outside a polygon and that is adjacent and supplementary to one of the polygon's interior angles. (p. 252)

**exterior points** (ek-STEER-ee-ur POYNTS) All points in a plane not part of an angle or its interior. (p. 242)

**extremes of a proportion** (ek-STREEMZ UHV UH pruh-POR-shuhn) In the proportion $a : b = c : d$, the terms $a$ and $d$. (p. 152)

## F

**faces** (FAY-siz) Flat surfaces of three-dimensional figures. (p. 302)

**factorial** (fak-TOR-ee-uhl) For a given integer, the product of all positive integers less than or equal to that integer. (p. 333)

**factors** (FAK-turz) Numbers that are multiplied to find a product. The factors of 8 are 1, 2, 4, and 8. (p. 108)

**factor tree** (FAK-tur TREE) A visual method for determining the prime factors of a composite number. (p. 108)

**fair** (FAIR) An experiment in which the number of favorable outcomes is equal to the number of unfavorable outcomes. (p. 339)

**favorable outcomes** (FAY-vur-uh-buhl OWT-*kuhms*) The outcomes that you are looking for in an event. (p. 334)

**formula** (FOR-myuh-luh) An equation, or rule, that shows a mathematical relationship between two or more quantities. (p. 46)

**frequency** (FREE-kwuhn-see) A record of the number of responses in a survey. (p. 209)

**frequency table** (FREE-kwuhn-see TAY-buhl) A method that can be used to organize the data gathered from surveys in order to show the number of times each type of answer occurs. (p. 209)

**function** (FUHNGK-shuhn) A one-to-one relation in which for each input value, there is only one output value. (p. 358)

**function rule** (FUHNGK-shuhn ROOL) A rule that relates an input ($x$) value to the corresponding output ($y$) value. (p. 359)

**function table** (FUHNGK-shuhn TAY-buhl) A table that helps to organize and display the $x$ and $y$ values of a function. (p. 360)

**Fundamental Counting Principle** (fuhn-duh-MEN-tuhl KOWNT-ing PRIN-suh-puhl) If an event can occur in $m$ ways and a second event can occur in $n$ ways, then the total number of possible ways that the events can occur together equals $m \bullet n$. (p. 332)

## G

**geometric constructions** (*jee*-uh-MET-rik kuhn-STRUHK-shuhnz) Drawings made with a compass and a straightedge. (p. 258)

**geometric sequence** (*jee*-uh-MET-rik SEE-kwuhnss) A sequence of numbers in which each term is found by multiplying the preceding term by a fixed number. (p. 353)

**greatest common factor (GCF)** (GRAYT-ist KOM-uhn FAK-tur) The greatest number that is a factor of two or more numbers. (p. 110)

**greatest possible error (GPE)** (GRAYT-ist POSS-uh-buhl ER-ur) A term that refers to one-half of the smallest unit that the measuring instrument can measure. (p. 272)

## H

**height of a parallelogram** (HITE UHV UH *pa*-ruh-LEL-uh-*gram*) The length of a perpendicular line segment from the base to the opposite side. (p. 282)

**height of a triangle** (HITE UHV UH TRYE-ang-guhl) The length of a perpendicular line segment from a base to the opposite vertex. (p. 284)

**hemisphere** (HEM-iss-*feer*) Half of a sphere. (p. 303)

**histogram** (HISS-tuh-*gram*) A graph that shows frequencies of data within equal intervals. (p. 218)

**hypotenuse** (hye-POT-uhn-ooss) The side opposite the right angle in a right triangle. (p. 280)

## I

**Identity Property of Addition** (eye-DEN-ti-tee PROP-ur-tee UHV uh-DISH-uhn) Adding 0 and any number does not change the value of the number.
$a + 0 = a$ or $0 + a = a$. (p. 14)

**Identity Property of Multiplication** (eye-DEN-ti-tee PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn) Multiplying 1 and any number does not change the value of the number. $a \bullet 1 = a$ or $1 \bullet a = a$. (p. 14)

**image** (IM-ij) The figure that results from a transformation. (p. 370)

**impossible event** (im-POSS-uh-buhl i-VENT) An event that cannot happen. (p. 330)

**improper fraction** (im-PROP-ur FRAK-shuhn) A fraction in which the numerator is greater than or equal to the denominator. (p. 413)

**independent event** (in-di-PEN-duhnt i-VENT) An event that does not depend on the outcome of another event. (p. 340)

**indirect measurement** (*in*-duh-REKT MEZH-ur-muhnt) A type of measurement that is used when the distance, height, or length of an object is difficult to measure directly. (p. 162)

**inequality** (*in*-i-KWOL-uh-tee) A mathematical sentence that compares two expressions using $<, >, \neq, \leq$, or $\geq$. (p. 54)

**input values** (IN-*put* VAL-yooz) Another name for **domain**, or $x$-values. (p. 358)

**inscribed angle** (in-SKRIBED ANG-guhl) An angle whose vertex is on the circle and whose sides intersect the circle at other points. (p. 263)

**inscribed circle** (in-SKRIBED SUR-kuhl) A circle inside a polygon whose sides are tangent to the circle. (p. 263)

**integers** (IN-tuh-jurz) Whole numbers and their opposites. (p. 2)

**interest** (IN-tur-ist) The amount of money earned or paid in exchange for the use of money. (p. 198)

**interior angles** (in-TEER-ee-ur ANG-guhlz) 1. Angles formed between parallel lines that are intersected by a transversal. (p. 246) 2. Angles that lie inside a polygon. (p. 252)

**interior points** (in-TEER-ee-ur POYNTS) All points in a plane between two rays that form an angle. (p. 242)

**interquartile range** (*in*-tur-KWOR-tile RAYNJ) The difference between the upper quartile and the lower quartile. (p. 223)

**intersecting lines** (in-tur-SEKT-ing LYENZ) Two lines that cross at exactly one point. (p. 241)

**intersecting planes** (in-tur-SEKT-ing PLAYNZ) Two planes that cross at exactly one line. (p. 241)

**inverse operations** (IN-vurss *op*-uh-RAY-shuhnz) Operations that "undo" one another; for example, $3 \bullet 6 = 18$ and $18 \div 6 = 3$ are inverse operations. (p. 12)

**Inverse Property of Addition** (IN-vurss PROP-ur-tee UHV uh-DISH-uhn) The sum of an integer and its additive inverse is 0. $a + (-a) = 0$. (p. 14)

**Inverse Property of Multiplication** (IN-vurss PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn) $\frac{1}{a}$ is the multiplicative inverse of $a$, or the reciprocal of $a$.
$a \bullet \frac{1}{a} = 1$ and $\frac{1}{a} \bullet a = 1$. (p. 130)

**inverse proportion** (IN-vurss pruh-POR-shuhn) A relationship in which an increase or decrease in one quantity causes the opposite kind of change in the other quantity. (p. 164)

**irrational numbers** (i-RASH-uh-nuhl NUHM-burz) Numbers that cannot be expressed as the quotient of two integers, $\frac{a}{b}$, where $b \neq 0$. (p. 279)

**isometric drawing** (eye-suh-MET-rik DRAW-ing) A pictorial view of a three-dimensional figure created on an isometric dot grid. It is made up of three types of lines: vertical lines, lines going 30° to the right, and lines going 30° to the left. (p. 304)

**isosceles triangle** (eye-SOSS-uh-leez TRYE-ang-guhl) A triangle with two congruent sides. (p. 254)

**iteration** (*it*-ur-AY-shuhn) A repeating step in a visual pattern. (p. 355)

## K

**kite** (KITE) A quadrilateral with two pairs of adjacent sides that are congruent. The measures of its angles are each less than 180°. (p. 260)

## L

**lateral area** (LAT-ur-uhl AIR-ee-uh) The sum of the areas of the lateral faces or lateral surfaces of a three-dimensional figure. (p. 308)

**lateral face** (LAT-ur-uhl FAYSS) Any face of a polyhedron that is not a base. (p. 308)

**lateral surface of a cylinder** (LAT-ur-uhl SUR-fiss UHV UH SIL-uhn-dur) The curved surface that is not the base. (p. 310)

**Law of Exponents for Division** (LAW UHV ek-SPOH-nuhnts FOR di-VIZH-uhn) $a^m \div a^n = a^{m-n}$, where $a \neq 0$. (p. 19)

**Law of Exponents for Multiplication** (LAW UHV ek-SPOH-nuhnts FOR *muhl*-tuh-pluh-KAY-shuhn) $a^m \bullet a^n = a^{m+n}$, where $a \neq 0$. (p. 19)

**Law of Exponents for Zero** (LAW UHV ek-SPOH-nuhnts FOR ZEER-oh) $a^0 = 1$, where $a \neq 0$. (p. 19)

**Laws of Exponents** (LAWZ UHV ek-SPOH-nuhnts) Laws used to simplify expressions that include exponents. (p. 19)

**least common denominator (LCD)** (LEEST KOM-uhn di-NOM-uh-*nay*-tur [EL SEE DEE]) The least common multiple of the denominators of two or more fractions. (p. 113)

**least common multiple (LCM)** (LEEST KOM-uhn MUHL-tuh-puhl [EL SEE EM]) The least nonzero common multiple of two or more numbers. (p. 112)

**legs** (LEGZ) The sides that form the right angle of a right triangle. (p. 280)

**likelihood of an event** (LIKE-lee-hud UHV AN i-VENT) The chance of an event happening. (p. 330)

**like terms** (LIKE TERMZ) Terms that have the same variables raised to the same power or terms that contain no variables (constants). (p. 32)

**line** (LINE) A continuous set of points in a straight path that extends without end in both directions. (p. 240)

**linear function** (LIN-ee-ur FUHNGK-shuhn) A function whose graph is a nonvertical line or part of a line. (p. 363)

**linear pair** (LIN-ee-ur PAIR) A pair of angles that are adjacent and supplementary. Their unshared sides form a straight angle. (p. 244)

**line of best fit** (LINE UHV BEST FIT) A line near the points of a scatter plot that clearly shows the trend or correlation between two sets of data. (p. 229)

**line of reflection** (LINE UHV ri-FLEK-shuhn) The line over which a figure is flipped in order to create a reflection. (p. 370)

**line of symmetry** (LINE UHV SIM-uh-tree) A real or imaginary line that divides the figure into mirror-image halves. (p. 290)

**line plot** (LINE PLOT) A graph that uses Xs to show data on a number line; a method that is especially useful for showing the mode and range of a set of data. (p. 211)

**line segment** (LINE SEG-muhnt) A part of a line; a line segment has two endpoints. (p. 240)

**line symmetry** (LINE SIM-uh-tree) A figure has line symmetry if a real or imaginary line, called the **line of symmetry**, divides the figure into mirror-image halves. (p. 290)

**list price** (LIST PRISE) The original price of an item. (p. 194)

**lower extreme** (LOH-ur ek-STREEM) The lower end of the "whisker" of a box-and-whisker plot; it represents the least value of a set of data. (p. 222)

**lower quartile** (LOH-ur KWOR-tile) The lower end of the "box" part of a box-and-whisker plot; it represents the median of the lower half of a set of data. (p. 222)

**lowest terms** (LOH-ist TERMZ) The form of a fraction (or a mixed number) when 1 is the only common factor of the numerator and denominator; also called **simplest form**. (p. 111)

## M

**major arc** (MAY-jur ARK) An arc that has a measure greater than 180°. (p. 262)

**marked price** (MARKT PRISE) The price of an item before sales tax is added. (p. 192)

**markup** (MARK-*uhp*) The difference between the wholesale price of an item and the list or retail price. (p. 195)

**markup rate** (MARK-*uhp* RAYT) The ratio that represents the percent increase in the wholesale price. (p. 195)

**mathematical expression** (*math*-uh-MAT-i-kuhl ek-SPRESH-uhn) A numerical or algebraic expression containing mathematics symbols. (p. 30)

**mean** (MEEN) A type of statistical average found by computing the sum of the data and dividing by the number of items in the data set. (p. 210)

**means of a proportion** (MEENZ UHV UH pruh-POR-shun) In the proportion $a : b = c : d$, the terms $b$ and $c$. (p. 152)

**measures of central tendency** (MEZH-urz UHV SEN-truhl TEN-duhn-see) The mean, median, and mode of a data set. (p. 210)

**measures of dispersion (measures of variation)** (MEZH-urz UHV di-SPUR-zhuhn [MEZH-urz UHV *vair*-ee-AY-shuhn]) Statistics that indicate how data are spread out or distributed. (p. 213)

**median** (MEE-dee-uhn) The middle value of a set of data that are arranged in order. If the set has an even number of data, the median is the mean of the two middle numbers. (p. 210)

**metric system of measurement** (MET-rik SISS-tuhm UHV MEZH-ur-muhnt) The system of measurement having the meter as its basic unit of length, the gram as its basic unit of mass, and the liter as its basic unit of capacity. (p. 100)

**midpoint** (MID-*poynt*) A point halfway between the two endpoints of a line segment that divides the line segment into two congruent parts. (p. 248)

**minor arc** (MYE-nur ARK) An arc that has a measure less than 180°. (p. 262)

**mixed numbers** (MIKST NUHM-burz) Fractional numbers greater than 1, with an integer part and a fraction part. (p. 116)

**mode** (MOHD) The most frequently occurring item in a set of data. (p. 210)

**monomials** (mon-OH-mee-uhlz) The terms in a polynomial. (p. 382)

**multiple** (MUHL-tuh-puhl) The product of a number and any whole number. (p. 112)

**multiple bar graph** (MUHL-tuh-puhl BAR GRAF) A graph that compares related sets of data. (p. 216)

**multiple line graph** (MUHL-tuh-puhl LINE GRAF) A graph that compares related sets of data that change over time. (p. 226)

**Multiplication Property of Equality** (*muhl*-tuh-pluh-KAY-shuhn PROP-ur-tee UHV i-KWOL-uh-tee) When you multiply both sides of an equation by the same number, you get a true statement. If $a = b$, then $ac = bc$. (p. 42)

**Multiplication Property of Inequality** (*muhl*-tuh-pluh-KAY-shuhn PROP-ur-tee UHV *in*-i-KWOL-uh-tee) If $a < b$ and $c$ is positive, then $ac < bc$. If $a < b$ and $c$ is negative, then $ac > bc$. Similar statements can be written for $a > b$, $a \leq b$, or $a \geq b$. (p. 62)

**multiplicative inverse** (*muhl*-tuh-PLIK-uh-tiv IN-vurss) The reciprocal of a number; if $a \neq 0$, then $\frac{1}{a}$ is the multiplicative inverse, or reciprocal, of $a$. (pp. 126, 130)

**Multiplicative Property of −1** (*muhl*-tuh-PLIK-uh-tiv PROP-ur-tee UHV NEG-uh-tiv WUHN) The product of −1 and any number is its opposite, or additive inverse. $a(-1) = -(a)$. (p. 130)

**mutually exclusive events** (MYOO-choo-uhl-ee eks-KLOO-siv i-VENTS) Two events that have no outcomes in common. Also called **disjoint events**. (p. 335)

## N

**natural numbers** (NACH-ur-uhl NUHM-burz) The counting numbers. (p. 279)

**negative correlation** (NEG-uh-tiv *kor*-uh-LAY-shuhn) In a scatter plot, the numbers for one data set decrease as the numbers for the other data set increase. (p. 228)

**negative exponent** (NEG-uh-tiv ek-SPOH-nuhnt) Exponents that are used to express fractions or decimals that have values between 0 and 1. For any integer $n$ and any number $a$, $a \neq 0$, $a^{-n} = \frac{1}{a^n}$. (p. 88)

**negative integer** (NEG-uh-tiv IN-tuh-jur) An integer that is less than 0. (p. 2)

**negative square root** (NEG-uh-tiv SKWAIR ROOT) The opposite of the positive square root of a number; a number that cannot represent the side length of a square. (p. 276)

**net** (NET) A two-dimensional shape that can be folded to form a three-dimensional object. (p. 306)

**noncollinear points** (*non*-koh-LIN-eer-ur POYNTS) Points that do not lie on the same line. (p. 240)

**nonlinear function** (non-LIN-ee-ur FUHNGK-shuhn) A function that does not have a constant rate of change. (p. 366)

**not equally likely** (NOT EE-kwuhl-ee LIKE-lee) Events that are not as likely to happen as other events. (p. 330)

**numerical equation** (noo-MER-uh-kuhl i-KWAY-zhuhn) An equation that contains numbers and operation symbols but no variables. (p. 34)

**numerical expression** (noo-MER-uh-kuhl ek-SPRESH-uhn) A mathematical expression that contains numbers and operation symbols but no variables. (p. 30)

## O

**obtuse angle** (uhb-TOOSS ANG-guhl) An angle with a measure that is greater than 90° and less than 180°. (p. 242)

**obtuse isosceles triangle** (uhb-TOOSS eye-SOSS-uh-leez TRYE-ang-guhl) A triangle with one obtuse angle and two congruent sides. (p. 254)

**obtuse triangle** (uhb-TOOSS TRYE-ang-guhl) A triangle with one obtuse angle. (p. 254)

**odds** (ODZ) A comparison of the number of favorable outcomes and the number of unfavorable outcomes. (p. 338)

**odds against** (ODZ uh-GENST) The chances an unfavorable outcome will occur: $\frac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}}$. (p. 338)

**odds in favor of** (ODZ IN FAY-vur UHV) The chances a favorable outcome will occur: $\frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$. (p. 338)

**open sentence** (OH-puhn SEN-tuhnss) An algebraic equation that contains one or more variables. Open sentences are neither true nor false. (p. 35)

**opposite integers (additive inverses)** (OP-uh-zit IN-tuh-jurz [AD-i-tiv IN-vurss-iz]) Integers that are the same distance from 0 on a number line but are located on opposite sides of 0. The sum of a pair of opposite integers is 0. (p. 2)

**ordered pairs** (OR-durd PAIRZ) Coordinates used to locate a point on a grid; the first number is the $x$-coordinate, and the second number is the $y$-coordinate. (p. 22)

**order of operations** (OR-dur UHV *op*-uh-RAY-shuhnz) A set of rules that is used to simplify mathematical expressions with more than one operation. (p. 20)

**origin** (OR-uh-jin) On the coordinate plane, the point of intersection of the $x$-axis and the $y$-axis; the point represented by (0, 0). (p. 22)

**orthographic drawings** (*or*-thuh-GRAF-ik DRAW-ingz) Two-dimensional views of the front, side, and top of a three-dimensional figure. (p. 305)

**outcome** (OWT-*kuhm*) The result of an experiment. (p. 330)

**outliers** (OWT-*lye*-urz) 1. Numbers in a data set that are much greater or much less than others in the set. (p. 212) 2. Points that are clearly separate from a set of data. (p. 223)

**output values** (OWT-*put* VAL-yooz) Another name for **range**, or $y$-values. (p. 358)

**overestimate** (OH-vur-ESS-ti-muht) An estimate greater than the actual value. (p. 312)

**overlapping events** (*oh*-vur-LAP-ing i-VENTS) Two events that have one or more outcomes in common. (p. 335)

## P

**parabola** (puh-RAB-uh-luh) A U-shaped curve on a coordinate plane that can open either up or down. (p. 367)

**parallel lines** (PA-ruh-*lel* LYENZ) Lines that are in the same plane but do not intersect. (p. 241)

**parallelogram** (*pa*-ruh-LEL-uh-*gram*) A quadrilateral with two pairs of parallel sides. Opposite sides and opposite angles are congruent. (p. 260)

**parallel planes** (PA-ruh-*lel* PLAYNZ) Planes that do not intersect. (p. 241)

**percent** (pur-SENT) A ratio or comparison of a quantity to 100. $\frac{\text{part}}{\text{whole}} = \frac{n}{100} = n\%$. (p. 174)

**percentage** (pur-SEN-tij) A number that is part of a whole. rate ($r$) • base ($b$) = percentage ($p$). (p. 180)

**percent change** (pur-SENT CHAYNJ) A ratio comparing a change in a quantity to an original amount. percent change $= \frac{\text{amount of change}}{\text{original amount}}$. (p. 188)

**percent decrease** (pur-SENT DEE-kreess) The amount of the decrease in a quantity divided by the original amount. (p. 190)

**percent increase** (pur-SENT IN-kreess) The amount of the increase in a quantity divided by the original amount. (p. 188)

**perfect square** (PUR-fikt SKWAIR) A number that is the square of a counting number; for example, 25 is the perfect square of 5. (p. 276)

**perimeter** (puh-RIM-uh-tur) The sum of the side lengths of a polygon. (p. 274)

**permutation** (*pur*-myoo-TAY-shuhn) An arrangement of items or objects in which order is important. (p. 342)

**perpendicular bisector** (pur-puhn-DIK-yuh-lur BYE-sekt-ur) A perpendicular line segment that bisects another line segment. (p. 248)

**perpendicular lines** (pur-puhn-DIK-yuh-lur LYENZ) Lines that intersect at right angles. (p. 241)

**perpendicular planes** (pur-puhn-DIK-yuh-lur PLAYNZ) Planes that intersect at right angles. (p. 241)

**pi** (PYE) An irrational number symbolized with the Greek letter $\pi$ (p. 279); the ratio of the circumference to the diameter of any circle (p. 286)

**plane** (PLAYN) A flat, two-dimensional surface that extends infinitely in all directions. (p. 241)

**point (**POYNT) An exact location in space. (p. 240)

**point symmetry** (POYNT SIM-uh-tree) A name for a special kind of **rotational symmetry**. The property of a figure that has a matching image after being rotated half a turn around its center point. (p. 291)

**polygon** (POL-ee-gon) A closed plane figure that has three or more sides. Its sides are line segments that only intersect at their endpoints. (p. 252)

**polyhedron** (*pol*-ee-HEE-druhn) A three-dimensional figure with faces that are all polygons. (p. 302)

**polynomial** (*pol*-ee-NOH-mee-uhl) An algebraic expression that is the sum or difference of terms called *monomials*. (p. 382)

**population** (pop-yuh-LAY-shuhn) A group of people or things. (p. 208)

**positive correlation** (POZ-uh-tiv *kor*-uh-LAY-shuhn) In a scatter plot, the numbers for one data set increase as the numbers for the other data set increase. (p. 228)

**positive integer** (POZ-uh-tiv IN-tuh-jur) An integer that is greater than 0. (p. 2)

**positive square root of a number** (POZ-uh-tiv SKWAIR ROOT UHV UH NUHM-bur) The positive number that, when multiplied by itself, equals the original number. Also called the **principal square root** of the number. (p. 276)

**possible outcome** (POSS-uh-buhl OWT-*kuhm*) A result of an experiment. (p. 334)

**powers** (POW-urz) Numbers that can be written using exponents. (p. 18)

**precision** (pri-SIZH-uhn) A term that refers to the smallest unit of measurement on a measuring instrument. (p. 272)

**prime factorization** (PRIME *fak*-tor-uh-ZAY-shuhn) A way of showing a composite number as the product of prime numbers. (p. 108)

**prime number** (PRIME NUHM-bur) A whole number greater than 1 that has exactly two factors, itself and 1. The only factors of 3 are 1 and 3, so 3 is a prime number. (p. 108)

**principal** (PRIN-suh-puhl) The amount of money borrowed or deposited in a bank. (p. 198)

**principal square root** (PRIN-suh-puhl SKWAIR ROOT) The positive square root of a number. (p. 276)

**prism** (PRIZ-uhm) A polyhedron with two congruent and parallel faces called *bases*. (p. 302)

**probability** (*prob*-uh-BIL-uh-tee) The measure of how likely it is that an event will occur. (p. 334)

**profit** (PROF-it) The money gained or realized when an item is sold above the cost. (p. 189)

**proportion** (pruh-POR-shuhn) An equation stating that two ratios are equivalent. (p. 152)

**protractor** (proh-TRAK-tur) A tool used to find the degree measure of an angle. (p. 243)

**pyramid** (PEER-uh-mid) A polyhedron with one base, which can be any polygon. (p. 302)

**Pythagorean Theorem** (pi-*thag*-uh-REE-uhn THEER-uhm) In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. $a^2 + b^2 = c^2$. (p. 280)

**Pythagorean Triples** (pi-*thag*-uh-REE-uhn TRIP-uhlz) Three positive integers that can form the side lengths of a right triangle. (p. 281)

## Q

**quadrants** (KWAHD-druhnts) The four sections into which the $x$-axis and $y$-axis divide the coordinate plane. (p. 23)

**quadratic function** (kwah-DRAT-ik FUHNGK-shuhn) When graphed on a coordinate plane, a function that takes the form of a parabola. (p. 367)

**quadrilaterals** (*kwahd*-ruh-LAT-ur-uhlz) Polygons with four sides and four angles. (p. 260)

**quartiles** (KWOR-tyelz) Values that divide data into fourths; they are visually represented on a box-and-whisker plot. (p. 222)

## R

**radical sign** (RAD-i-kuhl SINE) A symbol ($\sqrt{\phantom{x}}$) that stands for the positive square root of a number. (p. 276)

**radius** (plural *radii*) (RAY-dee-uhss [plural, RAY-dee-eye]) A line segment from the center of a circle to a point on the circle. (p. 262)

**random sample** (RAN-duhm SAM-puhl) A sample in which each member or part of the population has an equally likely chance of being chosen. (p. 208)

**range** (RAYNJ) 1. The difference between the greatest and least values in a data set. (p. 210) 2. The set of output values, or $y$-values, in ordered pairs. (p. 358)

**rate** (RAYT) A ratio that compares two unlike quantities. (p. 150)

**rate of interest** (RAYT UHV IN-tur-ist) The percent of interest earned or paid to the depositor on the principal. (p. 198)

**ratio** (RAY-shee-oh) A comparison of two like quantities, $a$ and $b$, by division, where $b \neq 0$. (p. 148)

GLOSSARY

**rational number** (RASH-uh-nuhl NUHM-bur) Any number that can be written in fractional form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$. (p. 72)

**ray** (RAY) Part of a line that has one endpoint and continues infinitely in the opposite direction. (p. 240)

**real numbers** (REEL NUHM-burz) The set of rational numbers and irrational numbers. (p. 278)

**reciprocal** (ri-SIP-ruh-kuhl) The multiplicative inverse of a number. $\frac{a}{b} \bullet \frac{b}{a} = 1$, when $a, b \neq 0$. (pp. 126, 130)

**rectangle** (REK-*tang*-guhl) A parallelogram with four right angles. (p. 260)

**rectangular prism** (rek-TANG-gyuh-lur PRIZ-uhm) A polyhedron that has bases that are rectangles. (p. 302)

**rectangular pyramid** (rek-TANG-gyuh-lur PEER-uh-mid) A pyramid that has a rectangular base. (p. 302)

**reduction** (ri-DUHK-shuhn) A dilation that is smaller than the original figure. (p. 374)

**reflection** (ri-FLEK-shuhn) A transformation that flips a figure over a line. (p. 370)

**reflex angle** (REE-fleks ANG-guhl) An angle whose measure is greater than 180° but less than 360°. (p. 242)

**regular polygon** (REG-yuh-lur POL-ee-gon) A polygon with congruent sides and congruent angles. (p. 252)

**regular polyhedron** (REG-yuh-lur *pol*-ee-HEE-druhn) A polyhedron with faces that are all congruent. (p. 302)

**regular prism** (REG-yuh-lur PRIZ-uhm) A prism with bases that are regular polygons. (p. 302)

**relation** (ri-LAY-shuhn) A set of ordered pairs that associates two quantities in a specific order. (p. 358)

**relative error** (REL-uh-tiv ER-ur) The absolute error of a measurement in relation to the correct value (or to the measured value if the correct value is not known). (p. 273)

**relatively prime** (REL-uh-tiv-lee PRIME) Two numbers are relatively prime if their only common factor is 1. (p. 111)

**repeating decimal** (ri-PEET-ing DESS-uh-muhl) A decimal in which a digit or sequence of digits repeats without end. (p. 72)

**replacement set** (ri-PLAYSS-muhnt SET) A set of numbers to be used as possible values for the variable in an equation or inequality. (p. 56)

**representative sample** (*rep*-ri-ZEN-tuh-tiv SAM-puhl) A sample that has characteristics similar to the entire population. (p. 208)

**rhombus** (ROM-buhss) A parallelogram with four congruent sides. (p. 260)

**right angle** (RITE ANG-guhl) 1. An angle that measures 90°. (p. 242) 2. The angle opposite the hypotenuse in a right triangle. (p. 280)

**right scalene triangle** (RITE skay-LEEN TRYE-ang-guhl) A triangle with one right angle and no sides congruent. (p. 254)

**right triangle** (RITE TRYE-ang-guhl) A triangle with one right angle. (p. 254)

**rotation** (roh-TAY-shuhn) A transformation that turns a figure around a point in either a clockwise or counterclockwise direction. (p. 372)

**rotational symmetry** (roh-TAY-shuhn-uhl SIM-uh-tree) The property of a figure that has a matching image after being rotated less than a full turn around a central point. (p. 290)

**rotation tessellation** (roh-TAY-shuhn *tess*-uh-LAY-shuhn) The use of rotated figures to create a pattern. (p. 292)

## S

**sale price** (SAYL PRISE) The difference between an item's list price and the discount. (p. 194)

**sales tax** (SAYLZ TAKS) The amount of tax added to the marked price of an item by a state or local government. (p. 192)

**sales tax rate** (SAYLZ TAKS RAYT) The ratio of the amount of sales tax to the marked price expressed as a percent. (p. 192)

**sample** (SAM-puhl) A part of a population. (p. 208)

**sample space** (SAM-puhl SPAYSS) The collection of all possible outcomes in an experiment. (p. 330)

**scale** (SKAYL) The ratio of a pictured measure to the actual measure. (p. 158)

**scale drawing** (SKAYL DRAW-ing) A two-dimensional drawing that is in proportion to an actual object. (p. 158)

**scale factor** (SKAYL FAK-tur) A scale written as a ratio in simplest form. (p. 159)

**scale model** (SKAYL MOD-uhl) A three-dimensional model that accurately represents a real object. (p. 159)

**scalene triangle** (skay-LEEN TRYE-ang-guhl) A triangle with no congruent sides. (p. 254)

**scatter plot** (SKAT-ur PLOT) A graph that compares two related sets of data on a coordinate plane. (p. 228)

**scientific notation** (sye-uhn-TIF-ik noh-TAY-shuhn) A number is written in scientific notation when it is the product of two factors. One factor is greater than or equal to 1 but less than 10, and the other factor is a power of 10 in exponent form. (p. 90)

**secant** (SEE-kant) A line that intersects a circle at two points. (p. 263)

**sector** (SEK-tur) The region in a circle bounded by two radii and their intercepted arc. (p. 262)

**selling price** (SEL-ing PRISE) The amount for which an item is sold. (p. 189)

**semicircle** (SEM-ee-*sur*-kuhl) An arc that connects the endpoints of a diameter. (p. 262)

**sequence** (SEE-kwuhnss) A list of numbers in a specific order. (p. 352)

**significant digits** (sig-NIF-uh-kuhnt DIJ-its) Digits that give a reasonable impression of the precision of a measurement. They are all nonzero digits, zeros between nonzero digits, and zeros following the last nonzero digit to the right of the decimal point. (p. 273)

**similar figures** (SIM-uh-lur FIG-yurz) Figures that have the same shape but that may be a different size. (p. 164)

**simple interest** (SIM-puhl IN-tur-ist) The amount of money earned or paid only on the principal for a stated period of time. (p. 198)

**simplest form** (SIM-plist FORM) 1. The form of an expression that has no like terms. (p. 33) 2. The form of a fraction or a mixed number in which 1 is the only common factor of the numerator and the denominator; also called **lowest terms**. (p. 111)

**simulation** (*sim*-yuh-LAY-shuhn) A mathematical experiment that is used to approximate the results of a real-life situation. (p. 337)

**skew lines** (SKYOO LYENZ) Lines that are in different planes and are neither intersecting nor parallel. (p. 241)

**slant height (of a cone)** (SLANT HITE [UHV UH KOHN]) The altitude of the lateral surface. (p. 311)

**slant height (of a pyramid)** (SLANT HITE [UHV UH PEER-uh-mid]) The height of each lateral face. (p. 308)

**slope** (SLOHP) A ratio that measures the slant, or steepness, of a line. (p. 364)

**solution of an equation** (suh-LOO-shuhn UHV AN i-KWAY-zhuhn) A value for a variable that makes an algebraic equation true. (p. 35)

**solution set** (suh-LOO-shuhn SET) A set that contains all the values for the variable that make the equation or inequality true. (p. 56)

**sphere** (SFEER) A three-dimensional figure with all points the same distance from the center. (p. 303)

**spreadsheet** (SPRED-*sheet*) A technological tool used to organize and analyze data. (p. 213)

**square** (SKWAIR) A parallelogram with four right angles and four congruent sides. (p. 260)

**square of a number** (SKWAIR UHV UH NUHM-bur) A term meaning to multiply a number by itself or to raise a number to the second power. (p. 276)

**square prism** (SKWAIR PRIZ-uhm) A rectangular prism whose faces are all squares. Also called a **cube**. (p. 302)

**square pyramid** (SKWAIR PEER-uh-mid) A pyramid that has a square base. (p. 302)

**square root of a number** (SKWAIR ROOT UHV UH NUHM-bur) A number that, when multiplied by itself, equals the original number. (p. 276)

**standard form** (STAN-durd FORM) 1. The form in which a number is usually written. (p. 18) 2. The form of a polynomial with one variable whose terms are written in order from greatest degree to least degree. (p. 383)

**stem-and-leaf plot** (STEM-AND-LEEF PLOT) A display that uses the digits of the numbers in a data set to show how the data are distributed. (p. 220)

**straight angle** (STRAYT ANG-guhl) An angle that measures 180°. (p. 242)

**Subtraction Principle** (suhb-TRAK-shuhn PRIN-suh-puhl) To subtract a rational number, add its opposite. $a - b = a + (-b)$. (p. 81)

**Subtraction Property of Equality** (suhb-TRAK-shuhn PROP-ur-tee UHV i-KWOL-uh-tee) When you subtract the same number from both sides of an equation, you get a true statement. If $a = b$, then $a - c = b - c$. (p. 36)

**Subtraction Property of Inequality** (suhb-TRAK-shuhn PROP-ur-tee UHV *in*-i-KWOL-uh-tee) If $a < b$, then $a - c < b - c$. This statement is also true if $<$ is replaced by $>$, $\leq$, or $\geq$. (p. 60)

**supplement** (SUHP-luh-muhnt) An angle of a pair of angles having a sum of 180°. (p. 244)

GLOSSARY

**supplementary angles** (*suhp*-luh-MEN-tuh-ree ANG-guhlz) Two angles with a sum of 180°. (p. 244)

**surface area of a cone** (SUR-fiss AIR-ee-uh UHV UH KOHN) The sum of the lateral area and the area of the circular base. (p. 311)

**surface area of a cylinder** (SUR-fiss AIR-ee-uh UHV UH SIL-uhn-dur) The sum of the lateral area and the area of the two congruent circular bases. (p. 310)

**surface area of a prism** (SUR-fiss AIR-ee-uh UHV UH PRIZ-uhm) The sum of the areas of the faces of a prism. (p. 306)

**surface area of a pyramid** (SUR-fiss AIR-ee-uh UHV UH PEER-uh-mid) The sum of the lateral area and the area of the base. (p. 308)

**survey** (SUR-vay) An examination of public opinion, attitudes, or behavior. (p. 208)

**symmetry** (SIM-uh-tree) See **line symmetry** and **point symmetry**.

## T

**tangent** (TAN-juhnt) A line in the plane of a circle that intersects the circle at exactly one point. (p. 263)

**terminating decimal** (TUR-muh-*nayt*-ing DESS-uh-muhl) A decimal that has a finite number of decimal places. Every terminating decimal can be written as a fraction having a denominator that is a power of 10. (p. 72)

**terms** (TURMZ) 1. The parts of an expression separated by plus or minus signs. (p. 32) 2. The two quantities being compared in a ratio. (p. 148) 3. The numbers in a sequence. (p. 352)

**tessellation** (*tess*-uh-LAY-shuhn) The covering of a plane with congruent copies of the same figure, with no overlap or gaps. (p. 292)

**tetrahedron** (*te*-truh-HEE-druhn) A polyhedron made of four triangles. Also called a **triangular pyramid**. (p. 309)

**theoretical probability** (thee-uh-RET-i-kuhl *prob*-uh-BIL-uh-tee) The number of favorable outcomes divided by the number of possible outcomes. (p. 334)

**three-dimensional figures** (THREE-duh-MEN-shuhn-uhl FIG-yurz) Figures that have length, width, and height. (p. 302)

**time** (TIME) A word that represents how long, in years, the principal is borrowed or left on deposit. (p. 198)

**tips** (TIPS) Gratuities for services provided in places such as restaurants and hotels. (p. 193)

**total cost** (TOH-tuhl KAWST) The sum of an item's marked price and the amount of sales tax. (p. 192)

**total sales** (TOH-tuhl SAYLZ) The total amount of goods or services sold. (p. 196)

**transformation** (*trans*-fur-MAY-shuhn) A change in orientation, shape, or size of a figure. (p. 370)

**translation** (trans-LAY-shuhn) A transformation that slides every point of a figure the same distance and in the same direction along a straight line without turning. (p. 370)

**translation tessellation** (trans-LAY-shuhn *tess*-uh-LAY-shuhn) The use of translations of a figure to create a pattern. (p. 292)

**transversal** (trans-VUR-suhl) A line that intersects two or more lines at different points. (p. 246)

**trapezoid** (TRAP-uh-zoyd) A quadrilateral with exactly one pair of parallel sides. (p. 260)

**tree diagram** (TREE DYE-uh-gram) A visual representation that shows all possible outcomes of one or more events. (p. 330)

**trial** (TRYE-uhl) The name given to each round of an experiment. (p. 336)

**triangle** (TRYE-ang-guhl) A polygon with three sides and three vertices. (p. 252)

**Triangle Inequality Theorem** (TRYE-ang-guhl *in*-i-KWOL-uh-tee THEER-uhm) The length of the third side of a triangle is always less than the sum of the lengths of the other two sides and greater than their difference. (p. 255)

**triangular numbers** (trye-ANG-gyuh-lur NUHM-burz) A sequence of whole numbers in which each number corresponds to an arrangement of dots in the shape of a triangle. (p. 354)

**triangular prism** (trye-ANG-gyuh-lur PRIZ-uhm) A prism that has bases that are triangles. (p. 302)

**triangular pyramid** (trye-ANG-gyuh-lur PEER-uh-mid) A pyramid, also called a **tetrahedron**, that has a base that is a triangle. (p. 302)

**trinomial** (trye-NOH-mee-uhl) A polynomial with three terms. (p. 382)

**two-step equation** (TOO-STEP i-KWAY-zhuhn) An equation that involves two operations. (p. 44)

## U

**underestimate** (*uhn*-dur-ESS-tuh-mit) An estimate less than the actual value. (p. 312)

**unfavorable outcomes** (un-FAYV-ur-uh-buhl OWT-*kuhmz*) Any outcomes that are not represented by an event; the complement of an event. (p. 338)

**unit cost** (YOO-nit KAWST) The price per unit of an item. (p. 150)

**unit rate** (YOO-nit RAYT) A ratio that compares an amount, $x$, to one unit: $\frac{x}{1}$. (p. 150)

**upper extreme** (UHP-ur ek-STREEM) The upper end of the "whisker" part of a box-and-whisker plot; it represents the greatest value of a set of data. (p. 222)

**upper quartile** (UHP-ur KWOR-tile) The upper end of the "box" part of a box-and-whisker plot; it represents the median of the upper half of a set of data. (p. 222)

## V

**variable** (VAIR-ee-uh-buhl) A symbol, usually a letter, used to represent a number. (p. 30)

**Venn diagram** (VEN DYE-uh-gram) A group of overlapping circles, each circle representing a single data set. (p. 224)

**vertex** (plural *vertices*) (VUR-teks [plural VUR-ti-*seez*]) 1. The common endpoint of two rays that form an angle. (p. 240) 2. A point where two line segments of a polygon intersect. (p. 252) 3. The point of intersection of three or more edges of a polyhedron. (p. 302)

**vertical angles** (VUR-tuh-kuhl ANG-guhlz) A pair of opposite angles formed by two intersecting lines. (p. 245)

**volume** (VOL-yuhm) The amount of space a three-dimensional figure occupies or contains. It is measured in cubic units. (p. 314)

## W

**wholesale price** (HOHL-*sayl* PRISE) The lower price that stores pay to buy an item. (p. 195)

## X

***x*-axis** (EKS-AK-siss) The horizontal number line in a coordinate plane. (p. 22)

***x*-coordinate** (EKS-koh-OR-duh-nit) The first number in an ordered pair; it locates a point by telling how many units to the left or right of the origin the point is. (p. 22)

## Y

***y*-axis** (WYE-AK-siss) The vertical number line in a coordinate plane. (p. 22)

***y*-coordinate** (WYE-koh-OR-duh-nit) The second number in an ordered pair; it locates a point by telling how many units above or below the origin the point is. (p. 22)

## Z

**zero pair** (ZEER-oh PAIR) A pair of algebra tiles (a 1 tile and a $-1$ tile); because $1 + (-1) = 0$, a zero pair can be joined to or removed from a group of algebra tiles without changing the value of the group. (p. 7)

**Zero Property of Multiplication** (ZEER-oh PROP-ur-tee UHV *muhl*-tuh-pluh-KAY-shuhn) The product of 0 and any number is 0. $0 \bullet a = 0$ or $a \bullet 0 = 0$. (p. 14)

# Symbols

## Numbers and Operations

| Symbol | Meaning |
|---|---|
| $+$ | plus or positive |
| $-$ | minus or negative |
| $a \cdot b, a(b), (a)b, ab$ | $a$ times $b$ |
| $a \div b, \frac{a}{b}$ | $a$ divided by $b$ |
| $\pm$ | plus or minus; positive or negative |
| $=$ | is equal to |
| $\neq$ | is not equal to |
| $\approx$ | is approximately equal to |
| $>$ | is greater than |
| $<$ | is less than |
| $\geq$ | is greater than or equal to |
| $\leq$ | is less than or equal to |
| $\varnothing, \{\ \}$ | the empty set |
| ... | continues without end |
| $10^2$ | ten squared |
| $0.\overline{3}$ | 0.333 … (repeating decimal) |
| % | percent |
| $a : b$ | the ratio of $a$ and $b$ |

## Geometry and Measurement

| Symbol | Meaning |
|---|---|
| $\cong$ | is congruent to |
| $\sim$ | is similar to |
| $^\circ$ | degree(s) |
| $\overleftrightarrow{AB}$ | line $AB$ |
| $\overline{AB}$ | segment $AB$ |
| $\overrightarrow{AB}$ | ray $AB$ |
| $\overset{\frown}{AB}$ | arc $AB$ |
| $\angle ABC$ | angle $ABC$ |
| $AB$ | length of $\overline{AB}$, distance between $A$ and $B$ |
| $ABC$ | plane $ABC$ |
| $\triangle ABC$ | triangle $ABC$ |
| $m\angle ABC$ | measure of angle $ABC$ |
| $\parallel$ | is parallel to |
| $\perp$ | is perpendicular to |
| $\pi$ | pi (approximately 3.14 or $\frac{22}{7}$) |
| $(x, y)$ | ordered pair |
| $\sin A$ | sine of angle $A$ |
| $\cos A$ | cosine of angle $A$ |
| $\tan A$ | tangent of angle $A$ |

## Probability and Logic

| Symbol | Meaning |
|---|---|
| $P(E)$ | probability of an event, E |
| $n!$ | $n$ factorial $[n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1]$ |
| $P(n, r)$ | permutation of $n$ things taken $r$ at a time |
| $C(n, r)$ | combination of $n$ things taken $r$ at a time |
| $\wedge$ | and, conjunction |
| $\vee$ | or, disjunction |
| $\rightarrow$ | if-then, implication |
| $\leftrightarrow$ | if and only if, biconditional |

## Algebra and Functions

| Symbol | Meaning |
|---|---|
| $a'$ | $a$ prime |
| $a^n$ | $a$ to the $n$th power |
| $a^{-n}$ | $a$ to the negative $n$th power(one over $a$ to the $n$th power) |
| $\lvert x \rvert$ | absolute value of $x$ |
| $\sqrt{x}$ | principal (positive) square root of $x$ |
| $f(x)$ | function, $f$ of $x$ |

# Percent Table

| | | | |
|---|---|---|---|
| $\frac{1}{2}\% = \frac{1}{200} = 0.005$ | $8\frac{1}{3}\% = \frac{1}{12} = 0.08\overline{3}$ | $20\% = \frac{1}{5} = 0.2$ | $66\frac{2}{3}\% = \frac{2}{3} = 0.\overline{6}$ |
| $1\% = \frac{1}{100} = 0.01$ | $9\frac{1}{11}\% = \frac{1}{11} = 0.\overline{09}$ | $25\% = \frac{1}{4} = 0.25$ | $70\% = \frac{7}{10} = 0.7$ |
| $2\% = \frac{1}{50} = 0.02$ | $10\% = \frac{1}{10} = 0.1$ | $30\% = \frac{3}{10} = 0.3$ | $75\% = \frac{3}{4} = 0.75$ |
| $3\frac{1}{3}\% = \frac{1}{30} = 0.0\overline{3}$ | $11\frac{1}{9}\% = \frac{1}{9} = 0.\overline{1}$ | $33\frac{1}{3}\% = \frac{1}{3} = 0.\overline{3}$ | $80\% = \frac{4}{5} = 0.8$ |
| $4\% = \frac{1}{25} = 0.04$ | $12\% = \frac{3}{25} = 0.12$ | $37\frac{1}{2}\% = \frac{3}{8} = 0.375$ | $83\frac{1}{3}\% = \frac{5}{6} = 0.8\overline{3}$ |
| $5\% = \frac{1}{20} = 0.05$ | $12\frac{1}{2}\% = \frac{1}{8} = 0.125$ | $40\% = \frac{2}{5} = 0.4$ | $87\frac{1}{2}\% = \frac{7}{8} = 0.875$ |
| $6\frac{1}{4}\% = \frac{1}{16} = 0.0625$ | $14\frac{2}{7}\% = \frac{1}{7} = 0.\overline{142857}$ | $50\% = \frac{1}{2} = 0.5$ | $90\% = \frac{9}{10} = 0.9$ |
| $6\frac{2}{3}\% = \frac{1}{15} = 0.0\overline{6}$ | $15\% = \frac{3}{20} = 0.15$ | $60\% = \frac{3}{5} = 0.6$ | $275\% = 2\frac{3}{4} = 2.75$ |
| $8\% = \frac{2}{25} = 0.08$ | $16\frac{2}{3}\% = \frac{1}{6} = 0.1\overline{6}$ | $62\frac{1}{2}\% = \frac{5}{8} = 0.625$ | $300\% = 3 = 3.0$ |

# Measurement Conversions

## Length

### Metric

| | |
|---|---|
| 1 millimeter (mm) = 0.001 meter (m) | 1 dekameter (dam) = 10 meters |
| 1 centimeter (cm) = 0.01 meter | 1 hectometer (hm) = 100 meters |
| 1 decimeter (dm) = 0.1 meter | kilometer (km) = 1000 meters |

### Customary

| | |
|---|---|
| 1 foot (ft) = 12 inches (in.) | 1 mile (mi) = 5280 feet |
| 1 yard (yd) = 3 feet | 1 mile = 1760 yards |
| 1 yard = 36 inches | |

### Customary to Metric

| | |
|---|---|
| 1 inch = 2.54 centimeters | 1 yard ≈ 0.914 meter |
| 1 foot ≈ 0.305 meter | 1 mile ≈ 1.61 kilometer |

## Capacity and Volume

### Metric

| | |
|---|---|
| 1 milliliter (mL) = 0.001 liter (L) | 1 kiloliter (kL) = 1000 liters |

### Customary

| | |
|---|---|
| 3 teaspoons (tsp) = 1 tablespoon (tbsp) | 1 quart (qt) = 2 pints |
| 1 cup (c) = 8 fluid ounces (fl oz) | 1 quart = 4 cups |
| 1 pint (pt) = 2 cups | 1 gallon (gal) = 4 quarts |

### Customary to Metric

| | |
|---|---|
| 1 fluid ounce ≈ 29.6 milliliters | 1 quart ≈ 0.946 liter |
| 1 pint ≈ 0.473 liter | 1 gallon ≈ 3.785 liters |

## Mass and Weight

### Metric

| | |
|---|---|
| 1 milligram (mg) = 0.001 gram (g) | 1 metric ton (t) = 1000 kilograms |
| 1 kilogram (kg) = 1000 grams | |

### Customary

| | |
|---|---|
| 1 pound (lb) = 16 ounces (oz) | 1 ton (T) = 2000 pounds |

### Customary to Metric

| | |
|---|---|
| 1 ounce ≈ 28.4 grams | 1 pound ≈ 454 grams |

## Temperature

### Metric

| | |
|---|---|
| 0° Celsius (C) …… Water freezes | 100° Celsius (C) ……………… Water boils |

### Customary

| | |
|---|---|
| 32° Fahrenheit (F) …… Water freezes | 212° Fahrenheit (F) ………….. Water boils |

## Time

| | | |
|---|---|---|
| 1 century (cent.) = 100 years (y) | 1 leap year = 366 days | 1 day = 24 hours (h) |
| 1 year = 12 months (mo) | 1 year = 52 weeks (wk) | 1 hour = 60 minutes (min) |
| 1 year = 365 days (d) | 1 week = 7 days | 1 minute = 60 seconds (s) |

# Formula Chart

| | | | | |
|---|---|---|---|---|
| **Perimeter & Circumference** | square<br>rectangle | $P = 4s$<br>$P = 2(\ell + w)$ or<br>$P = 2\ell + 2w$ | regular polygon<br>circle | $P = ns$<br>$C = 2\pi r$ or $C = \pi d$ |
| **Area** | square<br>rectangle<br>parallelogram<br>triangle | $A = s^2$<br>$A = \ell w$ or $A = bh$<br>$A = bh$<br>$A = \frac{1}{2}bh$ | trapezoid<br>regular polygon<br>circle | $A = \frac{1}{2}(b_1 + b_2)h$<br>$A = \frac{1}{2}aP$<br>$A = \pi r^2$ |
| **Lateral Area** | right prism<br>right cylinder | $LA = Ph$<br>$LA = 2\pi rh$ or<br>$LA = \pi dh$ | regular pyramid<br>right cone | $LA = n(\frac{1}{2}s\ell)$<br>$LA = \frac{1}{2}(2\pi r\ell)$ or $LA = \pi r\ell$ |
| **Surface Area** | cube<br>rectangular prism<br>right prism | $S = 6e^2$<br>$S = 2(\ell w + \ell h + wh)$<br>$S = LA + 2B$ | regular pyramid<br>right cylinder<br>right cone | $S = LA + B$<br>$S = LA + 2B$<br>$S = LA + B$ or $S = \pi r\ell + \pi r^2$ |
| **Volume** | cube<br>rectangular prism<br>prism | $V = e^3$<br>$V = \ell wh$<br>$V = Bh$ | pyramid<br>cylinder<br>cone<br>sphere | $V = \frac{1}{3}Bh$<br>$V = Bh$ or $V = \pi r^2 h$<br>$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$<br>$V = \frac{4}{3}\pi r^3$ |
| **Variation** | direct | $y = kx$ or $k = \frac{y}{x}, k \neq 0$ | inverse | $y = \frac{k}{x}$ or $k = xy, k \neq 0$ |
| ***n*th Term of a Sequence** | arithmetic | $a_n = a_{n-1} + d$ | geometric | $a_n = a_{n-1} \bullet r$ |
| **Pythagorean Theorem** | right triangle with hypotenuse $c$ and legs $a$ and $b$ | | $c^2 = a^2 + b^2$ | |
| **Trigonometric Ratios** | acute angle of a right triangle | | cosine (cos) $= \frac{\text{adjacent leg}}{\text{hypotenuse}}$<br>sine (sin) $= \frac{\text{opposite leg}}{\text{hypotenuse}}$<br>tangent (tan) $= \frac{\text{opposite leg}}{\text{adjacent leg}}$ | |
| **Slope** | a line with points $(x_1, y_1)$ and $(x_2, y_2)$ | | $m = \frac{y_2 - y_1}{x_2 - x_1}$ | |
| **Equation of a Line** | Standard Form<br>Slope-intercept form<br>Point-slope form | | $ax + by = c$<br>$y = mx + b$<br>$y - y_1 = m(x - x_1)$ | |
| **Probability** | Probability of a Single Event | | $P(\text{E}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$ | |
| **Other Formulas** | percentage = rate • base<br>Discount = Rate of Discount • List Price<br>Commission = Rate of Commission • Total Sales<br>Simple Interest = principal • rate • time (in years)<br>Compound Interest Balance | | $p = rb$<br>$D = R$ of $D \bullet LP$<br>$C = R$ of $C \bullet TS$<br>$I = prt$<br>$B = P(1 + \frac{r}{n})^{nt}$ | |

TABLES

**Key:** SU=Skills Update PB=Practice Book